DOUBLE-REGGE-POLE MODEL ANALYSIS OF $pp \to pn\pi^+$ AT 28.5 GeV/c; PION EXCHANGE AND THE 1400-MeV πN ENHANCEMENT*

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Experimental distributions in all kinematic variables for the reaction $pp \to pn\pi^+$ at 28.5 GeV/c are consistent with predictions of a double-Regge-pole model employing Pomeranchuk and pion exchanges with the pion required to couple in conspiracy fashion to np. The 1400-MeV πN enhancement seen in these data is well fitted.

The double-Regge-pole exchange model[1] has been applied to a sample of events of the three-body type $np\pi^+$ produced in 28.5-GeV/c pp interactions.[2] Good agreement is obtained with experimental distributions in all variables. In particular, the broad enhancement seen in the $n\pi^+$ invariant mass near 1400 MeV is well fitted. The computation employs a single double-exchange diagram involving pion and Pomeranchuk exchanges; the distribution in the momentum transfer to n is indicative of pion conspiracy.[3]

I. The kinematics is defined by means of the diagram given in Fig. 1. Symbols to be used below refer to that diagram. We set $S=(p_1+p_2)^2$, $S_{p\pi^+}=(q_1+q)^2$, $S_{n\pi^+}=(q_2+q)^2$, $t_n=(q_2-p_2)^2$, $t_p=(q_1-p_1)^2$.

II. Event selection.—(a) In order to compare the model with an unbiased data sample, it was essential to discard all events for which $|t_p|<0.06$ (GeV/c)^2 where detection efficiency is less than 100%.[2] (b) Because application of the double-Regge-pole exchange model presumes a three-body final state, events of a quasi-two-body type should be removed. A strong $\Delta^{++}n$ final state is seen in the data. To eliminate its effects, and for a model-dependent reason to be discussed below, a cut was taken to discard all events for which $M(p\pi^+)<2.0$ GeV. (c) There are 573 events in this remaining sample. Consideration is not restricted here to events for which $S_{n\pi^+}$ is also large; indeed one of the important results of this study is the demonstration that the model adequately fits the rather broad enhancement seen near $M(n\pi^+)\simeq 1400$ GeV which dominates the reaction.

III. Double-Regge model.—The contribution of baryon-exchange diagrams to $np\pi^+$ is deemed negligible for two reasons: The distribution in the momentum transfer to the π^+ from either proton shows no preference for values near the kinematic limit and, secondly, the Dalitz plot in $S_{n\pi^+}$ vs $S_{p\pi^+}$ gives no evidence for any accumulation of events at large values of either of these subenergies, such as one would expect (the "cornering" effect of Chan et al.[1]) for baryon exchange.

Attention is thus focused on that class of diagrams with π^+ emerging from the central vertex. Quantum-number considerations provide a number of alternatives for the pair of trajectories (α_1, α_2), but the following argument suggests a unique choice. Pomeranchuk exchange, if allowed, provides the dominant contribution to a

