NEW GEOMAGNETIC LIMIT ON THE MASS OF THE PHOTON

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On the basis of recent precise data on the earth's magnetic field, a new upper bound on the mass of the photon is obtained using a method that was originally proposed by Schrödinger. The limit is 1.15×10^{-10} cm^{-1} $\equiv 2.3\times10^{-15}$ eV $\equiv 4.0\times10^{-48}$ g, which is four orders of magnitude better than the best laboratory measurement, that of Plimpton and Lawton.

In conventional electrodynamics the mass of the photon is assumed to vanish. However, a finite photon mass may be accomodated in a unique way by changing the inhomogeneous Maxwell equation to the Proca equation,

$$\partial^\lambda F_{\lambda\nu} + \mu^2 A_\nu = (4\pi/c)J_\nu, \tag{1a}$$

where

$$F_{\lambda\nu} = \partial_\lambda A_\nu - \partial_\nu A_\lambda. \tag{1b}$$

If electric charge is conserved, we may rewrite Eq. (1) as

$$(\Box + \mu^2)A_\nu = (4\pi/c)J_\nu \tag{2a}$$

and

$$\partial^\lambda A_\lambda = 0. \tag{2b}$$

In the limit of static charge and current distributions, this yields

$$(\nabla^2 - \mu^2)\Phi = -4\pi\rho$$

$$(\nabla^2 - \mu^2)\vec{A} = -4\pi\vec{J}/c, \tag{3}$$

with $\vec{E} = -\nabla\Phi$ and $\vec{H} = \nabla\times\vec{A}$.

The mass μ introduces a natural scale of length for electrostatics. Obviously for a given precision of field measurement, the larger the distance between measurement points, the better is the mass limit obtained. The best laboratory limit of μ to date is that of Plimpton and Lawton,[1] who tried to detect a voltage difference ΔV between a conducting sphere raised to a high potential V and a second concentric uncharged sphere contained in the first one. From Eq. (3), one has

$$\Phi(r) \propto (e^{\mu r} - e^{-\mu r})/2\mu r \tag{4}$$

between the two spheres, and hence

$$\Delta V/V = \tfrac{1}{6}\mu^2(a^2 - b^2) + O((\mu a)^4), \tag{5}$$

where a and b are the radii of the two spheres. The result of Plimpton and Lawton gives

$$\begin{aligned}\mu &\lesssim 9.8\times10^{-7}\ \mathrm{cm}^{-1}\\ &\equiv 1.9\times10^{-11}\ \mathrm{eV}\\ &\equiv 3.4\times10^{-44}\ \mathrm{g}.\end{aligned} \tag{6}$$

Another method, first proposed by Schrödinger,[2] is to use measurements of the earth's magnetic field. From Eq. (3), the dipole ($\vec{m}$) contribution to the earth's field, measured from the center of the dipole, becomes

$$\vec{H} = (e^{-\mu r}/r^3)[(1 + \mu r + \tfrac{1}{3}\mu^2 r^2)\times(3\vec{m}\cdot\hat{r}\hat{r} - \vec{m}) - \tfrac{2}{3}\mu^2 r^2 \vec{m}]. \tag{7}$$

For magnetic field measurements near the surface of the earth ($r \simeq R =$ const) the principal observable effect of the photon mass is an apparent constant "external" magnetic field antiparallel to the direction of the dipole. The ratio of the "external" field (H_{ext}) to the dipole field at the equator ($H_{\mathrm{D.E.}}$) is

$$H_{\mathrm{ext}}/H_{\mathrm{D.E.}} = \tfrac{2}{3}(\mu R)^2/[1 + \mu R + \tfrac{1}{3}(\mu R)^2], \tag{8}$$

where R is the radius of the earth.

Recently Cain[3] has obtained a very careful fit to geomagnetic data from earthbound and satellite measurements. For epoch 1960.0, he obtains values[4,5] of

$$\begin{aligned}&m = 31\,044\gamma R^3,\\ &\vec{H}_{\mathrm{ext}}\cdot\hat{m} = (21\pm5)\ \gamma,\\ &\vec{H}_{\mathrm{ext}}\cdot\hat{y} = \vec{H}_{\mathrm{ext}}\cdot[(\hat{s}\times\hat{m})/|\hat{s}\times\hat{m}|] = (14\pm5)\ \gamma,\\ &\vec{H}_{\mathrm{ext}}\cdot\hat{y}\times\hat{m} = (8\pm5)\ \gamma,\end{aligned} \tag{9}$$

where $1\ \gamma = 10^{-5}$ G and $\hat{s}$ points towards the south geographic pole. The errors in the values of components of $\vec{H}_{\mathrm{ext}}$ are statistical.

