GENERALIZED DISPERSION SUM RULES AND THE A_2 TRAJECTORY IN PION PHOTOPRODUCTION

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Generalized dispersion sum rules are used for determining the A_2 trajectory and residue function in pion photoproduction. Our results favor the Gell-Mann mechanism of ghost elimination for the A_2 over the Chew and no-compensation mechanisms.

Generalized dispersion sum rules involving the real and imaginary parts of the amplitudes have been recently derived by Liu and Okubo,[1] who found them to be well satisfied for the $T^{(-)}$ amplitude in πN scattering. These relations have been used by Liu and Okubo[1] for determining the P and P' Regge parameters at $t=0$ in πN scattering, and by Olsson[2] to determine the ρ Regge parameters at $t=0$ in πN scattering. In this note we extend the use of such relations to give a systematic method for determining Regge trajectories and residue functions over a range of t; we here use this method for determining the A_2 trajectory and residue function in $\gamma+N\rightarrow\pi+N$.[3,4]

We consider the combination $G^{(-)}=A_1^{(-)}-2mA_4^{(-)}$ of photoproduction amplitudes,[5] which is expected to be dominated by the A_2 trajectory, and define

$$F^{(-)}(\nu,t)=e^{i\pi\gamma}(\nu^2-\nu_0^{\;2})^{-\gamma}G^{(-)}(\nu,t), \tag{1}$$

where γ is a real number less than +1, and $\nu_0=\mu+(t+\mu^2)/4m$. If $F^{(-)}$ has a suitable asymptotic behavior, then for sufficiently small t one may write a dispersion relation in ν for $F^{(-)}$, for fixed t. For small t, the dominance of the A_2 trajectory would imply the asymptotic behavior

$$F^{(-)}(\nu,t)\sim R_{A_2}(\nu,t)=\frac{\alpha(t)\beta(t)\frac{1}{2}[1+e^{-i\pi\alpha(t)}]}{\sin\pi\alpha(t)}\frac{\nu^{\alpha(t)-2\gamma-1}}{\nu_1^{\;\alpha(t)-1}}e^{i\pi\gamma} \tag{2}$$

for sufficiently large ν, where ν_1 is a scale factor, conveniently chosen to be 1 GeV, and $\alpha(t),\beta(t)$ refer to the A_2 trajectory. For $1>\gamma>\frac{1}{2}\alpha(t)$, for given t, $F^{(-)}$ is superconvergent; this gives a relation involving integrals over $\mathrm{Re}G^{(-)}$ and $\mathrm{Im}G^{(-)}$, and the nucleon pole terms. Assuming that $F^{(-)}(\nu,t)$ is of the form (2) for $\nu>\bar{\nu}$, we obtain the following relation:

$$\frac{1}{\pi}\int_{\nu_0}^{\bar{\nu}}d\nu(\nu^2-\nu_0^{\;2})^{-\gamma}[\cos\pi\gamma\,\mathrm{Im}G^{(-)}(\nu)+\sin\pi\gamma\,\mathrm{Re}G^{(-)}(\nu)]+r(\nu_0^{\;2}-\nu_P^{\;2})^{-\gamma}$$
$$=-\frac{\sin\pi(\frac{1}{2}\alpha-\gamma)}{2\pi\sin\frac{1}{2}\pi\alpha}\frac{\alpha(t)\beta(t)}{\alpha(t)-2\gamma}\frac{\bar{\nu}^{\alpha-2\gamma}}{\nu_1^{\;\alpha-1}}, \tag{3}$$

where $r=-(eG/4m)[1+2m(\mu_p-\mu_n)]$, and $\nu_P=(t-\mu^2)/4m$.[6] For $\gamma<\frac{1}{2}\alpha(t)$, we may write a superconvergence relation for $F^{(-)}-R_{A_2}$; this now gives (3) for $\gamma<\frac{1}{2}\alpha(t)$.