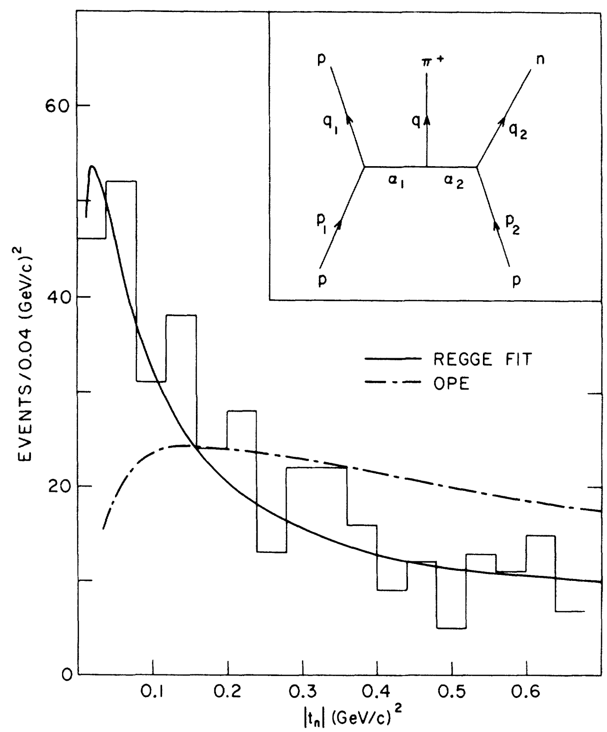

FIG. 1. Distribution in the square of the invariant four-momentum transfer to the final n for the process $pp\to pn\pi^+$ at 28.5 GeV/c. The plot contains 573 events for which $s_{p\pi^+}>4.0$ GeV^2 and $0.06<|t_p|<1.0$ (GeV/c)^2. Inset: Double-Regge-pole exchange diagram; the two trajectories are denoted by α_1 and α_2. The p_i and q_i are four-momenta.

given process when the energy is relatively large. Thus, if only events for which $S_{p\pi^+} > 4.0$ GeV^2 are selected, α_1 may be identified as the Pomeranchuk trajectory (P).

To couple properly at the middle $\pi P \alpha_2$ vertex, the trajectory α_2 must have G parity -1 and isospin 1; the candidates are π, A_1, and A_2. Various arguments favor π only. (a) A comparison of the on-mass-shell πp elastic and $\pi p \to Ap$ cross sections indicates that the strength of the π-π-P coupling is at least a factor of 10 greater than the π-A-P coupling. Because such coupling constants enter at the middle vertex of the proposed diagrams, that with π exchange would appear to be overwhelmingly favored. (b) A calculation of the contribution of the A_2-exchange diagram was performed. The distance of the A_2 singularity from the physical region plus the need for a ghost-eliminating mechanism[4] near $t_n \simeq -0.3$ (where $\alpha_{A_2} \simeq 0$), taken together with three-body kinematics, effectively rule out a significant A_2 contribution for $|t_n| < 0.4$ (GeV/c)^2. However, a glance at Fig. 1 demonstrates a substantial preference for very small values of t_n.

Therefore, for the remainder of this study, it will be assumed that a unique double-Regge-pole diagram with $\alpha_1 \equiv P$ and $\alpha_2 = \pi$ provides the dominant mechanism for producing $pn\pi^+$ with $S_{p\pi^+} > 4.0$ GeV^2.

IV. Matrix element.—The double-Regge-pole hypothesis[1] was adopted for the invariant matrix element squared, summed over final spins, and averaged over initial spins:

$$\sum |M|^2 = N_0 G_\pi(t_n) \left(\frac{S_{n\pi}\cdots}{S_{20}}\right)^{2\alpha_\pi} G(t_n, t_p, \omega) \left(\frac{S_{p\pi}\cdots}{S_{10}}\right)^{2\alpha_p} G_p(t_p). \quad (1)$$

In Eq. (1), $G_\pi(t_n)$ is the product of the Reggeized pion propagator, its signature factor, and the reduced residue function describing its coupling to the pn vertex. Similarly, $G_p(t_p)$ refers to the Pomeranchuk and its coupling to the pp vertex. In $G(t_n, t_p, \omega)$, describing the coupling of the two trajectories to the physical π at the central vertex, ω is the Toller rotation angle defined in Bali, Chew, and Pignotti's (BCP) work[1] or the angle Φ of Chan et al.[1] Explicitly, $(S_{n\pi^+}\cdots)$ is the numerator of the $\cosh\xi$ Toller variable of BCP, the denominator of which is absorbed into $G_\pi(t_n)$ as a kinematic factor. Likewise for $S_{p\pi^+}$:

$$(S_{n\pi^+}\cdots) = S_{n\pi^+} - t_p - M_p^2 - \tfrac{1}{2}(M_\pi^2 - t_p - t_n), \quad (2)$$

$$(S_{p\pi^+}\cdots) = S_{p\pi^+} - t_n - M_p^2 - \tfrac{1}{2}(M_\pi^2 - t_p - t_n). \quad (3)$$

For simplicity, P was chosen to be a fixed pole,[5,6] $\alpha_P \equiv 1$, and its associated $G_P(t_p)$ parametrized as $G_P(t_p) = \exp(8t_p)$. The model makes no prediction as to the dependence of $G(t_p, t_n, \omega)$ on ω; such dependence was assumed absent in this calculation, and the assumption seems justified by the resulting fits. See especially Fig. 2(b).

