INFINITELY RISING, NONLINEAR REGGE TRAJECTORIES

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By projecting s-channel partial waves from a t-channel Regge trajectory, families of resonances obeying $l\sim\sqrt{s}$ are obtained. It is proposed that these are Regge trajectories obeying $\alpha\sim\sqrt{s}$ for high s.

The discovery of many new resonances has suggested that for low energies, Regge trajectories rise linearly.[1] This is in contrast to potential scattering where Regge trajectories turn back. It is unknown whether trajectories rise indefinitely, and whether the rise continues to be linear. We will show that Regge behavior and crossing suggest infinitely rising though nonlinear trajectories.

Our approach is to study the s-channel resonances generated by t-channel Regge poles. Schmid[2] has recently shown that partial-wave projection from a t-channel Regge pole gives rise to circles in an Argand diagram which are interpreted as resonances.[3] This strongly reinforces the statement,[4] based on finite-energy sum rules, that Regge poles represent the smoothed-out resonance behavior even at low energy. The Schmid method has also been applied to investigate daughter trajectories.[5] We shall investigate the high-s and high-l behavior of resonances. Our main conclusions are the following: (1) Resonances exist even at very high energy. (2) The high-energy resonances can be grouped in families satisfying $l\sim\sqrt{s}$ (logarithmic terms are neglected). It is suggested that these correspond to infinitely rising, nonlinear Regge trajectories. (3) For $l\ll\sqrt{s}$, partial waves are nonresonant and large. (4) It is unlikely that the high-energy behavior can be described by an infinite number of linearly rising trajectories.

For simplicity we will consider the scattering of spinless particles, say π mesons. At sufficiently high s this scattering is dominated by two, well-separated, forward and backward peaks. We will represent the forward and backward peaks by the contribution from t- or u-channel Regge poles, respectively. At low energies where the two peaks overlap, it is unclear how to represent the contributions. Simple addition of t- and u-channel Regge poles is analogous to adding s- and t-channel trajectories. It may involve double counting,[4] similar to that done when the interference model is applied incorrectly.

It is useful to obtain approximate analytic expressions for the partial-wave amplitudes $a_l(s)$ in order to investigate the high-l and high-s behavior. Because the strong forward and backward peaks at small momentum transfer ($t\approx t_0$) dominate the high-energy scattering, only very small angles $\theta\approx(t_0/k^2)^{1/2}$ are important. We may thus use an approximate,[6] small-θ, high-l behavior of $P_l(\cos\theta)$:

$$P_l(\cos\theta)=J_0[(2l+1)\sin\tfrac{1}{2}\theta]+O(\sin^2\tfrac{1}{2}\theta). \tag{1}$$

This is the approximation used in the impact-parameter description of scattering. We parametrize the contribution of a Regge trajectory by

$$\beta(t)\left(\frac{s}{s_0}\right)^{\alpha(t)} =g\left[\frac{\pm1-e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\sin\pi\alpha(t)e^{+ct}\right]\left(\frac{s}{s_0}\right)^{\alpha(t)},$$
$$\alpha(t)=\alpha_0+\alpha_1 t. \tag{2}$$

This parametrization should not be considered to be of profound importance; it is clearly wrong for $t>0$. Only the first few zeros of $\beta(t)$ and the approximate linearity of $\alpha(t)$ for small t will influence our final results. In this region a polynomial in t may replace $\sin\pi\alpha(t)$ in the numerator. First, we proceed to project out partial waves from the contribution of a single trajectory:

$$a_l(s)=\frac{1}{2}\int_0^{-4k^2}\frac{dt}{2k^2}P_l\left(1+\frac{t}{2k^2}\right)\beta(t)\left(\frac{s}{s_0}\right)^{\alpha(t)}. \tag{3}$$

We use the parametrization in Eqs. (1) and (2). We measure s in units of s_0 to simplify equations. The upper limit of integration can be replaced[7] by $-\infty$ and we obtain[8]

$$a_l(s)=\pm\frac{gs^{\alpha_0}}{2k^2(\ln s+c)}\exp\left[-\frac{(l+\frac{1}{2})^2}{4k(\ln s+c)}\right]$$
$$-\frac{g}{2k^2}s^{\alpha_0}(c+\alpha_1\ln s-i\pi\alpha_1)^{-1}\exp[-b(c+\alpha_1\ln s)]\exp[-i\pi(b\alpha_1+\alpha_0)], \quad (4)$$

where

$$b=[(l+\tfrac{1}{2})^2/4k^2][(c+\alpha_1\ln s)^2+\alpha_1{}^2\pi^2]^{-1}. \quad (5)$$

The first term on the right-hand side of Eq. (4) is real and signature dependent; it also has a smooth energy dependence. The second term contains a phase factor which increases with energy. The term will describe circles in the Argand diagram, and these will commonly be interpreted as resonances. Figure 1 shows such a resonance with $l=40$.

