POSSIBLE MECHANISM FOR STRUCTURE IN AN INELASTIC RESONANCE*

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We suggest a situation in which one might observe a two-peak structure which is in fact a single, broad, inelastic resonance. Starting with an inelastic factor η which has a single smooth dip, but differs somewhat from a Breit-Wigner form, we find that the Ball-Frazer mechanism generates a two-peak resonance.

There has been much interest lately in the structure of the A_2 meson.[1,2] Various attempts have been made to parametrize the shape of this double-peaked resonance by assuming that there are actually two narrow, incoherent resonances,[2] each having a Breit-Wigner shape, or by using the more unusual "dipole" shape resulting from the mixing of two degenerate resonances.[3] Until the spin and parity of the lower peak are clearly established, nothing definite can be said about the true nature of this particular resonance.[2] However, either of these two models would lead us to expect to see the same two-peak structure in other members of the SU(3) multiplet[4] with almost surely a clearer separation of the peaks.

The purpose of this paper is to suggest that one might find a two-peak structure which is in fact a single, broad, inelastic resonance which results from the detailed shape of the inelastic cross section. In this case, we expect that other members of the SU(3) multiplet would probably not show the two-peak structure.

Consider the scattering of two spinless particles of equal mass m, which have an S-wave resonance above the inelastic threshold s_I (s = square of center-of-mass energy). The elastic partial-wave amplitude is

$$A=(S-1)/2i,$$
$$S=\eta e^{2i\delta}, \tag{1}$$

where δ is the real part of the phase shift and η is the inelastic factor. The elastic cross section is determined by $|A|^2$ while the total cross section is determined by $\mathrm{Im}A$:

$$|A|^2=\tfrac{1}{4}[(1-\eta\cos2\delta)^2+(\eta\sin2\delta)^2],$$
$$\mathrm{Im}A=|A|^2+\tfrac{1}{4}(1-\eta^2). \tag{2}$$

If the resonance has the usual single-peak structure, we attempt to parametrize it by using a Breit-Wigner fit

$$A=\tfrac{1}{2}\Gamma_e/[m_R^{\,2}-s-\tfrac{1}{2}i(\Gamma_e+\Gamma_i)], \tag{3}$$

where m_R is the mass of the resonance, Γ_e is the elastic width, and Γ_i is the inelastic width (including kinematic factors). We can now easily find expressions for η and δ in terms of Γ_e, Γ_i, and $m_R^{\,2}$.

For our calculation we will restrict ourselves to cases where $\Gamma_i>\Gamma_e$ at $s=m_R^{\,2}$. We find that $\delta(m_R^{\,2})=0$ and the S matrix has no zeros on the physical sheet near this resonance. The peak in the cross section is then entirely due to the Ball-Frazer mechanism[5] and δ may be represented as

$$\delta(s)=\frac{-k}{2\pi}\mathrm{P}\int_{s_I}^{\infty}\frac{\ln[\eta(s')]}{k'(s'-s)}ds', \tag{4}$$

where $k=\tfrac{1}{2}(s-4m^2)^{1/2}$ and P means that we must take the principal value integral. If we use the η corresponding to a given Breit-Wigner shape we can compute the δ for the same Breit-Wigner shape by using Eq. (4). This is shown in Fig. 1 where we plot δ, $|A|^2$, and $\mathrm{Im}A$ computed from a given Breit-Wigner expession for η.[6] δ and η are thus related by a dispersion integral based on the analytic properties of the S matrix. If $\Gamma_e>\Gamma_i$ then conjugate, S-matrix zeros do appear on the physical sheet and we can modify the dispersion relation to take this into account.[7] Then if we know η and the position of the zeros of a Breit-Wigner resonance, we can compute δ from the dispersion relation.[8]

Phase-shift analyses in πN scattering do reveal inelastic factors showing a pronounced minimum at the position of a resonance, but it is impossible to parametrize many of these resonances with a Breit-Wigner shape.[9,10] This is not surprising in view of the fact that these resonances are of the order of a hundred MeV wide whereas the Breit-Wigner resonance formula was designed to describe narrow resonances. We will therefore adopt the attitude that the phase-shift dispersion relation is more fundamental than the Breit-Wigner representation and can be used to compute the phase shift of a reso-