The pion trajectory was assumed linear, $\alpha_\pi = \alpha_\pi'(t - M_\pi^2)$, with slope α_π' left as an adjustable parameter. If one makes the assertion that a simple Reggeized pion is being exchanged, then in the limit $t_n \to M_\pi^2$, $G_\pi(t_n)$ must approach that for elementary one-pion exchange (OPE),

$$G_\pi(t_n) \to \frac{t_n}{(t_n - M_\pi^2)^2} \quad \text{(OPE)}. \quad (4)$$

This form predicts that the distribution in t_n should fall off for small t_n. The explicit character[7] of this effect is given by the dashed curve in Fig. 1. The falloff is clearly not seen in the data, a situation analogous to that in np charge exchange.[8] In order to maintain the assertion of a Regge-pole exchange mechanism, one must resort to a conspiracy[3] whereby the amplitude need no longer vanish at $t_n = 0$. This is accomplished here simply by removing the factor of t_n. Moreover, $G_\pi(t_n)$ was then assumed to be given by the product of the Reggeized propagator and signature factor, with no other vertex structure:

$$G_\pi(t_n) = \frac{(\pi\alpha_\pi')^2 M_\pi^2}{2(1 - \cos\pi\alpha_\pi)}. \quad (5)$$

The scale constants in Eq. (1) were treated as follows: S_{10} was fixed at 1.0 GeV^2 and S_{20} was left as the second variable parameter in the fit. The overall normalization constant, N_0, was determined by adjusting one of the calculated distributions (that in t_n) to the data, as shown in

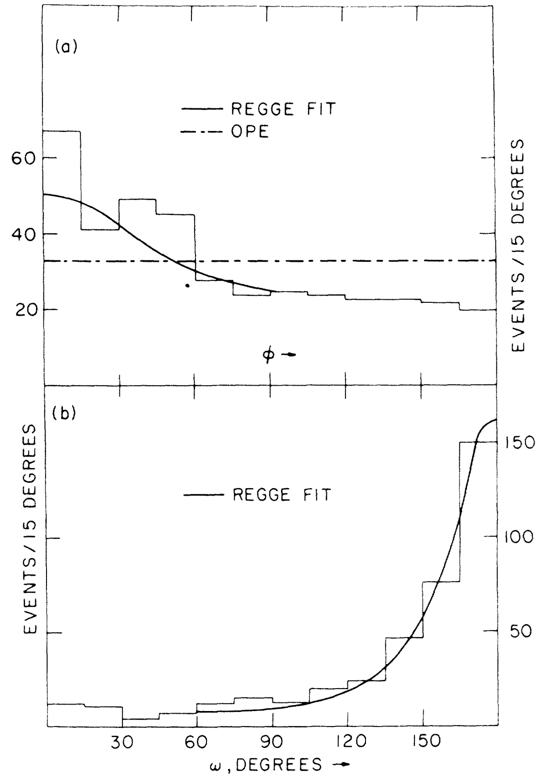

FIG. 2. (a) Distribution in the Trieman-Yang angle, defined in the $p\pi^+$ rest frame as

$$\varphi=\cos^{-1}[(\vec{p}_1\times\vec{q}_1)\cdot(\vec{p}_2\times\vec{q}_2)/|\vec{p}_1\times\vec{q}_1||\vec{p}_2\times\vec{q}_2|].$$

(b) Distribution in the Toller angle ω, defined in the π^+ rest frame as

$$\omega=\cos^{-1}[(\vec{p}_1\times\vec{q}_1)\cdot(\vec{p}_2\times\vec{q}_2)/|\vec{p}_1\times\vec{q}_1||\vec{p}_2\times\vec{q}_2|].$$

The Regge model adequately fits the distribution without requiring explicit ω dependence in the matrix element. Both (a) and (b) contain 391 events for which $s_{p\pi^+}>4.0$ GeV^2, $|t_n|<0.8$ (GeV/c)^2, and $0.06<|t_p|<1.0$ (GeV/c)^2.

