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[9]The angle usually denoted by θ_Y is given in the vector-mixing model by $\tan\theta_Y=(K_\omega/K_\varphi)^{1/2}\tan\theta$, whence $\theta_Y = 33.2°$.

[10]By the SU(6)-symmetric limit we mean $K_i=1$, $\sin\theta = 1/\sqrt{3}$, $\epsilon_i=0$. Some of the results obtainable in this limit have been given previously by, e.g., V. V. Anisovich et al., Phys. Letters 16, 194 (1965).

[11]Rosenfeld et al., Ref. 3.

[12]The ratios of these rates are, of course, in agreement with the ratios given in R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters 19, 1266 (1967).

[13]We remark that the ratio $\Gamma(K^*\to K\pi)/\Gamma(\rho\to\pi\pi)$ is also improved by the use of vector mixing. See, e.g., J. J. Sakurai, Phys. Rev. Letters 19, 803 (1967).

[14]It was already noticed that the calculation of the η lifetime using $\omega\to\pi\gamma$ [F. A. Berends and P. Singer, Phys. Letters 19, 249, 616(E) (1965)] gave a very different value from that obtained on the basis of the π^0 lifetime. The shorter η lifetime they predicted turned out to be confirmed experimentally.

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[16]We remind the reader that the calculation of rates for decays of the type $V\to P+P$ involves knowledge of $g_{VPP}(p^2)$, where $g=K_V^{1/2}g_{VPP}(0)$. Making a linear extrapolation in p^2 we obtain $g_{VPP}(p^2)=(g/\sqrt{K_V})(1-0.059p^2/m^2)$ by comparison with the well-determined K^* width. The resulting widths for ρ and φ are $\Gamma_\rho=130\pm26$ MeV, $\Gamma(\varphi-\bar{K}K)=5.1\pm1$ MeV.

THRESHOLD PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

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(Received 24 July 1968)

We present results pertaining to threshold π production in nucleon-nucleon collisions, obtained in the soft-pion approximation. A typical result is $\sigma_{tot}(pp\to np\pi^+)=24$ μb at $T_{lab}=310$ MeV, in good agreement with extrapolations from higher energy data.

It is possible to establish the expected[1] soft-pion connection between π production and nucleon-nucleon scattering by considering the algebra of currents as applied to the S-matrix element for the process $\gamma 2N\to\pi 2N$. Partial conservation of the axial-vector current (PCAC) is used to replace matrix elements of the pion source current by those for the axial-vector current. This facilitates the performance of certain low-energy expansions, which in lowest order, from pole terms, yield the production amplitude. The formal results are precisely consistent with those following from the Adler-Dothan theorem[2] for the weak axial-vector vertex. Calculations of the unbound process $pp\to np\pi^+$ are quite satisfactory from threshold ($T_{lab}=290$ MeV) to 320 MeV, in good agreement with the Mandelstam[3] and Rosenfeld[4] extrapolations from data existing at higher energies. The procedure can easily be extended to other processes of interest, such as $np\to pp\pi^-$ or $pp\to d\pi^+$. Our computations indicate that the situation at production threshold is compatible with results for other soft-pion processes.[5] This is in contradiction with recent work by Beder.[6]

The S-matrix element for the process $\gamma 2N\to\pi 2N$ is

$$S_{fi}=\langle\text{out};qp_3p_4|kp_1p_2;\text{in}\rangle\equiv\langle\text{out};q\beta|k\alpha;\text{in}\rangle. \tag{1}$$

