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LARGE-ANGLE NEUTRON-PROTON ELASTIC SCATTERING FROM 3.0 TO 6.8 GeV/c*

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We present extensive new data on cross sections for neutron-proton elastic scattering from 3.0 to 6.8 GeV/c. At the higher momenta the cross sections are found to be nearly symmetric about 90° in the c.m. system for $|\cos\theta| \lesssim 0.3$. This symmetry implies that the contribution to the cross section from interference terms between the isospin-0 and isospin-1 amplitudes is small in this angular region. Other implications of the data are also discussed.

This paper is a report of extensive new data on neutron-proton elastic scattering from 3.0 to 6.8 GeV/c, particularly in the 90° region in the barycentric system. Previous reports have described the experimental method and have given results based on one-quarter of the data now analyzed.[1,2] The new results also incorporate a more careful Monte Carlo calculation of effective solid angles. The experiment,[6] carried out at the Bevatron of the Lawrence Radiation Laboratory, used a liquid-hydrogen target, optical spark chambers, and a neutron beam with a continuous spectrum. The neutron spectrum was strongly peaked near the maximum momentum of 7 GeV/c.

The new results are presented in Fig. 1 which is a semilogarithmic plot of the differential cross section $d\sigma/dt$ vs $\cos\theta$, where θ is the c.m.-system scattering angle of the neutron. For simplicity we shall use σ for $d\sigma/dt$ in what follows. The square of the four-momentum transfer (t) from the incident to the scattered neutron is also shown at the top of each plot. t is given by $|t| = 2p^{*2}(1-\cos\theta)$ where p^* is the neutron momentum in the c.m. system. As discussed in Refs. 1 and 2, the events must be binned into ranges of incident momenta, and in Fig. 1 each plot gives the data for an incident momentum range of approximately ±0.25 GeV/c around the indicated central

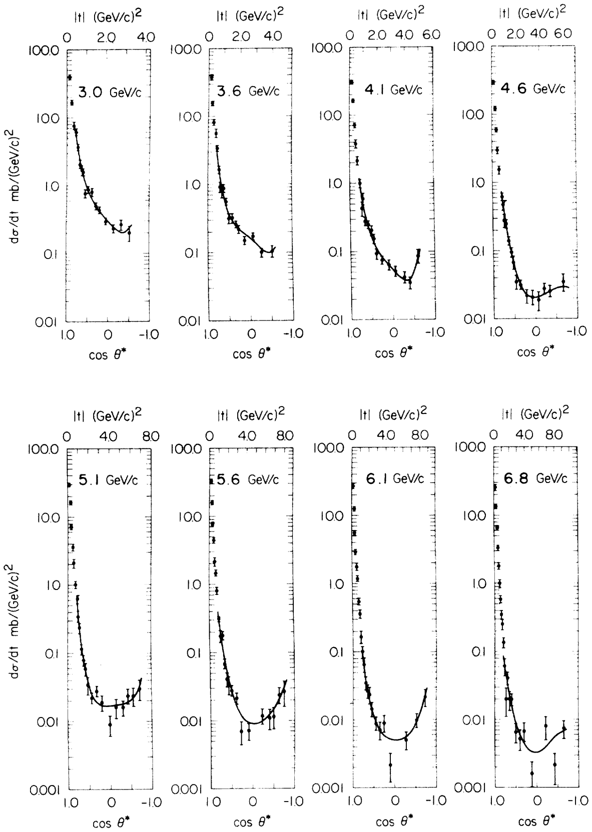

FIG. 1. The differential cross section ($d\sigma/dt$) vs $\cos\theta$; θ is the c.m.-system scattering angle of the neutron. See text for explanation of fit. $d\sigma/dt$ is called σ in the text.

momentum. The errors shown are statistical. The absolute normalization was made using the optical theorem. The total cross-section measurements used came from Bugg et al.,[3] and the real scattering amplitude was assumed to be zero.[4]

While all of the data are presented in Fig. 1, we shall only discuss here the behavior of the cross section outside the region of the diffraction peak in what we shall call the large-angle scattering region. To obtain a convenient parametrization of the data we have made a weighted least-squares fit for the angular range $|\cos\theta|<0.8$ with the equation

$$\sigma = c\exp\left[\sum_{n=1}^{4} a_n(\cos\theta)^n\right]. \qquad (1)$$

This equation allows symmetry effects about $\theta = 90°$ to be easily discerned. Higher powers of $\cos\theta$ do not meaningfully improve the fit. No attempt was made to obtain smooth variations of the parameters a_n with the incident momentum. We did not extend this equation to small angles because, in this region, the better statistics force the parameters to fit the diffraction peak. Therefore this is not an attempt to fit the cross section over the entire angular range but is primarily a means of smoothing the data in the large-angle region and obtaining a convenient parametrization. Table I presents the parameters for Eq. (1). The curves in Fig. 1 are the fits to Eq. (1).

