(b) The good fit to the $n\pi^+$ mass spectrum in the region of the 1400-MeV enhancement raises the question of the resonance interpretation of this phenomenon. One experimental difficulty clouds the issue. The rather small number of events and, in particular, the loss of events with $|t_p| < 0.06$ $(\mathrm{GeV}/c)^2$ makes it difficult for one to determine a slope for $d\sigma/dt_p$, having selected $M_{n\pi}$ in the 1400-MeV region. The missing-mass counter experiments suggest a dependence of $\exp(\lambda t_p)$ with $\lambda \simeq 14$-$18\ \mathrm{GeV}^{-2}$, if the missing mass is in the region of 1400 MeV.[9] However, the present data are consistent with a much smaller λ–viz., 8-10 $(\mathrm{GeV}/c)^{-2}$ for events with $M(n\pi^+) < 1500$ MeV, in agreement with the model.

Nevertheless, in line with recent work on Regge-pole theory,[10] the appropriate conclusion would seem to be that although the exchange model yields agreement with the data, such agreement fails to imply the absence of one or more resonances in that structure.

I am indebted to Richard Morrow for insight into three-body kinematics and deeply grateful to Robert Panvini and W. Edwin Ellis for help in understanding the data and for generously providing the various distributions essential to this work.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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[5] A somewhat better fit to the distribution in $S_{p\pi^+}$ can be achieved with a mild slope $\alpha_P' = 0.3$ $(\mathrm{GeV}/c)^2$ for the Pomeranchuk, as determined by R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

[6] Phillips and Rarita, Ref. 5.

[7] For the OPE computations, normalized to the total number of events,

$$\sum |M|^2 = t_n (t_n - M_\pi^2)^{-2} [s_{p\pi} - (m_p + m_\pi)^2] \times [s_{p\pi} - (m_p - m_\pi)^2] \exp(8t_p).$$

[8] For references, see Ref. 3 of this paper.

[9] E. W. Anderson et al., Phys. Rev. Letters 16, 855 (1966).

[10] G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

REGGE STRUCTURE OF THE $B^{(-)}$ AMPLITUDE IN πN SCATTERING*

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(Received 12 July 1968)

Applying the technique of finite-energy sum rules to the recent CERN πN phase-shift analysis, we investigate the Regge structure of $B^{(-)}$ at large momentum transfer. The features found are most easily explained by an additional fixed pole at $\alpha = -1$. Some of its implications are discussed.

We have redone the calculations explained in earlier work[1] and continued them to lower t values, using the recent phase-shift analysis of the CERN group.[2] A particularly interesting structure reveals itself in the S_n of $B^{(-)}$. The expected Regge expansion of this amplitude is

$$B^{(-)} \approx \sum_i \frac{\beta_i(t)\alpha_i \nu^{\alpha_i - 1}}{\Gamma(\alpha_i + 1)\sin\pi\alpha_i}(1 - e^{-i\pi\alpha_i}). \tag{1}$$

The sum is extended over the possible poles with the right quantum numbers. The high-energy data are believed to be accounted for by the ρ pole. The finite-energy sum rules (FESR) read

$$S_n \equiv \frac{1}{N^n}\int_{\mathrm{RHC}}^{N} d\nu\, \nu^n \operatorname{Im}B^{(-)}(\nu) \approx \sum_i \frac{\beta_i(t) N^{\alpha_i}}{(\alpha_i + n)\Gamma(\alpha_i)}. \tag{2}$$

The fact that the ρ pole itself is not enough in order to account for the various S_n was already noted in Ref. 1. In particular it was observed that S_0 has large values that can be explained in Regge language only by something similar to a fixed pole at $\alpha=0$. This being a nonsense wrong-signature point assures that this addition does not change anything in the Regge fit of $B^{(-)}$

Calculating the S_n with the phase shifts of Ref. 2 we find that they continue to oscillate down to low values of t. The various even FESR start to diverge (i.e., cease to oscillate) roughly near $t=-1.2$ whereas the odd FESR oscillate down to $t \simeq -2.4$. It seems therefore safe to assume that over a reasonable part of these regions one observes true effects that have to be explained within the context of Regge theory. Figure 1 shows the first three odd S_n as a function of t. The upper limit N is chosen here at 1.93 BeV, the highest point of Ref. 2. The most remarkable feature observed here is that $|S_1|$ is bigger than $|3S_3|$ and $|5S_5|$ over most of the plotted region. All along where this phenomenon holds there must be a dominant pole near $\alpha=-1$. From Eq. 2 we see that only such a pole can contribute a big amount to S_1 and practically nothing to the other S_n. In general a pole that lies around some $\alpha_i=-n$ will contribute only to S_n and the contribution will be $(-)^n n!\beta(t)N^{-n}$.