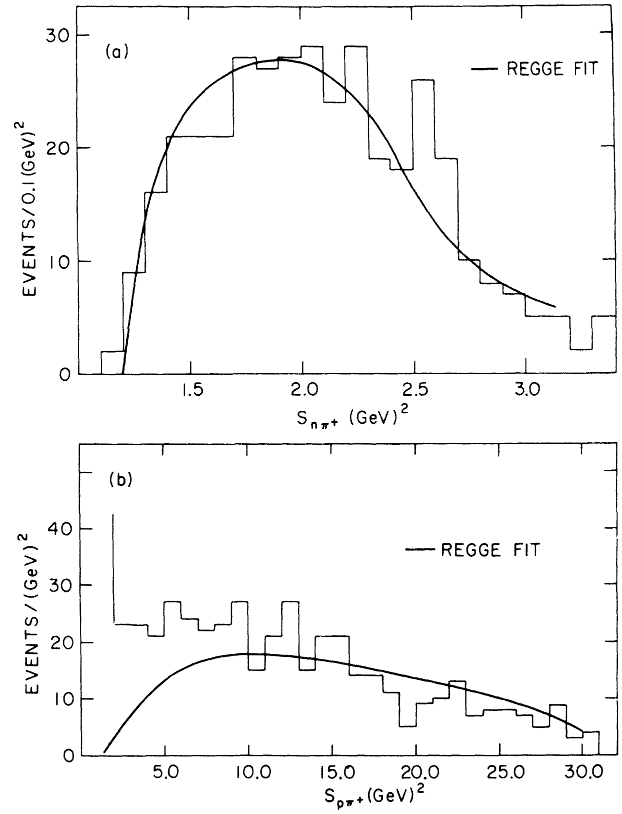

FIG. 3. (a) Distribution in the square of the invariant mass of the $n\pi^+$ system containing 404 events for which $S_{p\pi^+}>4.0$ GeV^2, $0.06<|t_p|<1.0$ (GeV/c)^2, and $|t_n|<1.0$ (GeV/c)^2. (b) Distribution of the square of the invariant mass of the $p\pi^+$ system containing 538 events for which $|t_n|<0.8$ (GeV/c)^2 and $0.06<|t_p|<1.0$ (GeV/c)^2. The prominent Δ^{++} peak containing 101 events is not shown.

Fig. 1. The values $\alpha_\pi'=1.2$ and $S_{20}=2M_pM_n$ provided the best overall fit; distributions calculated using these values are given in Figs. 1 through 3, with the same cuts made in the computations as taken in the data.

On the several distributions, only that in the Treiman-Yang angle, Fig. 2(a), is very sensitive to the value of α_π'. This is as expected because that distribution reflects the spin character of the trajectory which couples to the np vertex. If A exchange is included, contributing as discussed above mostly to the region $|t_n|>0.4$ (GeV/c)^2, best fits are realized with $\alpha_\pi'\simeq1.0$ GeV^{-2} and $S_{20}\simeq0.8$ GeV^2.

V. Conclusions.—(a) Conspiracy is not necessary in order to obtain good fit to the $S_{n\pi^+}$ distribution alone; a nonconspiring pion model or an OPE model (with form-factor in t_n) would also serve. However, neither would reproduce the t_n distribution nor the double-differential cross section $d^2\sigma/dt_n ds_{n\pi^+}$ over any range in $S_{n\pi^+}$. To the extent that a double-Regge model is correct, the presence of a conspiracy type of coupling of the pion to np is certainly evident. The possibility that the middle vertex is contributing a factor of t^{-1} to cancel the factor t_n of Eq. (4) cannot be entirely ruled out, of course. However this is deemed highly unlikely because the reaction $pp\to p\pi^-\Delta^{++}$ at this same energy should also be dominated by the combination of Pomeranchuk and pion exchanges whereas there is no evidence in fits to the data for an extra factor of, in this case, t_Δ^{-1} coming from the central vertex.