Alternatively, one may continue (3) as a function of γ below $\gamma=\frac{1}{2}\alpha$, as pointed out in a similar context by Olsson.[2] The relation (3), for fixed t, is therefore valid for values γ (less than +1) such that the non-Regge (background) part of the asymptotic amplitude $G^{(-)}$ falls off more rapidly than $\nu^{2\gamma-1}$ for large ν. As a function of t, the sum rule (3) may be extended directly to the region $-2m\mu<t<4\mu^2$,[7] if we assume that a Regge asymptotic behavior (2) is a good approximation in this region. For negative t, one may define a continuation of (3) down to[7] $t=(-4m\mu-\mu^2)$; however, in this paper we restrict ourselves to $t>-2m\mu$, where the situation is simpler. For positive $t>4\mu^2$, the derivation of (3) given above is no longer valid. However, if the absorptive part of $F^{(-)}$ in the t channel (which receives contributions from 3π, 5π, $K\bar{K}$, etc. states with $I^G = 1^-$) is small for a range of t above $4\mu^2$, and if $\mathrm{Im}\,\alpha(t) \ll \mathrm{Re}\,\alpha(t)$ in this range, then the sum rule (3), with α replaced by $\mathrm{Re}\,\alpha$, may be expected to be approximately valid for these values of t. We therefore examine the results obtained from (3) (with $\alpha \to \mathrm{Re}\,\alpha$) for values of t above $4\mu^2$ as well as in the range $-2m\mu < t < 4\mu^2$.

When, for given t, $\gamma = \gamma_A(t) \equiv \frac{1}{2}\alpha(t) - n$, with $n = 1, 2, 3, \cdots$, then the right-hand side of (3) vanishes. For $n = 1$, one obtains the sum rules

$$\mathscr{I}(\gamma_A, t; \bar{\nu}) = 0; \quad (4a)$$

$$\left.\frac{\partial \mathscr{I}}{\partial \gamma}\right|_{\gamma=\gamma_A} = -\tfrac{1}{4}\alpha(t)[\sin\tfrac{1}{2}\pi\alpha(t)]^{-1}\beta(t)\frac{\bar{\nu}^2}{\nu_1^{\alpha-1}}, \quad (4b)$$

where $\mathscr{I}(\gamma, t; \bar{\nu})$ is the function defined by the left-hand side of (3). For each value of t, solving (4a) for γ_A gives $\alpha(t)$ through the relation $\alpha(t) = 2\gamma_A + 2$. Using this in (4b) gives $\beta(t)$.

On the other hand, when $\gamma = \gamma_B = \frac{1}{2}\alpha(t)$, for given t, then (3) gives the sum rule

$$\mathscr{I}(\gamma_B, t; \bar{\nu}) = -\tfrac{1}{4}\alpha(t)[\sin\tfrac{1}{2}\pi\alpha(t)]^{-1}\beta(t)/\nu_1^{\alpha-1}. \quad (5)$$

Thus, for such values of $\gamma_B(t)$ for each t, the function $\mathscr{I}$ must be independent of the parameter $\bar{\nu}$ [which is the value of ν above which the Regge behavior (2) is assumed to become dominant].

To obtain numerical results, we approximate $F^{(-)}$ by the lowest few partial waves, which gives a parametrization of $F^{(-)}$ as a function of s and t. The coefficients in this expression for $F^{(-)}$ are determined by using the observed values of the multipole amplitudes at physical s and t; we have used the results of Walker et al.[8] for these multipoles. We now assume that for the range of t considered here this expression continues to give a good approximation for $F^{(-)}$.

The solutions obtained for $\mathrm{Re}\,\alpha(t)$ and $\mathrm{Re}\,\beta(t)$ are shown in Fig. 1 for $\bar{\nu}$ corresponding to a photon laboratory momentum $k_L = \bar{k}_L \equiv 1.2$ GeV/c, which is the highest value of k_L at which the multipoles are available at present.[8]

We have also obtained the solution for $\bar{\nu}$ corresponding to $k_L = 1.5$ GeV/c, obtaining the amplitudes above $k_L = 1.2$ GeV/c by extrapolation. It is found that the solutions do not vary much with $\bar{\nu}$ within this range. The main features of our results are the following.