Equation (4) should be investigated in three regions:

(i) $b\ll 1$, small impact parameter. In this region partial wave amplitudes are smooth. In the limit $s\to\infty$ a_l becomes independent of l.[9]

(ii) $b\gg 1$, large impact parameter. In this region partial waves are strongly suppressed by the negative exponentials appearing in Eq. (4). Semiclassically this corresponds to $l\gg kR$; therefore, the particle is outside the range of strong interaction forces so that no scattering occurs. Any resonance in this region must have vanishingly small elasticity.[10] The detailed predictions of Eq. (4) are not to be trusted. They strongly depend on terms which we have neglected in Eqs. (1) and (2), and on the shape of $\beta(t)$.

(iii) $b\approx 1$. This is the intermediate impact-parameter region in which the scattering process probes the outer regions of the interaction. In this region the phase in Eq. (4) rotates rapidly and gives rise to resonances.

We proceed to investigate the properties of resonances appearing in region (iii). The l dependence of the resonance energy can be obtained by equating the only rapidly varying phase in Eq. (4) to $\pm\frac{1}{2}\pi+2n\pi$:

$$-\pi[b\alpha_1+\alpha_0]=\pm\tfrac{1}{2}\pi+2n\pi. \quad (6)$$

The sign depends on whether g is positive or negative. Picking $g<0$, we obtain for the first resonance

$$b\alpha_1+\alpha_0=\tfrac{3}{2}\pi \quad (7)$$

or

$$(l+\tfrac{1}{2})^2=4(\tfrac{3}{2}-\alpha_0)k^2[(c+\alpha_1\ln s)^2+\alpha_1{}^2\pi^2] \quad (8)$$

which for a crude parametrization of the ρ trajectory, $\alpha_0=\frac{1}{2}$, $\alpha_1=1$, and $c=0$, gives

$$l+\tfrac{1}{2}=[s(\pi^2+\ln^2 s)]^{1/2}. \quad (9)$$

This analytic form should only be valid for large l and s; at lower s, trajectories can be and are approximately linear in s.[11]

Next the total width of the resonance can be obtained by considering the range in energy over which the phase changes from 0 to π. Neglecting logarithmic terms we obtain $\Gamma\sim\sqrt{s}$. This is consistent with the width of a trajectory with Reα $\sim\sqrt{s}$ as obtained in Regge theory.[12]

The elastic width of the resonance is connected to the radius of the circle in the Argand diagram. For the trajectory obtained in the above parametrization,

$$\Gamma_{\rm el}\sim s^{\alpha_0-1}s^{-\alpha_1 b}\Gamma_{\rm tot} \quad (10)$$

or $\Gamma_{\rm el}\sim s^{-1}$. This gives an elasticity which decreases like $s^{-3/2}$.

There are more apparent resonances for $b=\frac{7}{2}$, $\frac{11}{2}$, etc. These are suppressed in Eq. (4), by the factor $e^{-b\ln s}$ appearing in that equation. Their contribution is of the order of terms which we have neglected in Eqs. (1) and (2) and they may be cancelled by these correction terms. We cannot be certain about their interpretation.

It is unlikely that these resonances can be created by an infinite family of linearly rising trajectories. Such trajectories rise more steeply than $\sqrt{s}$. To create a series of resonances with l $\sim\sqrt{s}$ and no resonances for $l\ll\sqrt{s}$, linear trajec-

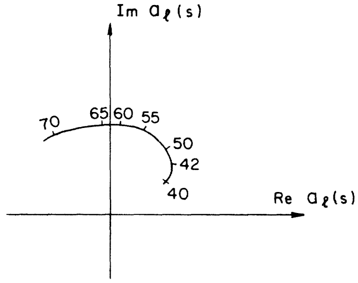

FIG. 1. Argand diagram, arbitrary scale, for $l=40$. The parameters used are $\alpha_0=0.5$, $\alpha_1=1$, $c=0$, $g<0$, and $s_0=1$. All units are in $(\mathrm{BeV})^2$.

tories should have no coupling to $\pi\pi$ at low l [region (i), $l \ll \sqrt{s}$], strong coupling to $\pi\pi$ when $l \sim \sqrt{s}$ [region (iii)], and no coupling again at $l \gg \sqrt{s}$ [region (ii)]. Such behavior seems unlikely. We can not rule out a few linearly rising trajectories which at high s will no longer couple to $\pi\pi$ because of their vanishing elasticity. These will not be noticed at all in our high-s analysis.

