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[17]The qualitative statement of the result of this theorem is due to Mittra and Ransom (Ref. 5) and is given in Ref. 12. A rigorous statement and proof of the theorem for $F(p,q)\in L_2$ has not been given previously.
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CURRENT ALGEBRA AND THE PION TRAJECTORY

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It is shown that one conventional interpretation of the success of current-algebra results requires the pion Regge trajectory to choose the Toller quantum number $M=0$ at zero energy.

There has been speculation that the Toller [O(3,1)] classification of the pion Regge trajectory gives restrictions on the Regge residue functions which can be compared with certain predictions of current algebra.[1,2] The purpose of this note is to show that one conventional interpretation of current-algebra results requires the pion trajectory to choose the Toller quantum number $M=0$.[3] The converse is not true; the $M=0$ assignment enforces no constraint, current algebraic or otherwise, on amplitudes involving pions.

We consider first the possibility that the pion trajectory belongs to an $M=0$ or $M=1$ conspiracy class.[4] The $P=(-1)^{J+1}$ (pion) member of the conspiracy contributes to the t-channel reactions, $\bar{N}N\to\sigma\pi$,[5] or $\bar{N}N\to\rho\pi$ with zero-helicity ρ's, through an amplitude which we denote in either case by $f_{1/2,1/2,0,0}$. As t approaches zero in either reaction there is a conspiracy condition,[6]

$$f_{\frac{1}{2},\frac{1}{2},0,0}(s,t) - (i/\sin\theta_t) f_{\frac{1}{2},-\frac{1}{2},0,0}(s,t)\sim\sqrt{t}. \quad (1)$$

One can easily see that in either the $M=0$ or the $M=1$ case the term involving $f_{1/2,-1/2,0,0}$ has no s^α contribution.[7] Thus, the amplitude $f_{1/2,1/2,0,0}$ behaves like $t^{\frac{1}{2}}s^\alpha$ for large s, small t. This leads to a behavior of the residue function for small t of the form

$$\beta_{1/2,1/2,0,0}(t)=\beta_{1/2,1/2}^{\bar{N}N}\beta_{0,0}^{\rho\pi\text{ or }\sigma\pi} \sim t^{-\frac{1}{2}\alpha+\frac{1}{2}}. \quad (2)$$

At the equal-mass $\bar{N}N$ vertex the normal couplings are known from O(3, 1) consideration for the $P=(-1)^{J+1}$ part of the $M=0$ and $M=1$ trajectories:

$$M=1:\ \beta_{1/2,1/2}\sim\text{const};$$
$$M=0:\ \beta_{1/2,1/2}\sim\sqrt{t}. \quad (3)$$

Thus, we read off the maximal behavior of the residue function in the $\sigma\pi$ (or $\rho\pi$) system as

$$M=1:\ \beta_{0,0}(t)\sim t^{\frac{1}{2}-\frac{1}{2}\alpha};$$
$$M=0:\ \beta_{0,0}(t)\sim t^{-\frac{1}{2}\alpha}. \quad (4)$$

Next we imagine a limit in which the pion mass approaches zero. In the limit $\alpha(0)\to 0$ and $t\to 0$ the β functions in (4) become the actual amplitudes for $\pi\pi\to\sigma$ and $\pi\pi\to\rho$, in our world in which (at least) one incident π has zero mass. We see that for the $M=1$ case the amplitudes vanish as the pion mass; for the $M=0$ case they remain finite. One can now take the position that current-algebraic results are to be read as exact on-shell results in a world with vanishing pion mass, and that pion scattering amplitudes (with certain pole terms excluded) in this world are approximately those in the real world. From this standpoint what we have proved is that for the $M=1$ case there is no zero-mass pion-pion scattering (at any energy, since the σ or ρ masses in our example are variable), and thus that whatever pion-pion scattering there may be in the real world with finite M_π is absolutely unconstrained by current algebra.

The result can be generalized to any process involving pions through the following theorem: If the pion trajectory is one member of an $M>0$ conspiracy, and if this trajectory couples to any single equal-mass channel with nonvanishing residues,[8] then kinematical constraints, plus O(3, 1) relations for the residues in the equal-mass channel, plus factorization of residues, lead to the vanishing of all residues in sense states at $t=0$ in the limit $\alpha\to 0$, even for unequal-mass channels.[9] For the equal-mass case this result follows from the Toller residue relations; it is the extension to the unequal-mass case which we need for the results of this paper.

