Phys. Rev. 162, 16 (1967).
[2]R. M. Freeman, Nucl. Phys. 26, 446 (1961).
[3]R. M. Steffen and H. Frauenfelder, in Perturbed Angular Correlations, edited by E. Karlsson, E. Matthias, and K. Siegbahn (North-Holland Publishing Company, Amsterdam, The Netherlands, 1964), p. 16.
[4]J. Christiansen, H.-E. Mahnke, E. Recknagel, D. Riegel, G. Weyer, and W. Witthuhn (to be published).
[5]E. Matthias, D. A. Shirley, M. P. Klein, and N. Edelstein, Phys. Rev. Letters 16, 974 (1966).

DYNAMICAL FORMATION OF A SMALL-SCALE FILAMENT

J. H. Marburger* and E. Dawes†
University of Southern California, Los Angeles, California 90007
(Received 25 August 1968)

We report a numerical analysis of the electromagnetic wave equation with a saturable, intensity-dependent, refractive index. The solutions show the dynamical self-focusing of an intense optical beam through the focus. The transverse intensity profile develops a complex ringed structure, the central maximum of which has many of the properties of the small-scale filaments observed experimentally.

Experimental studies of self-focusing of intense optical beams in liquids possessing intensity-dependent refractive indices have indicated two apparently distinct regions of importance.[1] In large-scale focusing, the beam contracts uniformly toward a point at some well-defined distance. Near this point however the contraction stops and one or more rather stable filaments of light are formed, the number of filaments depending on the power. These filaments are said to arise from a small-scale self-trapping mechanism. While large-scale self-focusing seems rather well understood theoretically, there has been considerable discussion about the origin of the small scale filaments.

In this paper we report a numerical analysis of the nonlinear wave equation which shows the dynamical formation of a small-scale "filament." We stress that it has not been possible to include in this analysis the plethora of physical phenomena known to be associated with small-scale self-trapping. The significance of this work is that filament formation is predicted by a model with very little physical structure: We include only the effects of saturation of the nonlinear index appearing in the wave equation.

Previous theoretical work has shown the existence of transverse intensity profiles which propagate in a medium with saturable index without changing shape.[2-6] These steady-state solutions, with characteristic radius of about 0.1 μ for CS_2, have been associated with the small-scale trapping, presumably because of their persistence. However the process of trapping is inherently dynamical and any theory of filament formation based upon the stationary profile solutions must explain how the beam evolves to such a profile from an arbitrary input shape. Moreover, approximate analyses of the dynamical self-focusing problem suggest that the beam radius oscillates producing periodic foci, rather than approaching a steady-state value.[2,7]

Dynamical focusing solutions of the nonlinear wave equation were found numerically by Kelley[8] for a nonsaturable, nonlinear refractive index. To reduce the number of independent variables in the problem, Kelley sought time-periodic, cylindrically symmetric solutions. To solve the resulting equation numerically with initial conditions (open boundary), he ignored the second derivative of the slowly varying amplitude E' with respect to longitudinal distance z (see below for notation). Unfortunately the approximations made in this procedure are not valid when E' changes rapidly within a wavelength, for example near the self-focus. This is a serious impediment to quantitative studies of filament formation.

When the nonlinear index is saturable however, an analysis based upon the paraxial-ray approximation[9] indicates that for "easy" saturation the first focus may be reached before the beam shrinks to wavelength dimensions. Thus for a medium which saturates rapidly, we may expect Kelley's equation (with saturation) to hold up to and beyond the first focus. Numerical computation is vastly simplified for this case because the criteria for a stable computing scheme, which require smaller step sizes for larger intensities, are easily satisfied even in the region of the focus.

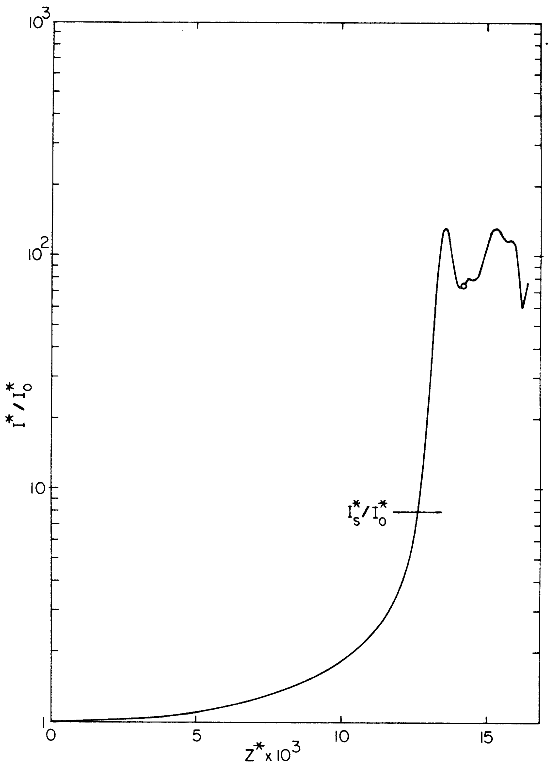

FIG. 1. Normalized on-axis intensity versus axial distance in units of the near field length $2ka_0^2$.

