the last point shown in Fig. 1, there are nine radial maxima in the transverse intensity profile.

It is clear from the oscillations in the on-axis intensity that the central maximum is not trapped in a stationary transverse mode. The behavior of this portion of the beam is more characteristic of the periodic focusing predicted for saturable media by the paraxial-ray analysis. This analysis predicts that for weak oscillations of the beam radius about its steady-state value, successive minima in the on-axis intensity should be separated by[9]

$$z_S = \pi k a_S^{\,2} \frac{(P/P_{\rm cr})^{1/4}}{[(P/P_{\rm cr})^{1/2}-1]^{3/2}},$$

where $(a_S/a_0)^2 = 0.273(P/P_{\rm cr}E_S{}^{*2})$. Fitting the central maximum with a Gaussian, for which this formula is valid, we find $z_S{}^* \approx 3\times10^{-3}$ which is to be compared with the distance $\approx 2\times10^{-3}$ between the two prominent minima in Fig. 1.

The central peak possesses many of the observed properties of small-scale trapping: It persists beyond the focus, is very intense, and contains somewhat more than critical power. To estimate its size for realistic saturation fields, we may use either the paraxial-ray theory or the "exact" stationary profile theory, both of which yield characteristic transverse radii which scale as $E_S{}^{*-1}$. Thus for $E_S{}^* = 10^4$ we expect a central peak of width $r^* \approx 5\times10^{-4}$ or about 0.5 μ for a 1-mm incident beam. This underestimate of observed filament sizes (~5-10 μ) seems endemic to theories including only saturation of the nonlinear index.[5] To the list of other mechanisms usually invoked to explain this discrepancy we may add that some of the rings near the center may be experimentally unresolved or blurred by other effects, giving an apparent filament size much larger than that of the central peak.

We are grateful to Dr. P. L. Kelley for providing us with information regarding his computing scheme and to Dr. W. G. Wagner for his advice and encouragement. Mr. L. Huff and Mr. R. Reynolds also assisted with the numerical computation.

---

*Work supported by the U. S. Army Research Office, Durham, Grant No. DA-ARO-D-31-124-G920 and Joint Services Electronics Program (U.S. Army, Navy, Air Force) Grant No. AF-AFOSR-496-67.

†National Science Foundation Fellow.

[1]R. Y. Chiao, M. A. Johnson, S. Krinsky, H. A. Smith, C. H. Townes, and E. Garmire, IEEE J. Quantum Electron. QE-2, 467 (1966).

[2]R. Y. Chiao, E. Garmire, and C. H. Townes, Phys. Rev. Letters 13, 479 (1964).

[3]H. A. Haus, Appl. Phys. Letters 8, 128 (1966).

[4]Z. K. Yankauskas, Zh. Eksperim. i Teor. Fiz. 5, 335 (1967) [translation: Soviet Phys.—JETP 5, 275 (1967)].

[5]T. K. Gustafson, P. L. Kelley, R. Y. Chiao, and R. G. Brewer, Appl. Phys. Letters 12, 165 (1968).

[6]J. D. Reichert and W. G. Wagner, IEEE J. Quantum Electron. QE-4, 221 (1968).

[7]Yu. P. Raizer, Zh. Eksperim. i Teor. Fiz. 4, 3 (1966) [translation: Soviet Phys.—JETP 4, 1 (1966)].

[8]P. L. Kelley, Phys. Rev. Letters 15, 1005 (1965).

[9]W. G. Wagner, H. A. Haus, and J. H. Marburger (to be published).

[10]We use for $P_{\rm cr}$ the definition of Ref. 2.

[11]Y. R. Shen, M. Y. AuYang, and M. L. Cohen, Phys. Rev. Letters 19, 1171 (1967).

