PARTICLE CREATION IN EXPANDING UNIVERSES

L. Parker
University of North Carolina, Chapel Hill, North Carolina
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We report results of an investigation of elementary particle creation in expanding universes.

If one considers the covariant quantized free-field equations of elementary particles in an expanding universe, then it becomes evident that the particle number is not a constant of the motion, but only an adiabatic invariant, even though the gravitational field is not quantized. Therefore, a creation of elementary particles will take place in a universe with no particles or a small particle density initially present. We wish to report here on the results of an extensive investigation into the particle creation in expanding universes.[1] This and related topics are presently being investigated also by other authors.[2-6] However, the present author's thesis[1] appears to be the earliest investigation in print of the particle-creation rate in expanding universes, and it is the results obtained there that we wish to report. We will present only the main ideas and results, since it would be impossible to do more in a short article. A detailed exposition will be published in a series of articles elsewhere, and may be found in Ref. 1.

The covariant equations for quantized fields of spin 0 and spin $\frac{1}{2}$ were considered for arbitrary mass, while for zero mass the covariant equations for particles of higher spin were also considered. The main investigation was carried out for metrics of the form

$$ds^2 = -dt^2 + R(t)^2(dx^2 + dy^2 + dz^2). \tag{1}$$

It was shown that the creation rate would be of the same order of magnitude for closed universes with

$$ds^2 = -dt^2 + R(t)^2(d\psi^2 + \sin^2\psi d\theta^2 + \cos^2\psi d\chi^2). \tag{2}$$

Particle creation occurs because, according to the covariant equations for the fields in the Heisenberg picture, the positive- and negative-frequency parts of the fields become mixed during the expansion of the universe, so that the creation and annihilation operators at one time t are linear combinations of those at an earlier time. The particle-creation rate was investigated mainly at the present stage of the expansion, for which $\dot{R}(t)/R(t) \sim H$, where H is Hubble's constant. Upper bounds were found for the creation rate per unit volume of various elementary particles. These are extremely small because the particle number is an adiabatic invariant and the present expansion rate is small. Because the creation rate is presently so small, the reaction of the particle creation back on the gravitational field could be neglected. The creation rate in the early stages of a Friedmann-type expansion is also under investigation. This primeval creation rate was certainly much larger than at present, although exact quantitative results are not yet available for the early stages of the expansion. Some results for the present stage of the expansion are reported below.

π-meson creation.—The equation governing the spin-0 field was the Klein-Gordon equation with covariant derivatives ∇_j in place of ordinary derivatives[7]:

$$(g^{jk}\nabla_j\nabla_k - m^2)\varphi = 0.$$

With g_{jk} given by Eq. (1) [or by Eq. (2)], the space- and time-dependent parts of the equation are separable. Making use of the spatial isotropy and homogeneity of the problem, we were able to define time-independent modes k, and to expand the field at any time in terms of its Fourier components with respect to k. As a consequence of the dynamical equations, the time-dependent Fourier coefficients could be consistently required to obey at all times the usual commutation relations for annihilation and creation operators, indicating a unitary time development.

To find the correct operator corresponding uniquely to the observable particle number in the present stage of the expansion, we required that the particle-number operator satisfy the following conditions: (a) It is Hermitian, its eigenvalues being the non-negative integers (since a direct measurement of the particle number generally involves a counting procedure, which can only yield a non-negative integer result). (b) When the expansion is stopped slowly, the operator becomes the well-defined particle-number operator for the static universe. (c) It should be measured in the slowly expanding universe by essentially the same apparatus as in the static case. Thus, for example, it should be free of extremely high-frequency oscillations which are in principle unobservable by such apparatus.

In order to satisfy these requirements, especially the third, we found it necessary to use an approximation procedure based upon a comparison of the separated time-dependent equation for the field with the differential equation satisfied exactly by the adiabatic solution of the first equation.[8] The method of approximation is also useful in other problems involving adiabatic invariants.[9] To within the degree of approximation, the particle-number operator could be shown to be unique. The creation rate can be determined by comparing the particle number at two times separated by an interval large with respect to the time needed for a single measurement of the particle number [which must be large compared with the inverse particle mass ($\hbar = c = 1$) to avoid creating particles in the measuring process]. Since the particle number is an adiabatic invariant of the motion, the particle-creation rate in the present slow expansion of the universe is extremely small, although not zero. We obtained an upper bound on the expectation value of the present creation rate per unit volume summed over all modes, which depends on the present density of matter in the universe ($\lesssim 10^{-29}$ g/cm^3), on the present rate of expansion ($H \sim 10^{-17}$ sec^{-1}), and on the mass of the particle involved. For π mesons the upper bound is 10^{-105} g cm^{-3}. This corresponds to less than one π meson per second in the observable universe ($\sim 10^{81}$ cm^3).

Some additional interesting results are these: (1) The particles are created in pairs with net momentum zero, equal amounts of matter and antimatter being created. (2) Attenuation of the momentum and energy of a free particle takes place as the universe expands, just as predicted by classical general relativity. (3) In the Friedmann universe with metric (1) in which radiation is predominant $[R(t) \propto t^{1/2}]$, exactly no creation of mass-zero mesons is predicted, while in the Friedmann universe in which matter is predominant $[R(t) \propto t^{2/3}]$ exactly no creation of massive mesons is predicted in the limit of infinite mass.

