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POSSIBLE ZERO AT A WRONG-SIGNATURE SENSE POINT IN BACKWARD π^-p ELASTIC SCATTERING*

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A possible zero at the wrong-signature sense point $\alpha_\Delta = \frac{1}{2}$ on the exchanged Δ_δ trajectory is suggested in connection with the new Cornell–Brookhaven National Laboratory experiments on the π^-p backward peaks.

Sharp diffraction peaks at high energy for the backward π^+p elastic scattering have been successfully explained in terms of a Reggeized-baryon-exchange model. In addition to this phenomenon, the marked dip observed in the π^+p cross section near $u \simeq -0.15$ $(\text{GeV}/c)^2$ has also been interpreted as the wrong-signature nonsense zero of the N_α Regge amplitude at $\alpha_N(\sqrt{u}) = -\frac{1}{2}$.[1-4]

On the other hand, the recent Cornell-BNL (Brookhaven National Laboratory) experiments[5] on backward π^-p elastic scattering show the following features: (i) The π^-p backward peaks are about twice as wide as most elastic forward diffraction peaks and about four times as wide as the π^+p backward peaks. (ii) The results may not be inconsistent with a tendency for flattening out of the π^-p backward peak at 180°. These are not accounted for in the usual parametrization[6] of the residue function of the exchanged Δ_δ trajectory.

The purpose of this Letter is to suggest a possible zero at the wrong-signature sense point $\alpha_\Delta = \frac{1}{2}$ on the exchanged Δ_δ trajectory. Such a zero gives us a new parametrization of the Δ_δ residue function, which would explain the new Cornell-BNL experiments[5] on the π^-p backward peaks in the framework of the Reggeized Δ_δ-exchange model with reasonable values of s_0 and reasonable extrapolated magnitude of the Δ_δ residue at the pole $\alpha_\Delta = \frac{3}{2}$.

In essence, the model utilizes the following assumptions regarding the behaviors of the trajectory and the residue function[7]:

(a) The Chew-Frautschi plot for the Δ_δ trajectory is a straight line,

$$\alpha_\Delta(\sqrt{u}) = 0.15 + 0.90u. \tag{1}$$

(b) The residue function $\gamma_\Delta(\sqrt{u})$ includes a factor $(1 \pm \delta u^{1/2}/M_\Delta)$, corresponding to the absence of a $\frac{3}{2}^-$ resonance. Here we put $\delta = 1$ from Eq. (1).

(c) The following four mechanisms[8] are considered at the wrong-signature point $\alpha_\Delta = \frac{1}{2}$ on the Δ_δ trajectory in the sense-sense amplitude: (i) choosing-sense mechanism, (ii) Chew's mechanism, (iii) Gell-Mann's mechanism, and (iv) no-compensation mechanism. Thus, the parametrization is taken to be

$$\gamma_\Delta(\sqrt{u}) = [\alpha_\Delta(\sqrt{u}) - \tfrac{1}{2}]^n (1 + u^{\frac{1}{2}}/M_\Delta)\gamma_0, \tag{2}$$

with $n = 0$ for case (i), $n = 1$ for case (ii) or (iii), and $n = 2$ for case (iv). The value γ_0 is then assumed to be constant.

(d) Properly written, the amplitude should contain[9] a factor $\Gamma(\alpha_\Delta+\frac{1}{2})^{-1}$ to cancel the poles from $(\cos\pi\alpha_\Delta)^{-1}$ at the negative half-odd integers. Since we are only interested in a small range of α_Δ, we can approximate $\Gamma(\alpha_\Delta+\frac{1}{2})^{-1}$ by $(\alpha_\Delta+\frac{1}{2})(\alpha_\Delta+\frac{3}{2})$ times an essentially constant factor that is lumped into the definition of the residue γ_Δ. Then, the differential cross section for backward π^-p elastic scattering is calculated to be

$$\frac{d\sigma}{du}=\frac{\pi}{k_s^2 s}\frac{\gamma_0^2}{M_\Delta^2}\{[(E_s+m)(s^{\frac{1}{2}}-2m-m_\Delta)]^2+[(E_s-m)(s^{\frac{1}{2}}+2m+M_\Delta)]^2+2\cos\theta k_s^2[s-(2m+M_\Delta)^2]\}$$
$$\times(\alpha_\Delta-\tfrac{1}{2})^{2n}(\alpha_\Delta+\tfrac{1}{2})^2(\alpha_\Delta+\tfrac{3}{2})^2\frac{2}{1+\cos\pi(\alpha_\Delta-\frac{1}{2})}\left(\frac{s-m^2-\mu^2}{s_0}\right)^{2\alpha_\Delta-1}. \quad (3)$$

It should be noted that our simplifying assumptions allow s_0 and γ_0 to be the only adjustable parameters.