We suppress the charge states for the moment but later specialize to π^+ production and initial, two-proton states. The standard reduction technique using PCAC gives

$$\begin{aligned} i(2k_0 2q_0)^{\frac{1}{2}}C\langle\text{out};\pi^\pm(q)\beta|j_\mu^{EM}(0)|\alpha;\text{in}\rangle \\ = -\int d^3z\, e^{-i\vec{q}\cdot\vec{z}}(-q^2+\mu^2)\langle\text{out};\beta|[j_\mu^{EM}(0),\ \bar{j}_0^{(\pm)}(0,\vec{z})]|\alpha;\text{in}\rangle \\ -iq^\nu\int dz\, e^{iqz}(-q^2+\mu^2)\langle\text{out};\beta|(j_\mu^{EM}(0)\bar{j}^{(\pm)}(z))_+|\alpha;\text{in}\rangle, \end{aligned} \tag{2}$$

where $j_\mu^{EM}=j_\mu^3+(\sqrt{3}/2)j_\mu^8$, $\bar{j}_\nu^{(\pm)}=\bar{j}_\nu^1\pm i\bar{j}_\nu^2$, in terms of SU(3) vector- and axial-vector currents, respectively. The constant C is related to the pion mass, μ, and the pion decay amplitude, F_π, through $C=(i/\sqrt{2})\mu^2F_\pi$. In writing (2), an overall factor involving a four-momentum-conserving delta function has been dropped, as has the photon polarization vector $\epsilon^\mu(k,\lambda)$. The specialization to pure timelike pions, $q=(q_0,\vec{0})$, generates the total axial charge $\bar{Q}$ from the space integral in (2). The SU(3)⊗SU(3) algebra[7] then states that [8]

$$[\bar{Q}^{(\pm)},j_\mu^{EM}]=\mp\bar{j}_\mu^{(\pm)},\quad \bar{Q}^{(\pm)}=\bar{Q}^1\pm i\bar{Q}^2. \tag{3}$$

If we now contract (2) with $\kappa_\mu=(k-q)_\mu$ and use

$$i\kappa^\mu\langle\text{out};\beta|\bar{j}_\mu^{(\pm)}|\alpha;\text{in}\rangle=\langle\text{out};\beta|\bar{D}^{(\pm)}|\alpha;\text{in}\rangle, \tag{4}$$

then, as the divergence of the axial current, $\bar{D}^{(\pm)}$, has a pion pole, we obtain our basic equation

$$-C\kappa^\mu\langle\text{out};\pi^\pm(q)\beta|j_\mu^{EM}(0)|\alpha;\text{in}\rangle=\mp C(-q^2+\mu^2)(-\kappa^2+\mu^2)^{-1}\langle\text{out};\pi^+(q)\beta|T|\alpha;\text{in}\rangle$$
$$+\kappa^\mu q^\nu\int dz\,e^{iqz}(-q^2+\mu^2)\langle\text{out};\beta|(j_\mu^{EM}(0)\bar{j}_\nu^{(\pm)}(z))_+|\alpha;\text{in}\rangle. \tag{5}$$

In the event that the integral does not have poles as q and $\kappa=k-q$ tend to zero, we have ostensibly a relation between π production in nucleon-nucleon collisions and the amplitude for photoproduction of pions on two nucleons. This is true but not quite useful because of lack of data. However, the contraction of κ^μ with the photoproduction amplitude produces, in lowest order in q and κ, the on-shell elastic nucleon-nucleon amplitude. Pion production is thus related to the elastic amplitude in lowest order. This is the obvious and expected result. One might have obtained it directly through the use of the Adler-Dothan theorem for the axial-vector vertex.[6] However, the corrections to such a statement, summarized by the integral in (5), would have not appeared in a way subject to systematic discussion.

A careful investigation of the limiting process ($\kappa\to0$ first, then $q\to0$) discloses that the integral does not have a double pole. It then becomes possible to obtain the invariant amplitude for production of zero-mass pions. Our prescription for determination of the amplitude at the physical mass is to assume that the nonphysical amplitude extrapolates smoothly.