Spin-$\frac{1}{2}$ particles. – The equation governing the free fields was the general relativistic Dirac equation:

$$\gamma^k \nabla_k \psi = \mu \psi$$

in the notation of Bargmann.[10] A calculation analogous to that in the scalar case, although somewhat more involved, showed that results (1) and (2) above also hold for spin-$\frac{1}{2}$ particles. An upper bound on the present expectation value of the creation rate per unit volume was also obtained. For protons the upper bound is 10^{-64} g cm^{-3} sec^{-1}, and for electrons it is 10^{-69} g cm^{-3} sec^{-1}. The largest of these creation rates is less than one particle per liter of volume every 10^{30} years. For massless neutrinos there is no creation through the mechanism considered here.

Particles of zero mass and nonzero spin. – The covariant equations governing the massless fields of nonzero spin s were expressed in the two-component spinor formalism following Penrose.[11] In his notation, the equation is

$$\nabla^{\nu_1 \dot{\sigma}} \xi_{\nu_1 \nu_2 \cdots \nu_{2s}} = 0. \tag{5}$$

These equations, which include Maxwell's equation for $s = 1$, are conformally invariant under the transformation

$$g_{jk} \to \tilde{g}_{jk} = \Omega^{-2} g_{jk},$$

$$\xi_{\nu_1 \cdots \nu_{2s}} \to \tilde{\xi}_{\nu_1 \cdots \nu_{2s}} = \Omega^{s+1} \xi_{\nu_1 \cdots \nu_{2s}},$$

where Ω is an arbitrary function of position and time.[11] As a consequence of the conformal invariance, it can be shown that the positive- and negative-frequency parts of the field are not mixed by an expansion of the universe, so that there is no creation of massless particles of nonzero spin.

Conclusion. – The particle creation in the expanding universe at the present time resulting from the mechanism considered here is quite negligible. However, for the early stages of a Friedmann expansion it may well be of great cosmological significance, especially since it seems inescapable if one accepts quantum field theory and general relativity. In considering the large amount of particle creation taking place in the early stages of an expansion, it is necessary to take into account the reaction of the matter created back on the gravitational field. Furthermore, it may be necessary to consider the effects of the quantization of the gravitational field. Therefore, no conclusive quantitative result can yet be reported here concerning the primeval creation. It should be emphasized that equal amounts of matter and antimatter would be created by this mechanism. This factor would tend to favor a cosmological theory with equal amounts of matter and antimatter, such as the theory of Alfvén and Klein.[12]

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LORENTZ POLES IN EQUAL-MASS SCATTERING FROM ANALYTICITY AND FACTORIZATION*

J. B. Bronzan and C. Edward Jones
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 24 June 1968)

It is shown, by means of factorization, that the daughter Regge trajectories implied by analyticity in unequal-mass scattering amplitudes constitute a single Lorentz pole when they couple to an equal-mass channel. At zero energy the equal-mass elastic amplitude has the form const $\times D_{-\alpha(0)-2}^{1}(-z)$.

In the past few years, two different approaches have been used to study the properties of Regge poles and S-matrix elements at zero total energy ($u=0$). The first involves analyticity[1] and the second uses group theory.[2] The analyticity approach can be employed when the particles in the u channel have unequal masses (like pion-nucleon scattering). In this case an apparent conflict arises between the presence of poles in the angular-momentum plane and the requirement of analyticity in the total energy at $u=0$. The conflict is resolved by introducing a set of daughter Regge trajectories at $\alpha(0)-k$ ($k=1, 2, \cdots$), where $\alpha(0)$ is the leading or parent trajectory.

If the masses in the u channel are pair-wise equal, the foregoing conflict with analyticity does not arise, but one may invoke the O(4) [or O(3, 1)] symmetry of the scattering amplitude which results from the vanishing at $u=0$ of Q_u, the total center-of-mass four-momentum in the u channel. Poles in the u channel are classified according to irreducible representations of O(4), and then each such Lorentz pole $n(0)$ gives rise to a sequence of daughter Regge poles with the parent trajectory located at $\alpha(0)=n(0)$. The daughter sequence lies below $\alpha(0)$ with a spacing of two units, $\alpha(0)-2n$ ($n=1, 2, \cdots$), if the scattering particles are spinless, and a spacing of one unit if the particles have spin. The ratios of the residues of the daughter trajectories are also determined by expanding the Gegenbauer function in terms of Legendre functions. On the other hand, the group theoretic approach cannot be immediately applied to the scattering amplitudes for unequal-mass particles, essentially because the center-of-mass three-momentum becomes infinite at $u=0$.

It is naturally desirable to attempt to bridge the gap between the analyticity and group theoretic treatments of daughter trajectories, and to see if the analyticity and group-theory daughters are really the same in some sense. Some insight on this question has been achieved by relying heavily on sophisticated group theoretic techniques.[3] However, it would be nice to have a discussion which embodies directly the tools of S-matrix theory, such as analyticity and factorization of residues. It is the purpose of this note to provide such a discussion and to prove that the daughters deduced from analyticity in unequal-