For small negative t, $\alpha(t) \approx \alpha_0 + \alpha_0't$, where $\alpha_0 \approx 0.52$, $\alpha_0' \approx 0.045$ for $\bar{k}_L \approx 1.2$ GeV/c. Varying $\bar{k}_L$ to 1.5 GeV/c decreases α_0 and α_0' by about

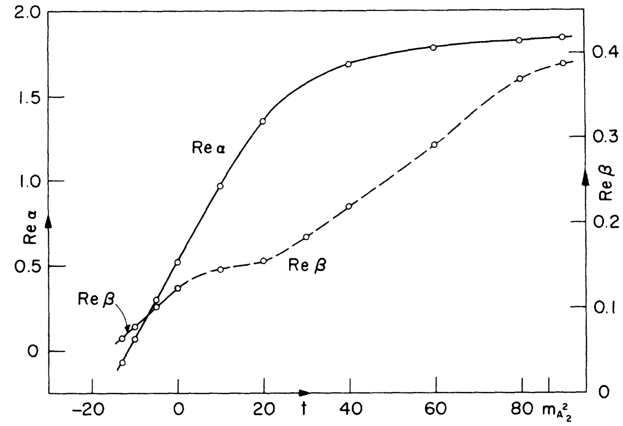

FIG. 1. $\mathrm{Re}\,\alpha(t)$ and $\mathrm{Re}\,\beta(t)$ for the A_2 trajectory as a function of t (in units of μ_π^2).

15 and 20%, respectively. Our estimates of $\alpha(t)$ for small negative t are of the same order as those of Ref. 3. $\alpha(t)$ is found to pass through zero at a value of t between about $11\mu^2$ and $14\mu^2$. To determine the exact position of this zero would require a more accurate knowledge of the low- and medium-energy multipoles, as well as a knowledge of the multipoles up to considerably higher energies, which would enable one to choose better values of $\bar{\nu}$.

For positive t, the rate of increase of $\mathrm{Re}\,\alpha$ with t falls off. Although strong assumptions have to be made to justify considering (3) (with $\alpha \to \mathrm{Re}\,\alpha$) as being approximately valid for fairly large positive t also, it is interesting to note that the function $\mathrm{Re}\,\alpha(t)$ thus obtained has the shape expected of the A_2 trajectory, and takes a value between 1.8 and 1.9 at $t = m_{A_2}^2$ (where it should be 2). These results for $\mathrm{Re}\,\alpha(t)$ are suggestive; we take them to indicate that for the amplitude $G^{(-)}$ considered here, the sum rule (3) does provide an approximate method for determining $\alpha(t)$ even for large positive t. Near $t = m_{A_2}^2$, our assumptions may introduce appreciable errors, and the discrepancy of 10 to 20% in this region is not unexpected.

Our results for positive t are in contrast to those of Ref. 3; the solution given by the finite-energy sum rules (FESR), if taken seriously for large positive t, gives $\mathrm{Re}\,\alpha > 2$ already at $t \approx 50\mu^2$.[3] This difference arises because our procedure allows γ to be varied so as to weight the most significant parts of the integrals separately for each t, whereas in the FESR, γ is kept fixed (at 0 or −1). Our results indicate that the best values of γ are probably in the region −1 to 0, and that the larger values of γ in this region are the better ones for more positive values of t.

For the residue function $\beta(t)$, we find that for small negative t, $\beta(t)\approx\beta_0+\beta_0't$, where $\beta_0\approx 0.26$ and $\beta_0'\approx 0.09$. The solutions for $\beta(t)$ obtained from (4b) and (5) agree to within about 20% for small negative t. For $t>4\mu^2$, this discrepancy increases; however, (5) is obtained from (3) with much larger values of γ [than is (4b)], and our overall results suggest that for positive t, $\mathrm{Re}\beta(t)$ as obtained with the smaller values of γ occurring in (4b) is more reliable. The results for $\beta(t)$ vary by about 20% as $\bar{k}_L$ is varied between 1.2 and 1.5 GeV/c.

