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THREE-BODY CALCULATION OF DEUTERON-ALPHA SCATTERING AND POLARIZATION*

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Three-body calculations using separable two-body interactions have been carried out for the deuteron-alpha system with the alpha particle assumed to be structureless. Phase shifts, angular distributions, and polarization have been calculated and found to be in substantial agreement with experiment.

Several recent studies[1] of the bound state and scattering properties of the three-nucleon system have shown that the major features of this system may be described by quite simple separable interactions if three-particle effects are treated exactly. There is considerable interest in applying these methods to problems involving complex nuclei, for example, the system neutron-proton-nucleus and, in fact, several model calculations employing s-wave two-body forces have been reported.[2-4] The aim of the present work is to apply these methods to the deuteron-alpha system,[5] considered as a three-body problem, and to compare the results with the wealth of experimental data that is available. In our approach the alpha particle must be treated as an elementary point particle with no internal structure. This assumption seems quite reasonable for this nucleus due to the compactness of the alpha particle and due to its large binding energy and lack of low-energy excited states.

We study the d-α system in terms of Amado's[6] formulation of the three-body problem generalized to include spin-dependent interactions. In this formulation two-body pair interactions are thought to proceed through a quasiparticle, or equivalently,[7] through a separable potential. In the neutron-proton system we introduce an s-wave interaction with parameters chosen[8] to fit the deuteron binding energy and the triplet scattering length. In the nucleon-alpha two-body subsystems we neglect Coulomb interactions and allow the neutron and proton to have the same interaction with the alpha particle. Studies[9] of neutron-alpha scattering below 10 MeV show that it is nonabsorptive and that only the partial waves $s_{1/2}$, $p_{3/2}$, and $p_{1/2}$ contribute significantly, with the scattering dominated by a $p_{3/2}$ resonance at 1.2 MeV. A separable interaction is introduced for each of these three partial waves with the parameters chosen to fit an effective-range analysis of low-energy n-α scattering carried out by Pearce and Swan.[10] The form factor in each partial wave is chosen to be

$$v(k)=k^l/(k^2+\beta^2)^{l+1}, \tag{1}$$

which gives two parameters per interaction, a coupling constant and the range parameter β. The s-wave nucleon-alpha potential is made repulsive in an effort to simulate the effect of the exclusion principle in the interaction of a nucleon with a doubly closed-shell nucleus.

In order to study d-α scattering, one writes a set of coupled integral equations for the amplitudes describing the processes $d+\alpha\to$ correlated pair + third particle, where the correlated pair may be the deuteron or any of the three quasiparticles coupled to nucleon plus alpha. The final state, so described, is said to be a channel. We write the integral equations in an L-S representation and after partial-wave analysis they have the form

$$T_{\alpha\alpha'}^{(J)}(k,k';E)=B_{\alpha\alpha'}^{(J)}(k,k';E)$$
$$+1/[(2\pi)^3]\sum_{\alpha''}\int_0^\infty n^2dn\,T_{\alpha\alpha''}^{(J)}(k,n;E)\tau_{\alpha''}(n;E)B_{\alpha''\alpha'}^{(J)}(n,k';E), \tag{2}$$

where T and B are the full amplitude and Born term (single-particle-exchange amplitude), k and k' are the initial and final center-of-mass momenta, and α represents the set of quantum numbers (clS), that is, the channel c and its associated orbital angular momentum l and spin S. τ denotes the propagator in the intermediate state. All partial-wave states α and α' that conserve the total angular momentum J and the parity are included. The number of coupled equations may be given in terms of l, the orbital angular momentum in the d-α channel, and we have nine coupled amplitudes if $l = J$ and ten if $l = J \pm 1$, with some simpler cases resulting if $J = 0$ or 1. The numerical solution of the equations has been carried out using methods similar to those previously described.[2]

We have solved (2) for partial waves through $l = 4$ at seven energies between 0 and 10 MeV, and in Fig. 1 we show the results for the real part of the d-α phase shifts for some of the low partial waves as a function of energy. Experimental results are also shown in the form of a smooth curve drawn through an energy-independent phase-shift analysis carried out by McIntyre and Haeberli.[11] The agreement is generally good and the theory reproduces the three D-wave resonances, although at somewhat higher energies. The S_1 phase shift falls from π because of the predominantly S-wave bound state that we find at 3.350 MeV relative to the three-particle threshold. This bound level corresponds to the $T = 0$ ground state of Li^6 which has a binding energy of 3.697 MeV.[12] Also of interest are the P-wave phase shifts of which only P_0 is shown. A phase shift analysis based on a limited set of experiments has been performed by Senhouse and Tombrello[13] who found resonances in the P waves in disagreement with the later work of McIntyre and Haeberli.[11] We find no such resonances and, in fact, the P-wave phase shifts become repulsive at higher energies. A recent calculation by Thompson and Tang[14] using the resonating group method also find no P-wave resonances.