So far we have discussed the contribution of a single trajectory for spinless particles. In most scattering processes there are additional complications. Consider first isospin: The resonances we have discussed have unique isospin in the t channel. By crossing, these correspond to a combination of several isospin states in the s channel which have identical energy and width. A single t-channel trajectory thus gives rise to a "generalized exchange degeneracy" in the s channel. In a similar way the condition that s-channel resonances are not exchange degenerate demands a cancellation between two t-channel trajectories and thus t-channel degeneracy.[13] If we demand that no $I=2$ resonances exist, we learn that the Pomeranchuk trajectory, which contributes to $I=2$ and cannot be cancelled by any other trajectory, must contain no resonances when projected in the s channel. It must therefore have no zeros in β, and thus no resonances arise.

Spin complications are similar to isospin because of the crossing of helicity amplitudes. We thus have either nontrivial relations between s-channel helicities and spins, or connections between t-channel helicities. We will elaborate these points further elsewhere. When no cancellation occurs we will also find degenerate trajectories of opposite signature.

In our discussion we have left open the question whether or not the resonances are "real." We can not continue our expressions to the unphysical sheet to check for poles in the S matrix. We are left, thus, with only two clues to the resonance nature.

(a) Do the residues of these resonances factorize in the s channel? This is not a trivial question and we can give no answer yet.[14]

(b) Can we produce these resonances in reactions like $\pi p \rightarrow \pi\pi p$? A recent paper by Chew and Pignotti[15] gives hopes that a double-Regge-exchange mechanism will give rise to such resonances in the final $\pi\pi$ state.

Our argumentation about curving trajectories does not exclude the possibility of approximate saturation, at low energies, with linear trajectories, as suggested by Mandelstam.[16] The curving trajectories we have discussed do not get into conflict with the boundedness of the amplitude as discussed by Khuri.[17]

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[1]For latest compilation see A. H. Rosenfeld et al., Rev. Mod. Phys. 40, 77 (1968).

[2]C. Schmid, Phys. Rev. Letters 20, 689 (1968).

[3]We assume that such circles correspond to resonances. Possible checks of this assumption are discussed later.

[4]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. Letters 19, 402 (1967), and Phys. Rev. 166, 1768 (1968).

[5]H. R. Rubinstein, A. Schwimmer, G. Veneziano, and M. A. Virasoro, to be published.

[6]Higher Transcendental Functions, edited by H. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 1, p. 147, Eqs. (3), (5), and (10).

[7]This will change the result by terms proportional to $s^{-\alpha(4k^2)}$ which are negligible.

[8]W. Magnus, F. Oberhettinger, and R. P. Soni, Formulas and Theorems of Special Functions (Springer-Verlag, Berlin, Germany, 1966), p. 93. The integral in Eq. (3) could be performed exactly, and expressed in terms of a combination of modified spherical Bessel functions. This gives the same result. The equations, however, become slightly more cumbersome.

[9]This was used as a foundation for a series of sum rules for partial wave amplitudes: M. Kugler, Phys. Rev. Letters 17, 1166 (1966), and Phys. Rev. 160, 1574 (1967).

[10]C. E. Jones and V. L. Teplitz, Phys. Rev. Letters 19, 135 (1967); N. N. Khuri, Phys. Rev. Letters 18, 1094 (1967).

[11]For numerical analysis see Refs. 2, 5, and P. D. B. Collins, R. C. Johnson, and E. J. Squires, Phys. Letters 27B, 23 (1968). This paper presents an intuitive argument for the linear rise of trajectories at low energy. This argument should not be applied at high s. I am grateful to Dr. Collins and Professor Squires for enlightening remarks.

[12]The width is given by $\Gamma = 2\,\mathrm{Im}\alpha\,\mathrm{Re}\alpha' |\alpha'|^{-2}$, where $\alpha(s)$ is the Regge trajectory and $\alpha'(s) = d\alpha/d\sqrt{s}$.

[13]For a detailed discussion from a different viewpoint, see H. Harari, to be published.

[14]This also involves details about the factorization of t-channel trajectories, an experimentally unverified point.

[15]G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

[16]S. Mandelstam, Phys. Rev. 166, 1539 (1968). M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. Letters 19, 1402 (1967).

[17]Khuri, Ref. 10. There still remains the difficulty associated with the analytic structure of $\alpha(s)$. Conceivably this is overcome by α having more than a right-hand cut.