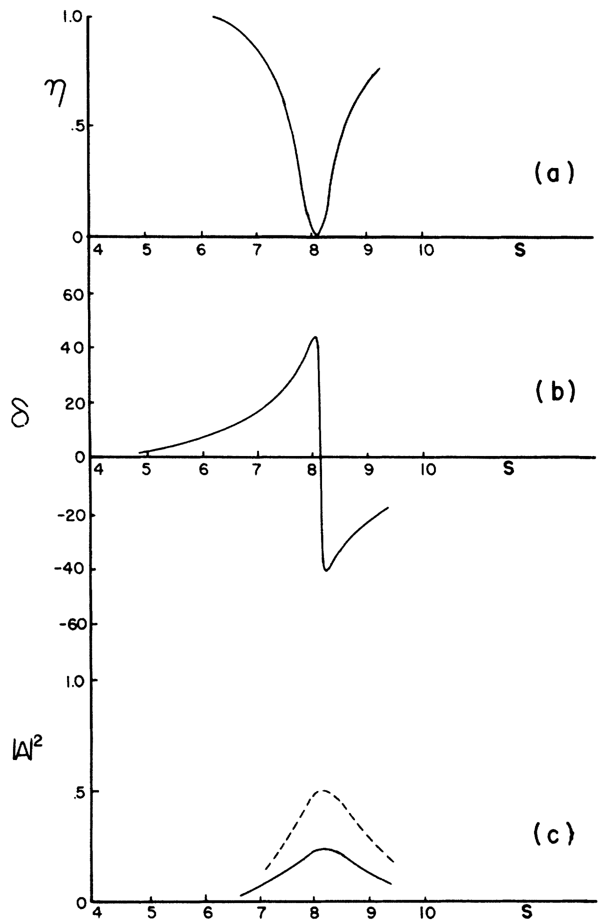

FIG. 1. (a) Inelastic factor resulting from a Breit-Wigner amplitude versus s. (b) Phase shift computed from (a) by using the dispersion relation Eq. (4) for the phase shift. (c) $|A|^2$ (solid line) and ImA (dashed line). We use units $\hbar = c = m = 1$.

nance and hence the amplitude if η is known (and there are no S-matrix zeros).

From Eq. (2) and the form of the phase shift in Fig. 1(b) we note that if we can make the maximum value of $\delta \gtrsim 45°$ and the minimum value of $\delta \lesssim -45°$, then we will see a double-peaked resonance since $|A|^2 > \frac{1}{4}$ at $|\delta| = 45°$ and $|A|^2 \le \frac{1}{4}$ at $\delta = 0$. Since the Breit-Wigner phase shift itself almost makes this possible, we expect to be able to produce this behavior with an inelastic factor not too different from a Breit-Wigner inelastic factor. We show such an inelastic factor graphically in Fig. 2(a). The crosses indicate the shape of a Breit-Wigner inelasticity for comparison. Both the elastic partial-wave cross section and total cross section shown in Fig. 2(c) exhibit a double peak even though there is only a single smooth dip in η.