The only way to obtain information on the nucleon-nucleon interaction for states with the total isospin $I=0$ is through np scattering. The amplitude for nucleon-nucleon scattering can in general be written in terms of a matrix M in spin space whose elements M_{jk} describe the scattering in the various spin states.[5] For np scattering each of the matrix elements is a linear combination of the form

$$M_{jk}^{np} = \tfrac{1}{2}(M_{jk}^{0} + M_{jk}^{1}),$$

where M_{jk}^{I} refers to the state with $I=0$ or 1. Each of the M_{jk}^{I} can be written as functions which are either symmetric or antisymmetric about $\theta = 90°$. By the generalized Pauli principle, for a given spin state M_{jk}^{0} is symmetric if M_{jk}^{1} is antisymmetric and vice versa. This makes it possible to isolate the cross section for the $I=0$ state and the total contribution to the cross section from terms due to interference between $I=0$ and $I=1$ states. The differential cross section σ^{np} can be written in the form[5]

$$\sigma^{np}(\theta) = \tfrac{1}{4}\sigma^{I=1}(\theta) + \tfrac{1}{4}\sigma^{I=0}(\theta) + (\text{interference term}),$$

where both σ^1 and σ^0 are symmetric relative to $\theta = 90°$ and the interference term is antisymmetric (since it is the sum of terms, each of which is a product of a symmetric and an antisymmetric function). The symmetry of the np cross section about 90° is therefore a measure of the contribution from interference between the $I=0$ and $I=1$ amplitudes.

In order to discuss this more quantitatively it is convenient to define the ratio $F(\theta) = \sigma(\theta)/\sigma(\pi-\theta)$. Values of F for $\cos\theta = 0.2$, 0.4, and 0.6 are given in Table I for incident momenta ≥ 4.1 GeV/c where the data are extensive enough to permit such a comparison. Another measure of symmetry about 90° is the value of $\theta_{\min}$, the angle at which the cross section attains its minimum.

Table I. P_0 is the incident neutron momentum. The coefficients from the equation

$$\sigma = c\,\exp\Big(\sum_{n=1,4} a_n(\cos\theta)^n\Big)$$

are least-squares fits to the differential cross section (σ) for $|\cos\theta| \leq 0.8$. The units of c are mb/(GeV/c)^2. $F(\theta)$ is the ratio of the forward to backward differential cross section for the indicated value of $|\cos\theta|$. $\sigma_{90°}$ is the differential cross section at 90° c.m. Cos$\theta_{\rm min}$ is the cosθ at which the differential cross section is smallest. w and w_t are measures of isotropy defined in the text.

P_0 GeV/c	Coefficients					$F(\theta)$			$\sigma(90°)$ mb/(GeV/c)^2	cos $\theta_{\rm min}$	w	w_t (GeV/c)^2
	c	a_1	a_2	a_3	a_4	cos θ=0.2	cos θ=0.4	cos θ=0.6				
3.0	.308	1.63	0.258	-1.07	5.23	1.7±.4	3.0±.7		.28 ±.04	-.35		
3.6	.165	1.28	-0.96	0.071	6.39	1.4±.3	2.5±.6		.155 ±.025	-.42		
4.1	.0542	1.65	0.225	-1.88	7.12	1.6±.5	2.5±.8	3.2±1.0	.056 ±.009	-.32	.78	2.3
4.6	.0207	-0.158	2.41	3.44	1.01	.9±.3	1.4±.4	4.2±1.5	.020 ±.004	+.05	>1.0	>3.5
5.1	.0170	-0.138	0.364	2.69	5.45	1.0±.3	1.2±.4	2.5±1.0	.015 ±.004	+.10	1.20	4.8
5.6	.00897	0.073	2.66	2.11	2.09	1.0±.3	1.3±.5	2.6±1.0	.0076±.0022	+.00	1.00	4.5
6.1	.00498	0.044	1.791	1.08	3.69	1.2±.5	1.2±.4	1.5±0.6	.0040±.0015	-.05	1.02	4.9
6.8	.00326	-0.281	3.40	2.91	-0.43	.8±.4	1.3±.7	3.5±1.9	.0029±.0014	+.05	1.02	5.5