Our results suggest that the zero of $\beta(t)$ would lie at a value of t considerably more negative than $-15\mu^2$ [and therefore more negative than the zero of $\alpha(t)$], unless the slope of $\beta(t)$ changes very rapidly below $t\approx -13\mu^2$.[9] Thus if the Regge behavior (2) is a good approximation above $\bar{k}_L\approx 1.2$ GeV/c and if $\beta(t)$ varies smoothly for t below $-13\mu^2$, then the residue function would not seem to have a zero at the ghost [where $\alpha(t)=0$]. This would favor the Gell-Mann mechanism[10] for ghost elimination rather than the Chew[11] or no-compensation mechanism,[12] in contrast to the conclusion of Ref. 3. This would agree with the result suggested by the absence of a dip in the angular distribution of $K^+p\to K^0\Delta^{++}$ and of $\pi^-p\to\eta n$.[13] We stress, however, that till multipole fits become available for $k_L>1.2$ GeV/c and enable us to test whether the Regge behavior (2) is a good approximation in this region, we cannot infer reliably the ghost-elimination mechanism for the A_2.

The sum rule (5) implies that its left-hand side must be independent of $\bar{\nu}$. In Table I we have shown the left-hand side of (5) as a function of t, for different $\bar{\nu}$; it is seen to be independent of $\bar{\nu}$ to an accuracy of 10 to 15%. Further, when $\alpha(t)$ and $\beta(t)$ as determined from (4) are substituted into (5), then for the negative values of t considered here, (5) is found to be satisfied to about 20 to 25%. These results support the validity of the sum rule (5) and the consistency of our basic assumptions. We believe that the main source of discrepancy in our results for $\alpha(t)$ and $\beta(t)$ is that the value of $\bar{\nu}$ up to which the multipoles are available at present may not be large enough for the Regge behavior (2) to dominate completely.

Table I. $\mathcal{I}(\gamma_B,t;\bar{\nu})$ as a function of $\bar{\nu}$ [See Eq. (5)]. (Note that for given t, $\bar{\nu}$ is determined by $\bar{k}_L$.)

t/μ^2	$\mathcal{I}(\gamma_B,t)$ as a function of $\bar{k}_L$: $\bar{k}_L=1.2$ GeV/c	$\bar{k}_L=1.5$ GeV/c
−13	−0.208	−0.234
−10	−0.223	−0.207
− 5	−0.154	−0.162
0	−0.125	−0.146
10	−0.267	−0.271
20	−0.487	−0.515
40	−0.64	−0.69
60	−0.68	−0.75
80	−0.745	−0.81
90	−0.75	−0.82

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[1]Y. Liu and S. Okubo, Phys. Rev. Letters 19, 190 (1967); and Phys. Rev. 168, 1712 (1968).

[2]M. Olsson, to be published.

[3]Recently finite-energy sum rules (FESR) have been used for studying the A_2 trajectory in photoproduction by S. Y. Chu and D. P. Roy, Phys. Rev. Letters 20, 958 (1968). We believe that the generalized dispersion sum rules (which include the FESR as a special case) provide a more reliable method for the determination of Regge trajectories.

[4]The A_2 trajectory and residue function in KN scattering have been studied by S. Matsuda and K. Igi, Phys. Rev. Letters 19, 928 (1967).

[5]For notation, see J. S. Ball, Phys. Rev. 124, 2014 (1961).

[6]Here G is the πNN coupling constant, and μ_p,μ_n are the proton and neutron anomalous magnetic moments. μ is the pion mass.

[7]Note that at $t=-2m\mu$, the crossed nucleon pole coincides with the s-channel threshold branch point, while at $t=t_L\equiv-(4m\mu+\mu^2)$, the s- and u-channel threshold branch points coincide. The question of continuing the relation (3) down to $t=t_L$ will be examined elsewhere.

[8]R. L. Walker, private communication, and R. L. Walker et al., to be published.