To obtain our mass limit from this number, we must take into account fields due to "true external" sources. Cole[6] has pointed out that there are three appreciable external contributions tending to decrease the "quiet" magnetic field at the equator, and hence produce an external field parallel to the dipole moment. He estimates that they are ~9 γ from the quiet-time proton belt, perhaps 15-30 γ due to currents in the geomagnetic tail,[7] but probably less,[6] and ~5 γ from the hot component of the plasma in the magnetosphere. Recently, Frank[8] has obtained the first reliable data on the hot plasma, showing that it is the primary contributor to the "ring current" which causes a decrease in the magnetic field at the equator during geomagnetic storms. He estimates the quiet-time decrease due to the proton component of this plasma at ~12 γ. Since one may infer an additional effect due to electrons, this changes the estimate of the plasma contribution to ~15 γ, instead of Cole's value of ~5 γ.

There is at least one true external field in the opposite direction (antiparallel). Parker[9] notes that the solar wind compresses the geomagnetic field, increasing the field at the equator by ~20 γ. Finally, the interplanetary field of[10] ~5 γ points in an unknown direction at the earth's surface.

Thus, the total external field parallel to $\vec{m}$ due to known sources is $\lesssim 40\ \gamma$. Subtracting this from $\hat{m}\cdot\vec{H}_{\text{ext}}$ in Eq. (9) gives us an upper limit on the antiparallel external field which could be due to a finite photon mass, $H_{\text{ext}}(\text{antiparallel}) \lesssim 20\ \gamma$.

The significance of this limit depends crucially on the reliability of the fit of Cain to the geomagnetic field. There are several ways of estimating this. First of all, the existence in Eq. (9) of components perpendicular to $\hat{m}$ is hard to explain. If we take them as spurious, the corresponding errors in the parallel component could be several tens of γ.[11] However, other features of the fit are more disturbing than this. There appears to be an irredicible "noise" in data from earthbound observatories of about 100 γ, in large part due to magnetic anomalies in the earth's crust.[12] Furthermore, the available earthbound data are very sparse in Asia and in much of the southern hemisphere.[13] Despite this, the fit is made by an expansion in spherical harmonics of a "potential" whose gradient gives $\vec{H}$, even though these harmonics form a complete orthonormal set only over an entire sphere. This fact clearly permits ambiguities in the fit. Spacecraft data cannot be trusted to clear up the picture even though they have a much lower noise level. (The rms error of a fit to the latest OGO satellite data is 7-8 γ.[12,14] This is because (1) these satellites only measure the magnitude of $\vec{H}$, not its direction,[12] and (2) there are peculiar secular (time) variations of the fits to the satellite data of the order of tens of γ.[14]

For these reasons, the possibility of systematic errors in the fits to a particular spherical-harmonic coefficient of many tens of γ cannot be excluded. To take account of this, and any errors in the estimates of true external fields, we add 100 γ to our estimate, meaning that the left-hand side of Eq. (8) is less than 4×10^{-3}. Simple numerical work then gives us a limit on the mass of the photon of

$$\begin{aligned} \mu &\leq 1.15\times10^{-10}\ \text{cm}^{-1} \\ &\equiv 2.3\times10^{-15}\ \text{eV} \\ &\equiv 4.0\times10^{-48}\ \text{g}, \end{aligned} \tag{10}$$

which is four order of magnitude better than the Plimpton-Lawton number. It is five times better than the number that Schrödinger suggested[15] on the basis of much less precise and very much less reliable data. (Note that the photon Compton wavelength is $2\pi/\mu = 5.5\times10^{10}\ \text{cm} = 81R$.)