(b) The good fit to the $n\pi^+$ mass spectrum in the region of the 1400-MeV enhancement raises the question of the resonance interpretation of this phenomenon. One experimental difficulty clouds the issue. The rather small number of events and, in particular, the loss of events with $|t_p| < 0.06\ (\mathrm{GeV}/c)^2$ makes it difficult for one to determine a slope for $d\sigma/dt_p$, having selected $M_{n\pi}$ in the 1400-MeV region. The missing-mass counter experiments suggest a dependence of $\exp(\lambda t_p)$ with $\lambda \simeq 14\text{-}18\ \mathrm{GeV}^{-2}$, if the missing mass is in the region of 1400 MeV.[9] However, the present data are consistent with a much smaller λ—viz., 8-10 $(\mathrm{GeV}/c)^{-2}$ for events with $M(n\pi^+) < 1500$ MeV, in agreement with the model.

Nevertheless, in line with recent work on Regge-pole theory,[10] the appropriate conclusion would seem to be that although the exchange model yields agreement with the data, such agreement fails to imply the absence of one or more resonances in that structure.

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[1]N. F. Bali, G. F. Chew, and A. Pignotti, Phys. Rev. Letters 19, 614 (1967), and Phys. Rev. 163, 1572 (1967); H. M. Chan, K. Kajantie, and G. Ranft, Nuovo Cimento 49A, 157 (1967); H.-M. Chan et al., Nuovo Cimento 51A, 696 (1967); E. L. Berger, Phys. Rev. 166, 1525 (1968); E. L. Berger et al., Phys. Rev. Letters 20, 964 (1968). These may be consulted for additional references.

[2]P. L. Connolly et al., Brookhaven National Laboratory Report No. BNL-11980, 1967 (unpublished); W. E. Ellis et al., preceding Letter [Phys. Rev. Letters 21, 697 (1968)].

[3]R. J. N. Phillips, Nucl. Phys. B2, 394 (1967); F. Arbab and J. Dash, Phys. Rev. 163, 1603 (1967).

[4]M. Gell-Mann, in Proceedings of the International Conference on High Energy Physics, CERN, 1962, edited by J. Prentki (CERN, European Organization for Nuclear Research, Geneva, Switzerland, 1962), p. 539.

[5]A somewhat better fit to the distribution in $S_{p\pi^+}$ can be achieved with a mild slope $\alpha_P' = 0.3\ (\mathrm{GeV}/c)^2$ for the Pomeranchuk, as determined by R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

[6]Phillips and Rarita, Ref. 5.

[7]For the OPE computations, normalized to the total number of events,

$$\sum |M|^2 = t_n (t_n - M_\pi^{\ 2})^{-2}[s_{p\pi} - (m_p + m_\pi)^2] \times [s_{p\pi} - (m_p - m_\pi)^2] \exp(8t_p).$$

[8]For references, see Ref. 3 of this paper.

[9]E. W. Anderson et al., Phys. Rev. Letters 16, 855 (1966).

[10]G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

REGGE STRUCTURE OF THE $B^{(-)}$ AMPLITUDE IN πN SCATTERING*

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Applying the technique of finite-energy sum rules to the recent CERN πN phase-shift analysis, we investigate the Regge structure of $B^{(-)}$ at large momentum transfer. The features found are most easily explained by an additional fixed pole at $\alpha = -1$. Some of its implications are discussed.

We have redone the calculations explained in earlier work[1] and continued them to lower t values, using the recent phase-shift analysis of the CERN group.[2] A particularly interesting structure reveals itself in the S_n of $B^{(-)}$. The expected Regge expansion of this amplitude is

$$B^{(-)} \approx \sum_i \frac{\beta_i(t)\alpha_i \nu^{\alpha_i - 1}}{\Gamma(\alpha_i + 1)\sin\pi\alpha_i}(1 - e^{-i\pi\alpha_i}). \tag{1}$$

The sum is extended over the possible poles with the right quantum numbers. The high-energy data are believed to be accounted for by the ρ pole. The finite-energy sum rules (FESR) read

$$S_n \equiv \frac{1}{N^n}\int_{\mathrm{RHC}}^{N} d\nu\, \nu^n \,\mathrm{Im} B^{(-)}(\nu) \approx \sum_i \frac{\beta_i(t) N^{\alpha_i}}{(\alpha_i + n)\Gamma(\alpha_i)}. \tag{2}$$