One thus concludes that if the pion trajectory were of class $M>0$, all reaction amplitudes for pions in the zero-mass limit would vanish.[10] We emphasize once more that although one cannot prove that such decoupling does not in fact take place in the $M_\pi=0$ limit, the decoupling would annihilate the interpretation of current-algebra results as being exact on-shell results in the fictitious world with $M_\pi=0$. If the entire observed pion scattering amplitudes (poles exempted) come from the effect of finite pion mass, then they are unconstrained by current algebra. Thus, the success of Weinberg's scattering length predictions is a compelling reason for rejecting the $M>0$ assignment for the pion.[11]

We can put the conclusion in another way, following Mandelstam. In Ref. 1 it was shown that the Adler self-consistency condition (the amplitude for emission of a pion vanishing as the pion four-momentum approaches zero) leads to the universality prediction: If $\frac{1}{2}\epsilon_{\alpha\beta\gamma}T_\gamma A^{(-)}(\nu)$ is the isoantisymmetric forward-scattering amplitude of pions from a target (with isospin matrix T), the coefficient of ν in an expansion of $A^{(-)}(\nu)$ in powers of ν is a constant C independent of the target. This is shown to be true in the zero-pion-mass world if pole contributions are excluded. The problem with the $M=1$ pion amplitude is the vanishing of the pion amplitude as k_μ^2 approaches zero, where k_μ is the pion four momentum. Linear vanishing as k_μ approaches zero (which is all that is demanded by the Adler self-consistency condition) is required in order to have a nonzero C. Thus, the constant C in the $M=1$ theory is zero in the limit $M_\pi\to 0$. Experimentally it is not equal to zero.[12]

If we desire to preserve a pure Regge-pole theory and we have admitted that the pion cannot be a member of an $M>0$ conspiracy, then something else must give the forward amplitudes in $pn\to np$ and $\gamma N\to\pi N$. Consider as an example the process $np\to pn$ with the exchange of an $M=0$ pion trajectory and an $M=1$ even-signature unmixed trajectory, all three leading Regge trajectories coinciding for simplicity. The near forward contribution to $pn\to np$ will look like[13]

$$T\to\frac{C_1\gamma_5(1)\gamma_5(2)}{\sin\pi\alpha(t)}s^\alpha+\frac{C_2\alpha(t)\sigma_{\mu\nu}(1)\sigma^{\mu\nu}(2)}{\sin\pi\alpha(t)}s^{\alpha-1}. \tag{5}$$

The first term comes from the exchange of the $M=0$ pion trajectory[14]; the second term from the $M=1$ trajectory with an $\alpha(t)$ factor removing the particles at $J=0$, in accord with the remark in footnote 12. If we tried to think of the combined contribution of the two $P=(-1)^{J+1}$ trajectories in (5) as the contribution of a single trajectory, we would obtain a rapidly varying residue, $b_1t+b_2(t-M_\pi^2)$, of exactly the type used in the data analysis.[15] Thus, the proliferation of trajectories is not really objectionable; with constant residues we can get what required rapidly varying residues in the single $M=1$ conspiracy fit; the $M=1$ trajectory would make particles first at $J=2$ in the 2-BeV region. Furthermore, the $M=0$ pion can be used to obtain a forward amplitude in the reaction $\pi+N\to\rho(\text{helicity zero})+\Delta$.

It should be emphasized that all the foregoing considerations are dependent on the existence of the limits $M_\pi\to 0$ [or $\alpha_\pi(0)\to 0$] and $t\to 0$ for the amplitudes under consideration. Of course the limits might well be very singular, as conjectured by Omnes.[16] However, to us the actual arguments given by Omnes in favor of such behavior are entirely unconvincing.

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[1]S. Mandelstam, Phys. Rev. 168, 1884 (1968).

[2]D. Z. Freedman and J. M. Wang, Phys. Rev. 160, 1560 (1967).

[3]The O(3, 1) classification is, of course, only valid at $t=0$.

[4]An $M=1$ conspiracy has opposite parity Regge trajectories intersecting at $t=0$. For details on Lorentz pole theory, see M. Toller, Nuovo Cimento 53A, 671 (1967), and 54A, 295 (1968).

[5]By σ we mean an S-wave state of two pions, of whatever isospin and energy.