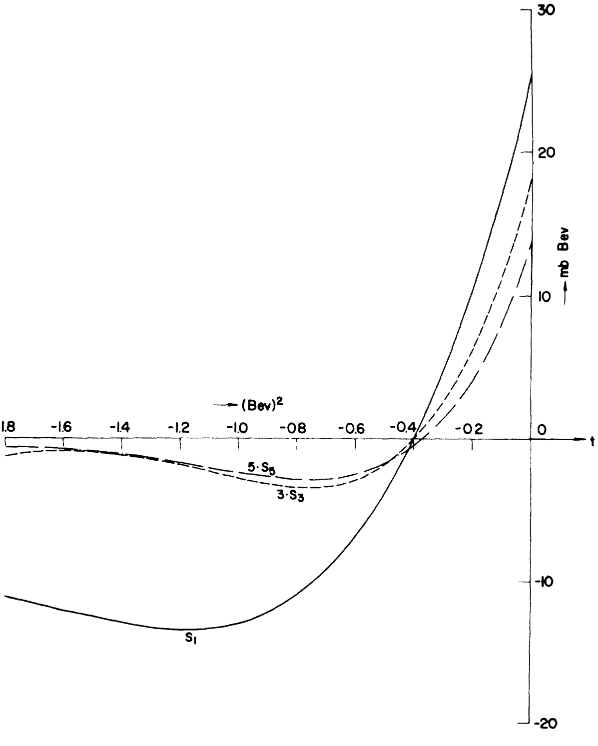

FIG. 1. Various S_n for $B^{(-)}$ calculated from the phase shifts of Ref. 2 with the choice $N=1.93$ BeV.

Now that we have established the existence of a pole at $\alpha \approx -1$ we can continue and ask which pole it is. Is that the usual ρ pole or an additional one? That calls for a further analysis and raises the question whether all the features of the S_n shown in Fig. 1 should be interpreted through the Regge representation. In particular one might think that the oscillations of S_3 and S_5 are spurious fluctuations. In order to settle this issue we varied N between 1 and 2 BeV and decided to consider as interesting those features that remain relatively unchanged. Thus it turns out, for example, that the relation $S_1>3S_3>5S_5$ at $t=0$ shown in Fig. 1 is not true for different choices of N. However, the zero point around -0.4 as well as the general shape of the S_n as shown in Fig. 1 remain essentially constant. Therefore we consider both this zero point and the shown maxima and minima as an input to a Regge analysis.

If one tries to fit the data with a ρ that tends to $\alpha=-1$ and a moving ρ', one finds that α_ρ should approach -1 quite rapidly and ρ' should then follow a trail that would form a continuation of that of the ρ. Therefore we find it more appealing to fit the data with one effective moving pole (ρ) plus a fixed pole at $\alpha=-1$. In order to fit the curves of Fig. 1 one has to introduce fixed poles at -3 and -5 as well because S_3 and S_5 do not oscillate around zero. We choose for the moving pole $\beta_\rho(t)=\beta_0\exp[\beta_1\alpha(t)]$ where β_1 is of the order of 0.1-0.3. That is consistent with the high-energy fit of Rarita et al.[3] The β's of the fixed poles are assumed to be constants. With these assumptions we try to deduce the form of $\alpha_\rho(t)$ as well as the values of the β's. In order to have an estimate of the errors involved we vary N between 1.56 and 1.93 BeV, and incorporate the results of this variation in the error bars of Fig. 2.

In the case of a single moving ρ pole one expects oscillations of S_n around zero. They stem from the factor $1/\Gamma(\alpha)$ in (2). Since we add fixed poles these oscillations do not occur around zero any longer; however, the place of their extrema should not be changed. The first minimum of S_1 is expected at $\alpha=-0.82\pm0.04$. From Fig. 1 we see that it corresponds to $t=-1.18\pm0.04$. For the other S_n ($2\leq n\leq 6$) we find that the first expected minimum is at $\alpha=-0.44\pm0.04$ and it occurs at $t=-0.72\pm0.04$. The first maximum of S_3 and S_5 should lie around $\alpha=-1.53\pm0.04$ which turns out to be at $t=-1.63\pm0.1$. For the other S_n the first maximum occurs at a point that is al-