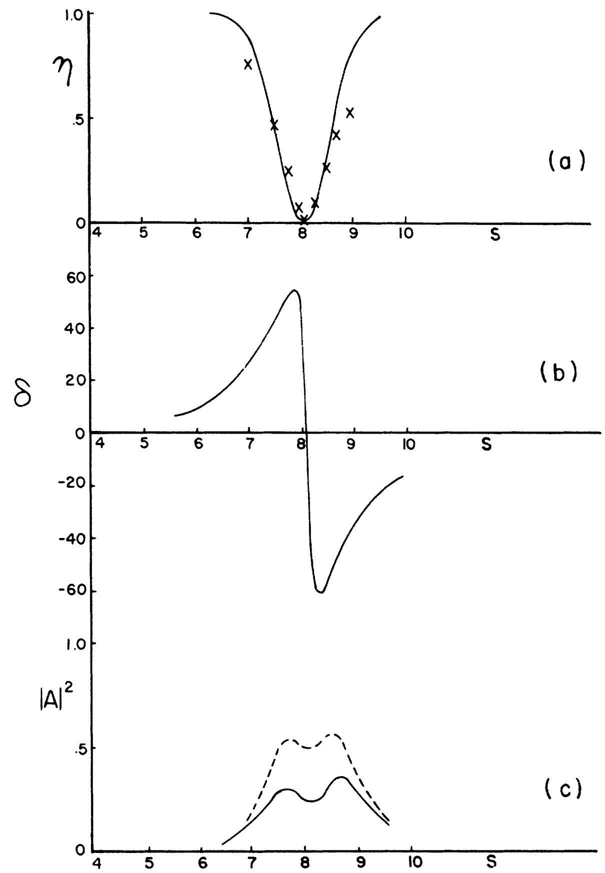

FIG. 2. (a) Inelastic factor which deviates from Breit-Wigner form. The crosses show a Breit-Wigner shape for comparison. (b) Phase shift computed from (a) by using the phase-shift dispersion relation. (c) $|A|^2$ (solid line) and ImA (dashed line).

We have not included the effects of S-matrix zeros because we have no means of determining the positions of the zeros from a knowledge of η for real s. We have done calculations where we chose the S-matrix zeros close to the minimum in η with a reasonable width and find that it is possible to enhance the dip greatly in the cross section shown in Fig. 2(c) and also make the branching ratio into the inelastic channel much smaller. There are, in addition, contributions to δ from singularities farther away such as the left-hand cut which could enhance or wash out this structure.

If the resonance structure seen in the region of the A_2 meson were actually due to two distinct resonances or a dipole-type resonance, we would expect to see the same type of structure in other members of the multiplet[4] with perhaps a clearer separation of the peaks. This calculation raises the possibility that the splitting may be an anomaly due to a deviation from the usual resonant shape which cannot be described adequately by a Breit-Wigner fit, but which can be accommodated by the phase-shift dispersion relation. If this analysis is relevant, we would not necessarily expect to see the same structure in other members of the multiplet. It is also clear that the structure would depend on the particular channels at which one looks. Thus the structure would depend on the production mechanism of the resonance.

We would suggest that one should closely examine all broad, highly inelastic resonances for this double-peak structure. In particular, likely candidates would have η's which are difficult to fit with a Breit-Wigner form. The presence of many open inelastic channels, some of whose thresholds occur in the region of the resonance, would be encouraging.

The calculation which we have presented is clearly contrived to produce a double peaked structure and we have left many questions unanswered. We do not know for certain if the departure of the shape of the inelastic factor from a Breit-Wigner shape implies the presence of more than one resonance. For example, if we take the inelastic factor corresponding to a dipole fit we can also compute a double-peaked structure from the phase-shift dispersion relation. On the other hand, for a broad resonance there are many factors which can affect the shape of the inelasticity and resonance such as new channels or a rapidly varying background, and it is not clear that one must assume that a broad, double-peaked structure necessarily implies the existence of two resonances or a dipole-type resonance. It is clear that the fine features of the inelastic factor of a broad resonance are not well described by a Breit-Wigner fit and we need to have a more general understanding of resonant shapes. A multichannel ND^{-1} model is being examined with the hope of clarifying some of these points.

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[1]G. Chikovani et al., Phys. Letters 25B, 44 (1967).

[2]D. Crennell et al., Phys. Rev. Letters 20, 1318 (1968).

[3]K. Lassila and P. Ruuskanen, Phys. Rev. Letters 19, 762 (1967).

[4]Actually, the presence of two poles means that there are two SU(3) multiplets.

[5]J. Ball and W. Frazer, Phys. Rev. Letters 7, 204 (1961).

[6]The position and width of a highly inelastic resonance are mainly determined by the minimum and shape, respectively, of the inelastic factor η. On the other hand, we also can relate the parameters of a resonance to a pole in the amplitude on an unphysical sheet. Consider the case where η reaches a minimum of ≈ 0 and the corresponding pole on one of the unphysical sheets in near the real axis. The width of the resonance can be broad however if this particular unphysical sheet cannot be directly reached from the position of the resonance on the physical sheet. See, e.g., Y. Fujii, Phys. Rev. 139, B472 (1965).

[7]P. Coulter, Phys. Rev. 167, 1352 (1968).

[8]See Fig. 3 of Ref. 7.

[9]P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. 165, 1730 (1968).

[10]C. Lovelace, in Conference on πN Scattering, Irvine, California December 1967 (unpublished).