We confine our attention to π^+ production in p-p collisions. This means that the limits are taken for the graphs shown in Fig. 1. The residues of the pole contributions then determine the threshold production amplitude. The various contributions are summarized by

$$\mathfrak{a}(pp;\pi^+)=\frac{g}{2M}\sum_\alpha F_\alpha^{np}\left\{\bar{u}_4\left[t_\alpha\frac{1}{\not{p}_2+\not{k}-M}\Gamma_0^p+\Gamma_0^p\frac{1}{\not{p}_4-\not{k}-M}t_\alpha\right]u_2\bar{u}_3t^\alpha\frac{1}{\not{p}_1-\not{q}-M}\tau_-\not{q}\gamma_5u_1\right\}$$
$$+\frac{g}{2M}\sum_\alpha F_\alpha^{pp}\left\{\bar{u}_3\not{q}\gamma_5\tau_-\frac{1}{\not{p}_3+\not{q}-M}\left(\Gamma_0^p\frac{1}{\not{p}_3+\not{q}-\not{k}-M}t_\alpha+t_\alpha\frac{1}{\not{p}_1+\not{k}-M}\Gamma_0^p\right)u_1\bar{u}_4t^\alpha u_2\right.$$
$$\left.+\bar{u}_4\left(\Gamma_0^p\frac{1}{\not{p}_4-\not{k}-M}t_\alpha+t_\alpha\frac{1}{\not{p}_2+\not{k}-M}\Gamma_0^p\right)u_2\bar{u}_3\not{q}\gamma_5\tau_-\frac{1}{\not{p}_3+\not{q}-M}t^\alpha u_1\right\}$$
$$+\frac{g}{2M}\sum_\alpha F_\alpha^{np}\bar{u}_4t_\alpha u_2\bar{u}_3t^\alpha\tau_-\gamma_5\frac{(-p_{10}+M\gamma_0)}{p_{10}}u_1-(1\leftrightarrow2). \tag{6}$$

The three terms correspond to the three graphs of Figs. 1(a), 1(b), and 1(c), respectively. The notation $(1\leftrightarrow2)$ means interchange nucleons 1 with 2 and serves to put the pion and photon on the 2 line as well. The electromagnetic current vertex function Γ_μ^N reduces, in the limit considered, to γ_μ for protons and zero for neutrons [which accounts for the lack of a term corresponding to Fig. 1(d)]. The pion-nucleon interaction is pseudovector coupling, with coupling constant $g/2M$ ($g^2/4\pi=14.6$, M = nucleon mass).[2] This implies the use of the inhomogeneous Klein-Gordon equation relating the pion source current and the interpolating field, the use of PCAC relating the axial-vector divergence to the interpolating field, and the use of the Goldberger-Treiman relation. Finally, the nucleon-nucleon scatter-

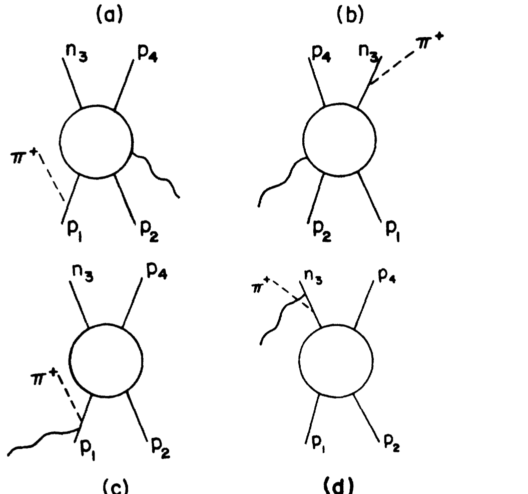

FIG. 1. Graphs contributing to $pp \to np\pi^+$ when limits are taken as described in text.

Table I. Comparison of $\sigma_{np\pi^+}$ cross sections.