$K_2{}^0 \to 2\pi^0$ DECAY RATE*

D. F. Bartlett, R. K. Carnegie, V. L. Fitch, K. Goulianos, D. P. Hutchinson, T. Kamae,† R. F. Roth,‡ J. S. Russ,§ and W. Vernon,‖

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received 31 May 1968)

The ratio $|\eta_{00}|^2$ of the rate of decay $K_2{}^0 \to 2\pi^0$ to the decay rate $K_1{}^0 \to 2\pi^0$ has been measured to be $|\eta_{00}|^2 = (-2\pm7)\times10^{-6}$.

The main interest in $|\eta_{00}|^2$, the ratio of the rate of decay $K_2{}^0 \to 2\pi^0$ to the decay rate $K_1{}^0 \to 2\pi^0$, is the extent to which it departs from $|\eta_{+-}|^2$, the corresponding ratio for the decay to $\pi^+\pi^-$. Any difference is a measure of the amount of $\Delta T \geq \frac{3}{2}$ amplitude present in CP nonconservation.[1] A survey[2] gives $|\eta_{+-}|^2 = (3.6\pm0.2)\times10^{-6}$. Recent measurements[3,4] of the neutral mode have given[5] $|\eta_{00}|^2 = (18^{+11}_{-6})\times10^{-6}$ and[6] $(24\pm5)\times10^{-6}$, apparently demonstrating a difference between $|\eta_{+-}|^2$ and $|\eta_{00}|^2$. We report a new measurement with the result $|\eta_{00}|^2 = (-2\pm7)\times10^{-6}$.

The general idea of this experiment, similar to that at CERN,[5] is to compare the number of $2\pi^0$'s originating from $K_2{}^0$ decay with the number from the $K_1{}^0$'s produced by regeneration. Under these circumstances delicate factors such as absolute detection efficiencies cancel out and one obtains the relative rates from

$$|\eta_{00}|^2 = I_R(\Lambda_1/L)E(N_F/N_R),$$

where I_R is the $K_1{}^0$ regeneration probability, Λ_1 is the mean decay length of the $K_1{}^0$, L is the length of the decay volume of the $K_2{}^0$, E is the relative efficiency for detecting the regenerated and free-decay events, N_F and N_R are the normalized number of free-decay and regenerated events, respectively. The main problem in the experiment is distinguishing the relatively rare $2\pi^0$ decay mode from the $3\pi^0$ mode which occurs with a branching ratio of 23%.

The experiment was performed at the Princeton-Pennsylvania 3-GeV accelerator. A neutral beam from the Pt target was defined by collimators and a sweeping magnet at 29° relative to the circulating protons. The beam was 2.5 in. wide and 11 in. high at the mean detector distance of 64 ft from the target. To detect the photons from π^0 decays, six spark chambers were arrayed on each side of the beam as shown in Fig. 1. Each chamber consisted of 11 sheets of 18-in. $\times$ 36-in. $\times$ $\frac{1}{16}$-in. stainless steel spaced at intervals of $\frac{3}{8}$ in. and was placed with its normal at 50° with respect to the beam. To detect photon-generated showers two scintillation counters (18 in. $\times$ 18 in. $\times$ $\frac{1}{4}$ in.) were placed following each of the chambers, one above and one below the median plane. The counters above and below and to the right and left of the beam form a quadrant array; the six counter outputs in each quadrant were added together. Photon anticoincidence counters made of alternate layers of Pb and scintillator surrounded most of the decay volume not covered by spark chambers. Charged-particle anticoincidence counters ($\frac{1}{4}$ in. thick) separated the decay volume from the spark chambers. To trigger the spark chambers we required signals from any three out of the four banks of shower counters, an integrated amplitude equivalent to seven or more particles in all quadrants, and no accompanying anticoincidence signal.