[6]H. Högaasen and P. Salin, Nucl. Phys. B2, 657 (1967).

[7]An s^α contribution to $f_{\frac{1}{2},-\frac{1}{2},0,0}$ would require an $M=0$ trajectory (A_1 type) at $J=\alpha+1$. Here we deal with $M=0$ or $M=1$ trajectories with leading $J=\alpha$.

[8]It must, if M is to be defined from on-shell considerations. We might mention that our special results for the $\pi\sigma$ and $\pi\rho$ couplings are easily derivable without the explicit use of conspiracy conditions from the off-shell invariant-amplitude approach given in R. F. Sawyer, Phys. Rev. 167, 1372 (1968). The argument given, leading to Eq. (4), is similar to one due to M. Le Bellac, Phys. Letters 25B, 524 (1967). Kinematic arguments leading to (2) and (3) can be found in L. Jones and H. K. Shepard (to be published).

[9]This theorem is implicit in the general residue formula for the unequal-mass case given in the paper by Cosenza, Sciarrino, and Toller (to be published),

$$\beta_{\lambda_1\lambda_3}^{\mu\sigma} \sim t^{\frac{1}{2}[-\lambda+1+|M-|\lambda_1-\lambda_3||]}.$$

[10]This is true up to certain pole terms for the pion amplitude (i.e., the Regge vertex), which correspond to bremsstrahlung graphs in the complete amplitude for Regge exchange. These graphs do not obey the conspiracy condition (1). But we can conclude that S-wave πN scattering, for example, vanishes in the $M_\pi \to 0$ limit.

[11]S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

[12]The $M=1$ pion trajectory raises an entirely distinct question: How can it possibly make a physical $J=0$ particle at an energy $E=M_\pi$ so near to the O(3, 1) symmetry point? In a model in which the symmetry breaking may be treated in a perturbation expansion in E it cannot. However, in an ingenious model due to Blankenbecler and Sugar it can, through mixing with a nearby $M=0$ trajectory. Frazer, Lipinski, and Snider have manufactured a similar model. It should be noted that neither of these models gives a dynamical understanding of the hypothesis of partially conserved axial-vector current. Furthermore, neither model is, in our opinion, even consistent with the interpretation of current-algebra results as exact results in a $M_\pi=0$ world, which are good approximate results in the $M_\pi=M_\pi$ world.

[13]We use the notation of R. F. Sawyer, Phys. Rev. 167, 1372 (1968).

[14]This type of pion coupling is frequently called evasive. But $M=0$ is the correct designation. An $M=0$ pion necessarily evades in the $\bar{N}N$ system, but it can couple to other systems at $t=0$. $M=0$ is a quality intrinsic to a trajectory; evasion is not.

[15]F. Arbab and J. Dash, Phys. Rev. 163, 1603 (1967).

[16]R. L. Omnes, Phys. Rev. 168, 1893 (1968).

FORM FACTOR RATIO ξ FROM A MEASUREMENT OF $K_{\mu3}^+:K_{e3}^+$ BRANCHING RATIO

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By measuring the branching ratio of $K_{\mu3}^+$ relative to K_{e3}^+ we have arrived at a value of $\xi(0)=f_-(0)/f_+(0)=-0.08\pm0.15$. This is in good agreement with the Callan-Treiman relation.

We describe a measurement of the branching ratio of $K_{\mu3}^+$ relative to K_{e3}^+. We deduce from this a value of the parameter ξ, the ratio of the form factors for vector coupling in K_{l3} decay, where

$$\xi(0)=f_-(0)/f_+(0),$$

$$f_\pm(q^2)=f_\pm(0)[1+\lambda_\pm(q^2/m_\pi^2)],$$

and

$$q^2=(p_K-p_\pi)^2=m_K^2+m_\pi^2-2m_KE_\pi.$$

There is a discrepancy between the values of ξ deduced from branching ratio and polarization methods.[1] Apart from a result given some time ago,[2] investigations of the relative muon and positron semileptonic rates indicate a comparatively high $K_{\mu3}:K_{e3}$ ratio, and hence a positive value, ~0.3, for ξ.[3] On the other hand, polarization experiments have shown a systematic shift towards negative values for ξ, averaging $\xi(0)=-1.25\pm0.32$.[4]

We have carried out a K^+ experiment at Nimrod in which positron- and muon-decay rates