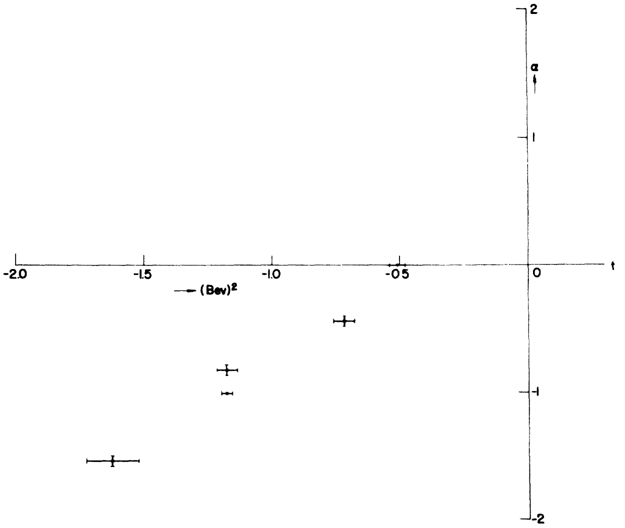

FIG. 2. The $\alpha_\rho(t)$ values deduced under the assumption of a moving ρ pole plus fixed poles. The error bars include the effect of the variation of N as well as the fact that most points incorporate information that comes from several S_n simultaneously.

ready far beyond the place where they start to diverge. All the points mentioned here are included in Fig. 2 which shows the deduced form of $\alpha_\rho(t)$ under the above-mentioned assumptions. We included also the $\alpha_\rho = 0$ and $\alpha_\rho = -1$ points that are obtained from S_3 and S_5 of Fig. 1 by finding the center of the oscillations. Fig. 2 gives a reasonable form for $\alpha_\rho(t)$ and therefore seems to be an a posteriori justification of our assumptions. The extracted value of β of the pole at $\alpha = -1$ is 8.9 ± 0.8 mb. For the poles at -3 and -5 we find β values of 1.0 ± 0.5 and 0.1 ± 0.05, respectively. Varying N we find the effect of the pole at -1 to decrease with increasing N as it should; however, the two other poles behave oppositely, namely, for N around 1 BeV S_3 and S_5 oscillate essentially around zero. Therefore, we cannot attach reasonable reliability yet to the poles at -3 and -5.

The introduction of the fixed poles explains why the dip in $d\sigma/dt$ is observed at a lower t value than the place of the zero at -0.4 in Fig. 1. We see from Fig. 2 that $\alpha_\rho = 0$ for t slightly below -0.5. Indeed, that should be the location of the dip even in the presence of the fixed poles. The latter contribute a real positive value to the B amplitude. Therefore, $|B|^2$ will not vanish at $\alpha_\rho = 0$, nevertheless it will still be minimal at that point! This is also true for the other predicted dips.

The situation of the FESR of the $A'^{(-)}$ is quite different. S_0 oscillates strongly around zero indicating an apparently rapidly varying $\beta_\rho(t)$. This makes it hard to decipher any fixed or other additional poles.

It has to be emphasized that the fixed pole we are talking about might also be a slowly moving pole or cut that cannot yet be distinguished from a fixed pole. However, it is interesting to speculate about the possibility that this is a genuine fixed pole. If so, it does not really appear as a pole in the j plane of any physical scattering amplitude.[4] It is rather a term that belongs to some asymptotic integer power series. Wherever such a term occurs it spoils the superconvergence relation for the amplitude minus the moving Regge poles.[1] It seems that such terms do occur in the analysis of $B^{(-)}$. How general this feature is only future experience will tell. Such a fixed pole could account for the π^+ photoproduction behavior.[5] Then it would have much in common with the fixed $\alpha = 0$ term of Ref. 1. In both cases it is the Born term in the direct channel which gives the dominant contribution to the amplitude and cannot be overpowered by the higher resonances. The role of the Born term in $B^{(-)}$ was explained in Ref. 1, and its importance in π^+ photoproduction is pointed out in Ref. 5. This is not to say that the Born term is the sole contributor to this or other fixed poles. The one at $\alpha = -1$ discussed in the present paper is a collective effect of all the s-channel structure. That is also true for the Pomeranchukon which might yet be the most prominent member of the class of terms discussed here. Difficulties raised by such an assumption[6] would then have to be resolved by providing different rules for the fixed poles compared to the moving poles.

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*Research sponsored by the Air Force Office of Scientific Research Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. E00AR-68-0010, through the European Officer of Aerospace Research.

†Visiting the Argonne National Laboratory, Argonne, Ill., during summer 1968.

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[4] Because of the ghost-eliminating factor $1/\Gamma(\alpha+1)$, the "fixed pole" loses all the properties of a pole in the j plane. Therefore, it will not lead to a contradiction with unitarity of the analytically continued partial waves in the t channel. Moreover, the usual proof of

factorization will not go through. The only Regge-pole property left is the asymptotic behavior in ν.