[9]In particular, a linear extrapolation [noting that $\beta(t)$ is roughly linear between $t=0$ and $t\approx-13\mu^2$] would suggest that $\beta(t_0)=0$ for a value of t_0 between $-25\mu^2$ and $-30\mu^2$. A rapid variation in the slope of $\beta(t)$ below $t\approx-13\mu^2$ seems unlikely.

[10]M. Gell-Mann, in Proceedings of the International

Conference on High-Energy Physics, CERN, 1962, edited by J. Prentki (CERN, European Organization for Nuclear Research, Geneva, Switzerland, 1962), p. 539.

[11]G. F. Chew, Phys. Rev. Letters 16, 60 (1966).

[12]C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. 161, 1563 (1967).

[13]M. Krammer and U. Maor, Nuovo Cimento 52A, 308 (1967); and L. Bertocchi, in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967, edited by H. Filthuth (North-Holland Publishing Company, Amsterdam, The Netherlands, 1968), p. 218.

POSSIBLE ZERO AT A WRONG-SIGNATURE SENSE POINT IN BACKWARD π^-p ELASTIC SCATTERING*

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A possible zero at the wrong-signature sense point $\alpha_\Delta = \frac{1}{2}$ on the exchanged Δ_δ trajectory is suggested in connection with the new Cornell–Brookhaven National Laboratory experiments on the π^-p backward peaks.

Sharp diffraction peaks at high energy for the backward π^+p elastic scattering have been successfully explained in terms of a Reggeized-baryon-exchange model. In addition to this phenomenon, the marked dip observed in the π^+p cross section near $u \simeq -0.15$ (GeV/c)^2 has also been interpreted as the wrong-signature nonsense zero of the N_α Regge amplitude at $\alpha_N(\sqrt{u}) = -\frac{1}{2}$.[1-4]

On the other hand, the recent Cornell-BNL (Brookhaven National Laboratory) experiments[5] on backward π^-p elastic scattering show the following features: (i) The π^-p backward peaks are about twice as wide as most elastic forward diffraction peaks and about four times as wide as the π^+p backward peaks. (ii) The results may not be inconsistent with a tendency for flattening out of the π^-p backward peak at 180°. These are not accounted for in the usual parametrization[6] of the residue function of the exchanged Δ_δ trajectory.

The purpose of this Letter is to suggest a possible zero at the wrong-signature sense point $\alpha_\Delta = \frac{1}{2}$ on the exchanged Δ_δ trajectory. Such a zero gives us a new parametrization of the Δ_δ residue function, which would explain the new Cornell-BNL experiments[5] on the π^-p backward peaks in the framework of the Reggeized Δ_δ-exchange model with reasonable values of s_0 and reasonable extrapolated magnitude of the Δ_δ residue at the pole $\alpha_\Delta = \frac{3}{2}$.

In essence, the model utilizes the following assumptions regarding the behaviors of the trajectory and the residue function[7]:

(a) The Chew-Frautschi plot for the Δ_δ trajectory is a straight line,

$$\alpha_\Delta(\sqrt{u}) = 0.15 + 0.90u. \tag{1}$$

(b) The residue function $\gamma_\Delta(\sqrt{u})$ includes a factor $(1 + \delta u^{1/2}/M_\Delta)$, corresponding to the absence of a $\frac{3}{2}^-$ resonance. Here we put $\delta = 1$ from Eq. (1).

(c) The following four mechanisms[8] are considered at the wrong-signature point $\alpha_\Delta = \frac{1}{2}$ on the Δ_δ trajectory in the sense-sense amplitude: (i) choosing-sense mechanism, (ii) Chew's mechanism, (iii) Gell-Mann's mechanism, and (iv) no-compensation mechanism. Thus, the parametrization is taken to be

$$\gamma_\Delta(\sqrt{u}) = [\alpha_\Delta(\sqrt{u}) - \tfrac{1}{2}]^n (1 + u^{\frac{1}{2}}/M_\Delta)\gamma_0, \tag{2}$$

with $n = 0$ for case (i), $n = 1$ for case (ii) or (iii), and $n = 2$ for case (iv). The value γ_0 is then assumed to be constant.

(d) Properly written, the amplitude should con-