Figure 2 shows d-α angular distributions at 4, 6, and 10 MeV compared with experimental results. The agreement is generally very good. Coulomb interactions have not been introduced

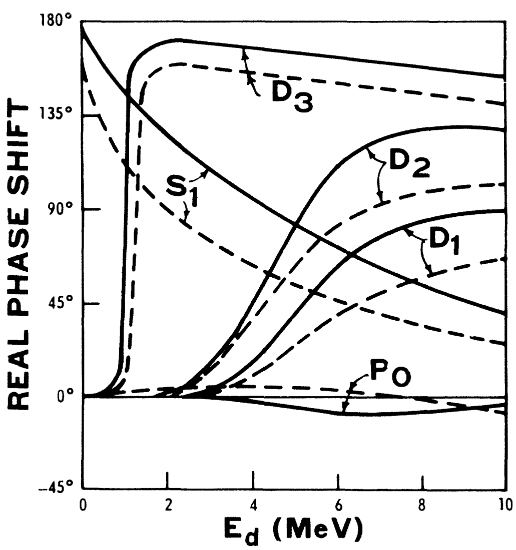

FIG. 1. Real d-α phase shifts versus laboratory deuteron energy for some low partial waves. The solid curves are the experimental phase shifts of Ref. 11. The broken curves are the calculated phase shifts.

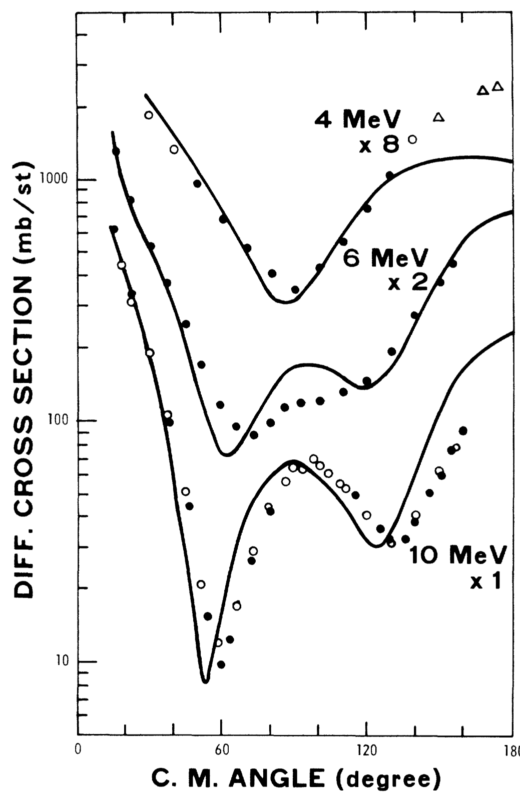

FIG. 2. Experimental and theoretical differential cross sections versus angle. At 4 MeV the data are from the following sources: Ref. 13 (dots), G. G. Ohlsen and P. G. Young, Nucl. Phys. 52, 134 (1964) (circles), and A. Galonsky, R. A. Douglas, W. Haeberli, M. T. McEllistrem, and H. T. Richards, Phys. Rev. 98, 586 (1955) (triangles); at 6 MeV from Ref. 13; and at 10 MeV from L. Stewart, J. E. Brolley, Jr., and L. Rosen, Phys. Rev. 128, 707 (1962) (dots), and G. G. Ohlsen and P. G. Young, Nucl. Phys. 52, 134 (1964) (circles).

into the three-body dynamics but in order to compare angular distributions they have been included in the standard way of combining nuclear and Coulomb amplitudes.[15] Our angular distributions are shifted slightly to smaller angles relative to the experimental results because of the lack of sufficient attraction noted in the discussion of the phase shifts. At the higher energies, the effect of deuteron break-up on the elastic cross sections is considerable and at 10 MeV the calculated reaction cross section amounts to 375 mb, compared with an experimental value[16] of 460 ± 80 mb.

In Fig. 3 we show a comparison of our results with experiment for the first- and second-rank