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RADIATIVE MESON DECAYS IN BROKEN SU(3)*

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(Received 8 July 1968)

We consider vector and pseudoscalar meson decays using vector gauge fields with current mixing. The processes $\omega \to 3\pi$, $\omega \to \pi\gamma$, and $\pi \to 2\gamma$ are fitted with one value of the gauge field's coupling g, which also determines leptonic decay rates of vector mesons. Octet breaking of the underlying VVP strong interaction introduces four parameters. The available experimental information on π, η, ω, and φ decays gives relations among these parameters which predict rates for decays such as $K^* \to K\gamma$.

The vector-dominance approach to meson decays[1] gives a good qualitative account of a number of strong and radiative decays, but the unbroken SU(3) version of this approach has certain quantitative failures. Examples are the ratios $\Gamma(\omega \to \pi\gamma)/\Gamma(\omega \to 3\pi)$ and $\Gamma(\eta \to \pi\pi\gamma)/\Gamma(\eta \to 2\gamma)$ which differ markedly from the predicted values.[2] Another experimental disagreement with the unbroken SU(3) prediction, which has been recently established,[3] is the ratio $\Gamma(\eta \to 2\gamma)/\Gamma(\pi^0 \to 2\gamma)$, the disagreement in this ratio being about six. We study these and other decays here, using octet breaking as well as some recent improvements in the theory of vector gauge fields within the effective-Lagrangian framework.[4]

The fundamental strong-interaction term responsible for the processes we study has the form

$$\mathcal{L}_{PVV} = \tfrac{1}{4}\epsilon_{\alpha\beta\mu\nu}(hD^{abc}V_{\alpha\beta}{}^{a}V_{\mu\nu}{}^{b}P^{c} + \lambda D^{ab}V_{\alpha\beta}{}^{a}P^{b}V_{\mu\nu}{}^{0}), \qquad (1)$$

where $V_\mu{}^0$ is an SU(3)-singlet vector meson, and $V_\mu{}^a$ and P^b are, respectively, vector and pseudoscalar octets: $a, b, c = 1, \cdots, 8$. In octet-broken SU(3) the D's have the general form[5]

$$D^{abc} = d^{abc} + \sqrt{3}\epsilon_1 d^{abd}d^{d8c} + \tfrac{1}{2}\sqrt{3}\epsilon_2(d^{acd}d^{d8b} + d^{bcd}d^{d8a}) + (\epsilon_3/\sqrt{3})\delta^{ab}\delta^{c8}, \qquad (2)$$

$$D^{ab} = \delta^{ab} + \sqrt{3}\epsilon_4 d^{ab8}. \qquad (3)$$

The vector fields are described by the generalized Yang-Mills Lagrangian

$$\mathcal{L}_V = -\tfrac{1}{4}K^{ab}V_{\mu\nu}{}^{a}V_{\mu\nu}{}^{b} + \tfrac{1}{2}m^2V_\mu{}^aV_\mu{}^a - \tfrac{1}{4}K^{00}V_{\mu\nu}{}^{0}V_{\mu\nu}{}^{0} + \tfrac{1}{2}m^2V_\mu{}^0V_\mu{}^0 - \tfrac{1}{2}K^{80}V_{\mu\nu}{}^{8}V_{\mu\nu}{}^{0}, \qquad (4)$$

where

$$V_{\mu\nu}{}^{a} = \partial_\mu V_\nu{}^a - \partial_\nu V_\mu{}^a - gf^{abc}V_\mu{}^bV_\nu{}^c$$

and where the K's are responsible for the observed mass splittings among the nine vector mesons and for the ω-φ mixing in the vector-mixing model[6-8]; K^{ab} is the diagonal matrix

$$K^{ab} = \delta^{ab} + \sqrt{3}\epsilon_0 d^{ab8}. \qquad (5)$$

Diagonalizing (4) in terms of physical ω and φ, we obtain

$$V_\mu{}^{1,2,3} = \frac{1}{\sqrt{K_\rho}}\rho_\mu{}^{1,2,3};$$

$$V_\mu{}^{4,5,6,7} = \frac{1}{\sqrt{K_{K^*}}}K_\mu{}^{*4,5,6,7};$$

$$V_\mu{}^{8} = -\frac{\sin\theta}{\sqrt{K_\omega}}\omega_\mu + \frac{\cos\theta}{\sqrt{K_\varphi}}\varphi_\mu;$$

$$V_\mu{}^{0} = \frac{\cos\theta}{\sqrt{K_\omega}}\omega_\mu + \frac{\sin\varphi}{\sqrt{K_\varphi}}\varphi_\mu, \qquad (6)$$

with

$$K_i = m^2/m_i{}^2 \quad (i = \rho, K^*, \omega, \varphi),$$