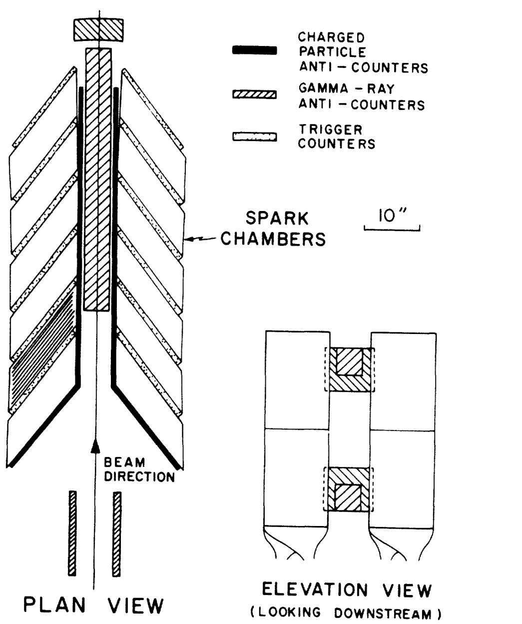

FIG. 1. Plan view of the experimental apparatus. An elevation view of the counter arrangement is shown in the insert.

The momentum of the K meson was determined using its time of flight from the target. The arrival of the proton bunches on the target (~1 nsec duration) was signaled by a Cherenkov counter placed 4 ft away. The K meson at the detector was signaled by the shower counters. The elapsed time was digitized and displayed. As a check of internal consistency, two independent timing measurements were made using the shower detectors on the right and left sides of the beam. From the spark-chamber photographs the times were corrected according to the particular counter in which the shower first appeared and for the position of the shower in the counter. The overall timing resolution of ±1.2 nsec gave a momentum resolution of 5% at 730 MeV/c, the mean momentum of the detected K's.

All photographs were measured in which four showers, two on each side of the beam, appeared to come from a common origin. The angle of each of the photons and the momentum of the K permit a two-constraint fit to the $K^0 \to 2\pi^0$ hypothesis. We transform to the K-meson c.m. system and assign a π^0 to the two photons on each side of the beam, an assignment correct 80% of the time for valid decays. We characterize the decay by the smaller of the opening angles of the two pairs of γ rays and by the angle between the possible pion directions.

The distribution of the smaller of the two opening angles, θ_S, is sharply peaked at the minimum angle, 1.15 rad. With infinite resolution, 50% of the decays have θ_S between 1.15 and 1.2 rad and 80% between 1.15 and 1.3 rad. Furthermore, the opening angles are relatively insensitive to momentum (i.e., timing) errors.

There are two possible directions in the plane of the photons for each π^0, taking its energy to be $m_K/2$. On each side that direction was chosen which was closest to the line of intersection of the two planes, a unique solution for $2\pi^0$ decay. If the opening angle was less than 1.15 rad, the bisector was used as the pion direction. We define the collinearity to be the cosine of the angle, $\theta_{\pi\pi}$, between the π^0 directions determined separately on the two sides.

The regeneration data, obtained by placing a 4-in.-thick piece of tungsten at five successive positions in the decay volume, are shown in Fig. 2. Figure 2(a) gives the distribution in θ_S, where the collinearity has been restricted to $\cos\theta_{\pi\pi} < -0.90$. The collinearity distribution for $1.0 < \theta_S < 1.3$ is shown in Fig. 2(b). Both distributions are completely consistent with Monte Carlo simulations when we take into account the diffraction regeneration. From Fig. 2(a) we obtain, after background subtraction, 69 ± 9 events with $1.0 < \theta_S < 1.3$ rad. The coherent regeneration probability from tungsten and the total cross section were measured at the same momentum in the same beam by observing the regenerated $K_1^0 \to \pi^+\pi^-$ in a magnetic spectrometer.[7] The incoherent regeneration was computed following the method of Good et al.[8] using the coherent data for input. The coherent and incoherent contributions (56% and 44%, respectively, after final data cuts) are combined to give $I_R = 3.5\times10^{-3}$. From the data in Fig. 2, using Eq. (1) with $L/\Lambda_1 = 33.6$ and $E = 0.93$ and taking into account the relative monitor counts [(free decay)/regeneration = 5.23], we expect, after the same cuts, 94 events in the free decay data if $|\eta_{00}|^2 = 25\times10^{-6}$.