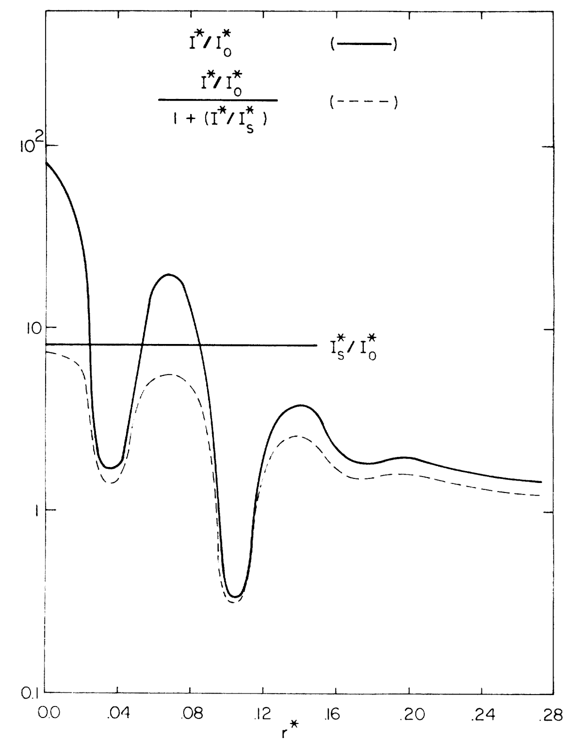

FIG. 2. Normalized intensity (solid line) and refractive index (dashed line) versus radial distance in units of the input radius a_0.

Using Kelley's notation,[8] the equation in question is

$$i\frac{\partial E^*}{\partial z^*}+\frac{\partial^2 E^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial E^*}{\partial r^*}+\frac{|E^*|^2 E^*}{1+|E^*/E_s^*|^2}=0, \tag{1}$$

where $r^*=r/a_0$, $z^*=z/2ka_0^2$, $E^*=(\epsilon_2/\epsilon_L)^{1/2}ka_0E'$, and $2\vec{E}=\hat{e}\{E'\exp[i(kz-\omega t)]+\text{c.c.}\}$. Here a_0 is the variance of the Gaussian intensity distribution of the plane-wave input at $z=0$, and E_s^* is a dimensionless saturation field which is about 10^4 for CS_2. Choosing the unrealistically low value $E_2^*=10^2$ and a very high initial input power $P=333P_{cr}$ to cause rapid focusing,[10] we find numerical solutions (obtained from a CDC 6600 computer) with the following features: (1) The on-axis intensity rises steeply to a maximum and then fluctuates about a high average value. This is shown in Fig. 1 in which the on-axis intensity $I^*=E_s^{*2}$, normalized to that at the input, is plotted versus z^*. (2) Near the focus the transverse intensity profile develops a ringed structure which becomes increasingly complex beyond the focus as shown in Fig. 2, which corresponds to $z^*=0.0141$ (open circle in Fig. 1). Considerable pains were taken to ensure that none of this structure is simply a result of instabilities in the computing scheme. The total computed beam power remained constant to within 0.1% for all z^*. Details of the mesh technique as well as numerical solutions for other cases will be published elsewhere. (3) The intensity profile has a central peak for all distances computed (up to $z/z_f\approx 1.2$). The power in this peak at the focus is about an order of magnitude greater than the critical power for self-trapping and decreases slowly at greater distances.

This behavior may be understood as follows. As the beam propagates, the refractive index near the axis rises at first but then becomes linear upon saturation. The resultant induced "convex lens" is flat in the center and therefore tends to focus incoming (still nearly parallel) rays into a ring. The rays initially bent toward the axis continue inward and give rise to a central maximum. The intensity in the ring also rises until a new "flat" region is formed in the induced lens, whereupon a new ring begins to form. This tendency to form additional rings is consistent with the result of Shen, AuYang, and Cohen[11] (inferred from a somewhat different model) that it is energetically favorable to form regions of high intensity separated by regions of low intensity. At the last point shown in Fig. 1, there are nine radial maxima in the transverse intensity profile.