One might hope to use satellite measurements at varying altitudes to detect the exponential decay of $\vec{H}$. From Eq. (7), the magnitude of the dipole field for $\mu \neq 0$ is

$$\begin{aligned} H_D(\mu) &= \{1+\tfrac{1}{2}(\mu r)^2 \\ &\quad\times[(1-5\cos^2\theta)/(1+3\cos^2\theta)]\}H_D, \\ &\equiv F(\mu, r, \theta)H_D, \end{aligned} \tag{11}$$

where $\cos\theta = \hat{m}\cdot\hat{r}$, H_D is the magnitude of the orthodox dipole field, and cubic terms in μr are neglected. Ginzburg,[16] in an otherwise excellent paper, used the factor $F = 1-(\mu r)^2$ instead of the above in order to obtain a mass limit from magnetic measurements at varying altitudes by Vanguard, Explorer, and Pioneer satellites. He gives a limit $\mu < 3\times10^{-48}$ g, and states that this may be too low by a factor of 2 or 3. After reconsidering his numbers in the light of Eq. (11), we feel that the same conservative error estimation that we have used would make Ginzburg's limit $(8\text{-}10)\times10^{-48}$ g. Thus, it is nearly a geometric mean between the old and new results of the Schrödinger method. The main limitation on the altitude-dependent method is that external perturbations become quite significant beyond ~$3R$.[8] Clearly, a combination of altitude- and latitude-dependent data would yield a more stringent limit than either alone, and might even be used to separate the mass effect, which falls off as r^{-1}, from true external fields, which are nearly constant.

One may ask what the ultimate limits of the present and other methods of measuring μ are. It seems quite likely that improvements in geomagnetic data could reduce our result by a factor 2, and perhaps down to 10^{-48} g.[17] Such efforts should be encouraged.

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[4]In consulting the literature, care should be taken to differentiate between the sign conventions used for the spherical harmonics and between "north-seeking" and "north" poles. See, for example, Table 3 in P. F. Fougère, J. Geophys. Res. 70, 2171 (1965). Also see J. C. Cain, S. J. Hendricks, R. A. Langel, and W. V. Hudson, J. Geomag. and Geoelect. 19, 335 (1967), Appendix. This is a later fit than that of Ref. 3, but it does not calculate possible external terms.

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[9]E. N. Parker, in Physics of Geomagnetic Phenomena II, edited by S. Matsushita and W. H. Campbell (Academic Press, Inc., New York, 1967), p. 1154, Sec. 4.1.

[10]Pioneer V data showed that the quiet interplanetary field at ~1.0 A.U. is $5.0 \pm 0.5\ \gamma$. See E. W. Greenstadt, Astrophys. J. 145, 270 (1966). Also, it is well known that the interstellar field is much less than $1.0\ \gamma$. See R. D. Davies et al., Nature 187, 1088 (1960), and F. G. Smith, Nature 218, 325 (1968).

[11]This is borne out by the somewhat less precise fit of L. Hurwitz, D. G. Knapp, J. H. Nelson, and D. E. Watson, J. Geophys. Res. 71, 2373 (1966). This is the only recent fit aside from Ref. 3 to include parameters for an external field. The results for the three components of the external field defined in Eq. (3) were +85, −17, and +8.

[12]Cain et al., Ref. 4.

[13]Cain, Ref. 3.

[14]J. C. Cain, private communication.

[15]In his original paper (Ref. 2) Schrödinger referred to a photon rest mass of 10^{-47} g. However, he later felt that the deviations from Gauss's laws in the surveys he used were possibly spurious, and so he took twice this, $\sim 2.0 \times 10^{-47}$ g, as a safe upper limit to the mass. See L. Bass and E. Schrödinger, Proc. Roy. Soc. (London), Ser. A 232, 1 (1955).

[16]M. A. Ginzburg, Astron. Zh. 40, 703 (1963) [translation: Soviet Astron.−AJ 7, 536 (1964)]. In this paper, Ginzburg also suggested a method based on the observation of long-wavelength magnetohydrodynamic oscillations within the magnetosphere. This method was applied by V. L. Patel, Phys. Letters 14, 105 (1965). He obtained $\mu \lesssim 10^{-47}$ g with an uncertainity of one or two orders of magnitude.

[17]Ginzburg (Ref. 16) gives a fine summary of methods based on detecting frequency dispersion in the velocity of light by astronomical objects within our galaxy. None of these methods can compete with the Plimpton-Lawton result. This applies even to the precisely timed signals from pulsars, if only because dispersion is also introduced by electrons in the interstellar medium. See B. S. Tanenbaum, G. A. Zeissig, and F. D. Drake, Science 160, 760 (1968).