In order to determine the best values of s_0 and γ_0 for each case ($n=0$, 1, and 2), a least-squares fit of Eq. (3) to the backward π^-p differential cross section[5] for $u \geq -0.8$ $(\mathrm{GeV}/c)^2$ has been carried out. In Table I, the best χ^2 values for s_0 and γ_0 are listed together with the extrapolated magnitude of the Δ_δ residue at the pole, $\alpha_\Delta=\frac{3}{2}$ ($\sqrt{u}=1236$ MeV). In Fig. 1, the resultant fits are compared with the experimental data[5] at four representative momenta, 5.9, 9.9, 13.7, and 16.3 GeV/c.

In order to discriminate a possible ghost-eliminating mechanism at the wrong-signature sense point, $\alpha_\Delta=\frac{1}{2}$, we will take the criterion that the extrapolated magnitude of the Δ_δ residue at $\alpha_\Delta=\frac{3}{2}$ should be the same order of magnitude as the experimental (3, 3) width, 120 MeV.

Mechanism (i): Choosing-sense case ($n=0$). The experimental cross section can be reproduced by the parameter listed in Table I. However, the extrapolated magnitude of the Δ_δ residue $\alpha_\Delta=\frac{3}{2}$ is much too small:

$$\Gamma_\Delta^{\mathrm{calc}}\approx(1/60)\Gamma_\Delta^{\mathrm{exp}}. \quad (4)$$

Mechanism (ii) or (iii): Chew's or Gell-Mann's case ($n=1$). These cases are most favorable for the reason that they reproduce not only the experimental shape of π^-p backward peaks but also give us a reasonable extrapolated magnitude[10] of the Δ_δ residue at $\alpha_\Delta=\frac{3}{2}$,

$$\Gamma_\Delta^{\mathrm{calc}}\approx\tfrac{1}{2}\Gamma_\Delta^{\mathrm{exp}}. \quad (5)$$

Mechanism (iv): No-compensation case ($n=2$). We have to choose an unreasonably small value of s_0 and unreasonably large value of $\Gamma_\Delta^{\mathrm{calc}}$:

$$\Gamma_\Delta^{\mathrm{calc}}\approx 20\Gamma_\Delta^{\mathrm{exp}}. \quad (6)$$

In conclusion, we would like to point out the following:

(1) To reproduce the experimental shape and

Table I. The best χ^2 values (for 40 degrees of freedom) and the calculated width of the (33) resonance.

Case	n	s_0 (GeV^2)	γ_0 (GeV^{-1})	χ^2	$\Gamma_\Delta^{\mathrm{calc}}$ (MeV)
(i)	0	4.2	0.10	61	2.0
(ii) or (iii)	1	0.7	0.5	58	60
(iv)	2	0.10	2.6	114	2200

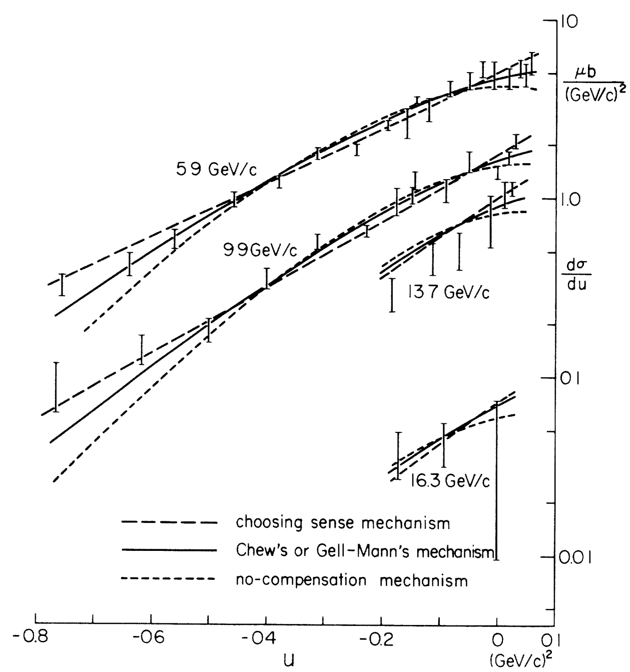

FIG. 1. Backward π^-p differential cross-section data compared with the Regge fits in terms of the four mechanisms mentioned in the text. The data are taken from Ref. 5. The 16.3-GeV/c results are plotted one decade lower.