It is clear from the oscillations in the on-axis intensity that the central maximum is not trapped in a stationary transverse mode. The behavior of this portion of the beam is more characteristic of the periodic focusing predicted for saturable media by the paraxial-ray analysis. This analysis predicts that for weak oscillations of the beam radius about its steady-state value, successive minima in the on-axis intensity should be separated by[9]

$$z_S = \pi k a_S^2 \frac{(P/P_{\rm cr})^{1/4}}{[(P/P_{\rm cr})^{1/2}-1]^{3/2}},$$

where $(a_S/a_0)^2 = 0.273(P/P_{\rm cr}E_S^{*2})$. Fitting the central maximum with a Gaussian, for which this formula is valid, we find $z_S^* \approx 3\times10^{-3}$ which is to be compared with the distance $\approx 2\times10^{-3}$ between the two prominent minima in Fig. 1.

The central peak possesses many of the observed properties of small-scale trapping: It persists beyond the focus, is very intense, and contains somewhat more than critical power. To estimate its size for realistic saturation fields, we may use either the paraxial-ray theory or the "exact" stationary profile theory, both of which yield characteristic transverse radii which scale as E_S^{*-1}. Thus for $E_S^* = 10^4$ we expect a central peak of width $r^* \approx 5\times10^{-4}$ or about 0.5 μ for a 1-mm incident beam. This underestimate of observed filament sizes (~5-10 μ) seems endemic to theories including only saturation of the nonlinear index.[5] To the list of other mechanisms usually invoked to explain this discrepancy we may add that some of the rings near the center may be experimentally unresolved or blurred by other effects, giving an apparent filament size much larger than that of the central peak.

We are grateful to Dr. P. L. Kelley for providing us with information regarding his computing scheme and to Dr. W. G. Wagner for his advice and encouragement. Mr. L. Huff and Mr. R. Reynolds also assisted with the numerical computation.

*Work supported by the U. S. Army Research Office, Durham, Grant No. DA-ARO-D-31-124-G920 and Joint Services Electronics Program (U.S. Army, Navy, Air Force) Grant No. AF-AFOSR-496-67.

†National Science Foundation Fellow.

[1]R. Y. Chiao, M. A. Johnson, S. Krinsky, H. A. Smith, C. H. Townes, and E. Garmire, IEEE J. Quantum Electron. QE-2, 467 (1966).

[2]R. Y. Chiao, E. Garmire, and C. H. Townes, Phys. Rev. Letters 13, 479 (1964).

[3]H. A. Haus, Appl. Phys. Letters 8, 128 (1966).

[4]Z. K. Yankauskas, Zh. Eksperim. i Teor. Fiz. 5, 335 (1967) [translation: Soviet Phys.—JETP 5, 275 (1967)].

[5]T. K. Gustafson, P. L. Kelley, R. Y. Chiao, and R. G. Brewer, Appl. Phys. Letters 12, 165 (1968).

[6]J. D. Reichert and W. G. Wagner, IEEE J. Quantum Electron. QE-4, 221 (1968).

[7]Yu. P. Raizer, Zh. Eksperim. i Teor. Fiz. 4, 3 (1966) [translation: Soviet Phys.—JETP 4, 1 (1966)].

[8]P. L. Kelley, Phys. Rev. Letters 15, 1005 (1965).

[9]W. G. Wagner, H. A. Haus, and J. H. Marburger (to be published).

[10]We use for $P_{\rm cr}$ the definition of Ref. 2.

[11]Y. R. Shen, M. Y. AuYang, and M. L. Cohen, Phys. Rev. Letters 19, 1171 (1967).

$K_2^0 \to 2\pi^0$ DECAY RATE*

D. F. Bartlett, R. K. Carnegie, V. L. Fitch, K. Goulianos, D. P. Hutchinson, T. Kamae,† R. F. Roth,‡ J. S. Russ, § and W. Vernon, ‖
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 31 May 1968)

The ratio $|\eta_{00}|^2$ of the rate of decay $K_2^0 \to 2\pi^0$ to the decay rate $K_1^0 \to 2\pi^0$ has been measured to be $|\eta_{00}|^2 = (-2\pm7)\times10^{-6}$.

The main interest in $|\eta_{00}|^2$, the ratio of the rate of decay $K_2^0 \to 2\pi^0$ to the decay rate $K_1^0 \to 2\pi^0$, is the extent to which it departs from $|\eta_{+-}|^2$, the corresponding ratio for the decay to $\pi^+\pi^-$. Any difference is a measure of the amount of $\Delta T \geq \frac{3}{2}$ amplitude present in CP nonconservation.[1] A survey[2] gives $|\eta_{+-}|^2 = (3.6\pm0.2)\times10^{-6}$. Recent measurements[3,4] of the neutral mode have given[5] $|\eta_{00}|^2 = (18^{+11}_{-6})\times10^{-6}$ and[6] $(24\pm5)\times10^{-6}$, apparently demonstrating a difference between $|\eta_{+-}|^2$ and $|\eta_{00}|^2$. We report a new measurement with the result $|\eta_{00}|^2 = (-2\pm7)\times10^{-6}$.