T_{lab} (MeV)	$\sigma_{np\pi^+}$ (μb) This work	Beder[a]	Mandelstam[b]	Rosenfeld[c]
300	11	~9	10	10
305	17	20	13	17
310	24	36	17	28
315	31[d]	63	31	42
320	39[d]	92	41	66

[a]From Fig. 6 of Ref. 6.
[b]Ref. 3.
[c]Ref. 4.
[d]These values correspond to the use of the np effective-range formula for the laboratory energies greater than 25 MeV and hence is an underestimate.

ing involves the invariant functions F_α^{NN}.[9,10] The quantities t_α comprise the basic set of five 4×4 Dirac matrices describing this interaction. The F_α^{NN} have arguments $\nu = p_1\cdot p_2 + p_3\cdot p_4$ and $\Delta = p_1\cdot p_3 + p_2\cdot p_4$ appropriate to on-shell scattering. Originally the functions were those pertaining to the off-shell situation, e.g., with arguments $\nu+\kappa\cdot p_4$ and $\Delta+\kappa\cdot p_1$. Upon performing a Taylor's series expansion and retaining the lowest order, we obtain the (experimentally known) functions given.

The three terms in (6) have been left in differing stages of evaluation for the purpose of discussing the limits. The last term has the limits taken in the order $\kappa_0\to0$, then $q_0\to0$. The same must be done for the first two terms. In this limit only the last term, pre-emission of the pion, and the first part of the second term, postemission of the pion, survive. In both cases photon and pion are on the same nucleon line. Near production threshold the final pair of nucleons is taken at rest or nearly so, and the postemission contributions are exceedingly small. We drop them from further discussion.

After performing all the limits in (6) we obtain the threshold production amplitude to lowest order in μ/M as

$$\mathfrak{a}(pp,\pi^+) = (g\sqrt{2}/2M)\sum_\alpha F_\alpha^{np}\{\bar{u}_4 t_\alpha u_2 \bar{u}_3 t^\alpha \gamma_5(-1+\gamma_0)u_1 - \bar{u}_4 t_\alpha u_1 \bar{u}_3 t^\alpha \gamma_5(-1+\gamma_0)u_2\}. \tag{7}$$

The mathematical simplifications for threshold kinematics were pointed out by Beder.[6] The pair exchange in (7) reflects the fact that the π^+ has been emitted from both initial protons (pre-emission) as well as their antisymmetry. Squaring (7), performing the spin average, and doing the phase-space integration, we obtain for the total π^+-production cross section at threshold

$$\sigma_{np\pi^+}(W) = \left(\frac{g^2}{4\pi}\right)\frac{\mu}{M}\frac{1}{4\sqrt{2}\pi}\frac{1}{M^2}(I_s + 2I_t), \tag{8a}$$

$$I_{s,t} = \int_0^\epsilon \sigma_{s,t}(\eta)[\eta(\epsilon-\eta)]^{1/2}d\eta, \tag{8b}$$

$$W = 2M+\mu+\epsilon = 2M(1+T_{\text{lab}}/2M)^{1/2}, \tag{8c}$$

where $\sigma_{s,t}(\eta)$ is singlet (triplet) total np cross section at total center-of-mass kinetic energy η.

We have evaluated (8) and find the results listed, together with those of Beder[6] and the extrapolations of Mandelstam[3] and Rosenfeld,[4] in Table I. It is our conclusion that the soft-pion technique gives results not larger than experimental values in its region of validity. Overall, there needs to be data taken in the region 290 MeV $< T_{\text{lab}} <$ 340 MeV. It seems clear to us that the present technique does provide a suitable description of threshold production. This is consistent with the current-algebra calculations of π production in π-N collisions.[5] Details of our calculations, together with complete numerical results for all production cross sections of interest, will be published elsewhere.

We are grateful to R. F. Peierls for originally describing Beder's work to us during the course of our investigation. F. E. Low, M. Gell-Mann, I. J. Muzinich, R. L. Arnowitt, and M. Bolsterli provided many helpful suggestions and comments. The invariant functions for N–N scattering were supplied us in numerical form by E. Nyman.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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