Approximate values of $\cos\theta_{\rm min}$ are also given in Table I. At 4.6 GeV/c and above, $\cos\theta_{\rm min}$ is statistically in agreement with $\theta_{\rm min} = 90°$. It is clear from the curves in Fig. 1 and the values of $F(\theta)$ and $\cos\theta_{\rm min}$ that the cross sections become more nearly symmetric in the region $|\cos\theta| < 0.4$ at the higher incident momenta. This near symmetry implies that the phases between the $I=0$ and $I=1$ amplitudes are generally near 90° throughout this angular range or, what is more likely, that the amplitudes which are antisymmetric about 90° all remain relatively small for $|\cos\theta| \lesssim 0.4$ at the higher momenta. A similar interference between $I=0$ and $I=1$ amplitudes leads to a deviation of the np polarization from being purely antisymmetric about 90° so it is likely that the polarization in np scattering will be found small over this angular range. In the region near $|\cos\theta| \approx 1$ it is known that $\sigma(0°) \approx 30\sigma(180°)$.[6] This implies that the interference term is comparable with $\sigma^{I=0}$ and $\sigma^{I=1}$ in magnitude there.

The fits in Fig. 1 were made on the assumption that the fluctuations in $\sigma(\theta)$ were statistical. If, in fact, there is some structure in the region of the fit, an appreciable degree of asymmetry is not ruled out.

The near symmetry of the np cross section about 90° for $|\cos\theta| \lesssim 0.3$ was first predicted by Wu and Yang.[7] This symmetry is consistent with the expectations of the statistical model.[8] This model predicts that the cross section near 90° should fall off as $(\sigma_{\rm inel}/4p^{*2})\exp[h(E+g)]$, where $\sigma_{\rm inel}$ is the np total inelastic cross section and E is the total energy of the np system. The values of $\sigma(90°)$ are presented in Table I. These were calculated using only the data points closest to 90° and are therefore slightly different from the parameter c in Table I and Eq. (1). The values of $\sigma(90°)$ are found to agree with the statistical-model prediction with $g = -2.0$ GeV and $h = -3.9$ GeV^{-1}.

Białas and Czyzewski[9] and Kastrup[10] have discussed the symmetry of the np cross sections about 90°. The former analysis was based on our previously published data[1] and applies only to the region where the cross sections are asymmetric about 90°. It therefore appears to be applicable only for $|\cos\theta| \gtrsim 0.4$. The reader is referred to the original papers for more details.[9,10]

Another interesting feature of the data is that the cross sections at the higher momenta appear to be nearly independent of θ for a rather large range of θ near 90°. As a measure of this isotropy we list in Table I a width $w = |\cos\theta_1 - \cos\theta_2|$, where θ_1 and θ_2 are the two angles at which the cross section reaches twice the value at 90°. The corresponding width in four-momentum transfer, $w_t = |t(\theta_1) - t(\theta_2)|$, is also given. w_t is seen to increase steadily with increasing momentum. No theoretical explanation for this behavior seems to be available.

One of the purposes of this experiment was to look for deviations from smooth behavior of the cross section outside the small-$|t|$ region. In particular it is interesting to look for structure in the region $|t| \approx 1$ $(\mathrm{GeV}/c)^2$ where dips and shoulders have been found for πp and $\bar{p}p$ elastic scattering.[11,12] It is clear from Fig. 1 that our results show no marked structure. There is, however, the possibility of some structure, narrow in t, at the lower momenta. If this were so, the structure would have to be in the $I=0$ state, since no structure has been found in pp scattering at comparable momenta.[13] The data at large angles show several points which deviate significantly from the smooth curves. This could be due to systematic errors of an unknown nature that could also affect the data near $|t| \approx 1$ $(\mathrm{GeV}/c)^2$. If these indications of structure were correct it would be of considerable interest. Further measurements in the intermediate and large-angle region are clearly desirable.

The building and operation of this experiment and the early analysis were greatly aided by F. Martin.

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