In this experiment 10 600 free-decay events were measured. Of these 2400 events were retained after a cut with $\theta_S > 0.8$ rad and $\cos\theta_{\pi\pi} < -0.7$. The collinearity distribution for all these events is shown in Fig. 3(a). We can reasonably expect (an expectation completely justified by a detailed Monte Carlo simulation) that this collinearity distribution of the $3\pi^0$ background should be nearly linear on a semilog plot. There is no

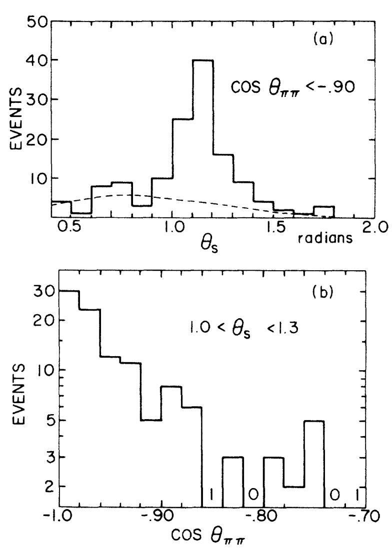

FIG. 2. Data obtained from a 4-in.-thick tungsten regenerator placed at five different positions in the decay volume. (a) Distribution of the smaller of the opening angles of the two pairs of γ rays for events with collinearity $\cos\theta_{\pi\pi} < -0.90$. The dashed line is the background estimated from the free-decay distribution. (b) Collinearity distribution for those events with the smaller opening angle between 1.0 and 1.3 rad.

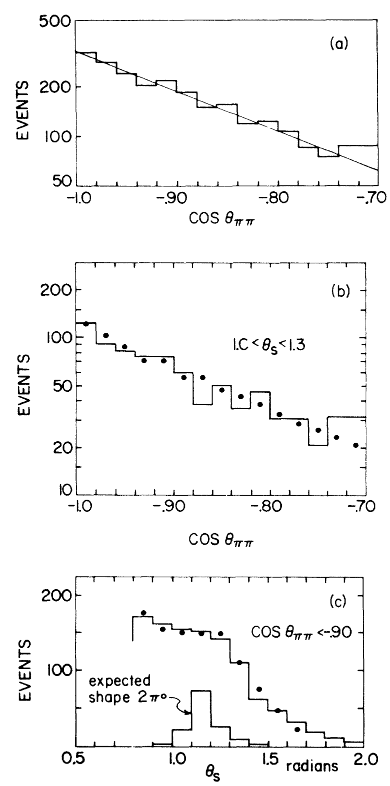

FIG. 3. Free-decay data. (a) Collinearity distribution of all events with the smaller opening angle of the two pairs of γ rays > 0.8 rad. (b) Collinearity distribution for events with the smaller opening angle θ_S between 1.0 and 1.3 rad. (c) Distribution of the smaller opening angle for events with $\cos\theta_{\pi\pi} < -0.90$. The dots in (b) and (c) represent the expected background distribution for the $K_2 \to 3\pi^0$ events normalized outside the region of the expected $2\pi^0$ events.