magnitude of the backward π^-p differential cross section with a reasonable choice of the parameter s_0 and the extrapolated magnitude of the Δ_δ residue at the pole, $\alpha_\Delta=\frac{3}{2}$ ($\sqrt{u}=1236$), the Δ_δ trajectory should favor the Chew or the Gell-Mann mechanisms.

(2) The wide backward peaks[5] for the π^-p scattering can be explained as a consequence of the wrong-signature sense zero[11] at $\alpha_\Delta=\frac{1}{2}$ on the exchanged Δ_δ trajectory while a wrong-signature nonsense zero at $\alpha_N=-\frac{1}{2}$ is responsible for the sharp backward peaks for the π^+p scattering.[5]

(3) Future experiments on the backward π^-p charge-exchange scattering would confirm the first evidence for the wrong-signature sense zero in the Regge-pole theory, since the results are sensitive to the relative sign between the N_α and Δ_δ residues.[3]

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Note added in proof.—After completing our paper, we learned that the Carnegie-Brookhaven data have been published.[12] A least-squares fit of Eq. (3) to the Carnegie-Brookhaven data on backward π^-p differential cross section has also been carried out in terms of the four mechanisms mentioned in the text. The best χ^2 values for 17 degrees of freedom for s_0 and γ_0 are listed together with the calculated width of the (33) resonance as follows:

Case	n	s_0 (GeV^{-1})	γ_0 (GeV^{-1})	χ^2	$\Gamma_\Delta^{\text{calc}}$ (MeV)
(i)	0	6.4	0.1	15	1.3
(ii) or (iii)	1	1.0	0.5	19	43
(iv)	2	0.16	2.6	35	1430

Therefore, we can also conclude from the Carnegie-Brookhaven data that the Chew or Gell-Mann mechanisms are most favorable.

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[6]See Ref. 3. Barger and Cline have fitted the experiments in terms of the choosing-sense mechanism of the Δ_δ trajectory at $\alpha_\Delta=\frac{1}{2}$. The extrapolated magnitude of their parametrized Δ_δ residue gives an unreasonably small width for the (33) resonance.

[7]We use essentially the same notation as Ref. 2.

[8]C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. 161, 1563 (1967); L. Bertocchi, in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967, edited by H. Filthuth (North-Holland Publishing Company, Amsterdam, The Netherlands, 1968), p. 197. In Table III of above Proceedings, the behaviors of the residuum functions and the amplitudes near the wrong-signature point, $\alpha=j_0$, in a sense-sense transition, are listed as follows:

Mechanism	Residue functions γ_{ss}	Amplitudes —wrong signature T_{ss}
(i)	1	const
(ii) or (iii)	$\alpha-j_0$	$\alpha-j_0$
(iv)	$(\alpha-j_0)^2$	$(\alpha-j_0)^2$

We essentially follow the above convention.

[9]The explicit form of the u-channel amplitude is given in Eq. (15) of Ref. 2. This definition of γ_Δ coincides with that given in Eq. (14) of Ref. 2.

[10]See footnote 22 in Ref. 2. If we retain all the zeros at $\alpha_\Delta=-n-\frac{1}{2}$ ($n=0,1,2,\cdots$), $\Gamma_\Delta^{\text{calc}}$ increases to as large as 280 MeV. Anyway, $\Gamma_\Delta^{\text{calc}}$ is in agreement with $\Gamma_\Delta^{\text{exp}}$ within a factor of 2 for this mechanism.

[11]Using the finite-energy sum rule (FESR), we can investigate whether there exists a wrong-signature sense zero or not. When we perform this program, we must calculate the integral of the imaginary part of the scattering amplitude with $I=0$ exchange in the t channel. But there is much ambiguity in estimating the contribution to this amplitude from $f(1260)$. Moreover, the convergence of the partial-wave expansion is not so good. Therefore, we cannot draw any definite conclusion from the FESR.

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