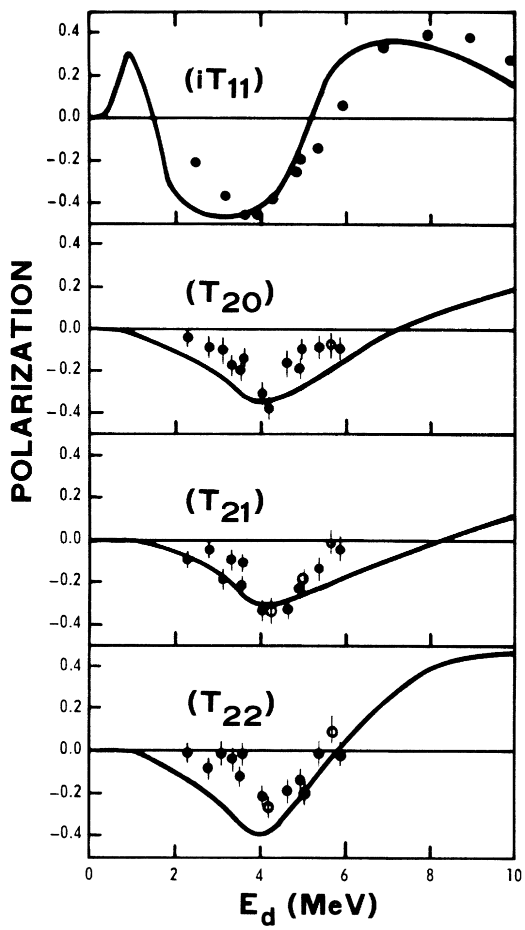

FIG. 3. Experimental and theoretical first-rank $\langle iT_{11}\rangle$ and second-rank $\langle T_{20}\rangle$, $\langle T_{21}\rangle$, and $\langle T_{22}\rangle$ polarizations versus energy at a center-of-mass angle of 66°. The data points for $\langle iT_{11}\rangle$ are from A. Trier and W. Haeberli, Phys. Rev. Letters 18, 915 (1967). The other data are from Ref. 11, dots, and P. G. Young, G. G. Ohlsen, and M. Ivanovich, Nucl. Phys. A90, 41 (1967), circles.

deuteron polarizations as a function of energy for fixed angle. The calculated polarizations follow the main trends of the experiments although there are some discrepancies.

In conclusion, it is evident that the major features of the d-α system have been reproduced by our three-body treatment and that the alpha particle is largely inert at the energies we are considering. One step in improving the calculation would be the inclusion of a tensor force in neutron-proton two-body interaction. This extension would not add to the number of coupled channels and the effect of such an interaction on the polarization would be of considerable interest. Two other problems which may be studied by the methods outlined here are the break-up reaction $d+\alpha\rightarrow n+p+\alpha$ and also deuteron elastic scattering and stripping on other light nuclei.

The computations described in this paper were performed on a CDC 6600 computer at the Lawrence Radiation Laboratory at Berkeley. I wish to thank the Laboratory and its staff for making these facilities available to me. I also wish to acknowledge helpful discussions with R. Aaron, J. V. Corbett, L. R. Dodd, G. L. Strobel, and M. T. Vaughn.

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HIGH-RESOLUTION STUDY OF LOW-ENERGY HEAVY COSMIC RAYS WITH LEXAN TRACK DETECTORS

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The response (track-etching rate) of Lexan polycarbonate detectors increases exponentially with the ionization rate of heavy nuclei ($Z \geqslant 10$). These detectors have been used to measure the charge composition and flux in 1967 of low-energy (200- to 400-MeV/nucleon) cosmic rays from $12 \leqslant Z \leqslant 28$.

We have discovered that sheets of ordinary Lexan polycarbonate[1] have a remarkable property that enables one to identify heavily ionizing particles with uniquely high resolution. This high resolving power may be explained by the assumption that the chemical reactivity along particle tracks in Lexan increases exponentially with ionization rate, in contrast to the linear response of conventional detectors such as nuclear emulsions and semiconductor crystals. Using Lexan detectors, we have identified 218 low-energy cosmic rays with $Z \geqslant 10$, of which 147 fall in the category of *VH* nuclei ($Z \geqslant 20$). The results show that the abundances of Mn and Cr are higher at low energy than at high energy[2] and thus provide new astrophysical information. Our results represent half the world data on *VH* nuclei with good charge resolution and the only data on the charge composition of low-energy nuclei with $Z \gtrsim 14$.[3] They are particularly valuable because the detecting plastics were flown at a much higher altitude (2 g/cm^2) than were previous detectors (4 to 10 g/cm^2),[2] so that the background introduced by fragmentation in air was small.

Figure 1 describes our method of particle identification in plastic sheets. In a recent paper[4] we showed that over small areas cellulose nitrate is sufficiently uniform in response that light isotopes such as B^{10} and B^{11} can be resolved. In a note added in proof we pointed out that Lexan polycarbonate, which records etchable tracks of nuclei with $Z \gtrsim 10$, has an even higher resolution

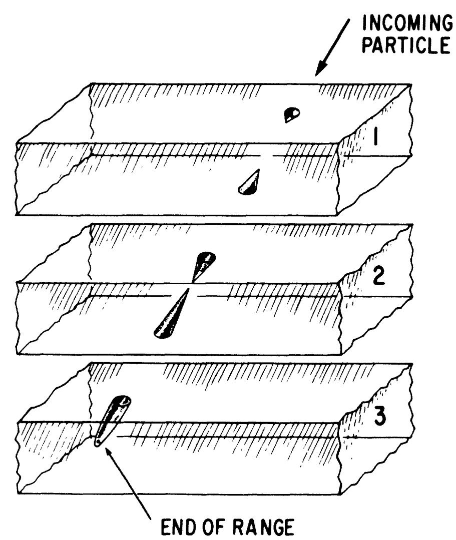

FIG. 1. NaOH etching solution attacks Lexan polycarbonate more rapidly along the trajectory of a heavily ionizing particle than elsewhere. The lengths of the hollow, etched cones increase with ionization rate. The last cone (sheet 3) has a rounded tip where the nucleus came to rest.