evidence for an excess of ~100 events in those channels close to $\cos\theta_{\pi\pi} = -1$. The collinearity distribution in the interval $1.0 < \theta_S < 1.3$ is shown in Fig. 3(b) and again there is no obvious peaking in the early channels. A quantitative evaluation of the number of $2\pi^0$ events was done in two ways. First, the distribution in the interval $1.0 < \theta_S < 1.3$ was compared with the combined distribution between $0.9 < \theta_S < 1.0$ and $1.3 < \theta_S < 1.5$ and normalized in the region of $\cos\theta_{\pi\pi} > -0.86$. The subtraction yields -5 ± 25 events. Secondly, the distribution in $\cos\theta_{\pi\pi}$ in the restricted range $1.0 < \theta_S < 1.3$ was computed with Monte Carlo techniques using as input the observed distributions (from all the data) of the opening angle of one pair of photons and the angle between the bisectors of the two opening angles. The assumption of no correlation between these parameters is justified for $3\pi^0$ decay to a high degree. The pseudo events so generated should faithfully reproduce the observed $\cos\theta_{\pi\pi}$ and θ_S distributions if they are $3\pi^0$, but completely destroy any peaking due to $2\pi^0$ events. The latter should then be revealed by subtracting the "pseudo" from the observed distribution. The background determined in this way is shown by the dots in Figs. 3(b) and 3(c). The subtraction gives -11 ± 25 events in Fig. 3(b) and -4 ± 25 events in Fig. 3(c). An even more restrictive cut, $1.1 < \theta_S < 1.2$, where we expect 50% of the events, gives the same result. Using $N_{\rm free} = -7 \pm 25$, we obtain

$$|\eta_{00}|^2 = (-2 \pm 7) \times 10^{-6}.$$

This result is substantially smaller than those previously reported. The clear observation of $K_1 \to 2\pi^0$ originating from regeneration at the expected rate gives us confidence in the performance of the detector.[9] The present result is entirely consistent with $|\eta_{00}|^2 = |\eta_{+-}|^2$.

We wish to thank the staff of the Princeton-Pennsylvania accelerator for their extraordinarily helpful cooperation and Mr. Mark Strovink for designing and implementing the Cherenkov counter used in timing the arrival of the protons on the target. The meticulous work of our laboratory and scanning staff is much appreciated. This work made use of computer facilities supported in part by National Science Foundation Grant No. NSF-Gp 579.

---

*Work supported by the U. S. Office of Naval Research, Contract No. N0014-67-A-0151-0001.

†Present address: Department of Physics, University of Tokyo, Tokyo, Japan.

‡Present address: Commission on College Physics, Ann Arbor, Mich.

§Present address: Department of Physics, Carnegie Mellon University, Pittsburgh, Pa.

||Present address: Department of Physics, University of California at San Diego, La Jolla, Calif.

[1]T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964); T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 16, 511 (1966).

[2]V. Fitch, R. Roth, J. Russ, and W. Vernon, Phys. Rev. 164, 1711 (1967).

[3]We have chosen to quote the relative rates rather than the relative amplitudes because (a) they are the measured quantities and (b) the errors are much more precisely represented, particularly when the central value is unphysical as it is in this experiment.

[4]It is interesting to note that in evaluating the background in an experiment devoted to $K_2^0 \to 2\gamma$ decay, L. Criegee et al. [Phys. Rev. Letters 17, 150 (1966)] deduced $|\eta_{00}|^2 = (6_{-6.0}^{+7.5}) \times 10^{-6}$.

[5]J. M. Gaillard, F. Krienen, W. Galbraith, A. Hussri, M. Jane, N. Lipman, G. Manning, T. Ratcliffe, P. Day, A. Parham, B. Payne, A. Sherwood, H. Faissner, and H. Reither, Phys. Rev. Letters 18, 20 (1967).

[6]J. Cronin, P. Kunz, W. Risk, and P. Wheeler, Phys. Rev. Letters 18, 25 (1967). Professor Cronin kindly informs us that a preliminary analysis of the data from a new experiment shows that his earlier data contained substantial background. We look forward to the final results of this new experiment.

[7]R. Carnegie, V. Fitch, T. Kamae, R. Roth, J. Russ, and W. Vernon, to be published.

[8]R. H. Good, R. Matsen, F. Muller, O. Piccioni, W. Powell, H. White, W. Fowler, and R. Birge, Phys. Rev. 124, 1223 (1961).

[9]This account of a complicated experiment and analysis is necessarily highly abbreviated. A detailed account can be found in T. Kamae, thesis, Princeton University Elementary Particles Laboratory Technical Report No. 45, 1968 (unpublished).