Hungerford, III, and R. W. Rutkowski for their able assistance during the cyclotron runs.

†Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.

*U. S. Atomic Energy Commission Postdoctoral Fellow under appointment from Oak Ridge Associated Universities.

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PERTURBED ANGULAR CORRELATION OF NUCLEI COHERENTLY EXCITED BY PULSED-BEAM TECHNIQUES

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(Received 10 June 1968)

A new resonance method is discussed which is suited for the investigations of hyperfine interactions of excited nuclear states with lifetimes longer than 1 μsec. A first experiment was performed on a 4-μsec state of ^{69}Ge. The g factor was determined as $g = -0.222 \pm 0.001$.

We have developed a new method for studying hyperfine interactions of excited nuclear states. An excited nuclear state is produced by a pulsed particle beam with a repetition time shorter than the lifetime of the nuclear state. Then the method depends on the observation of a resonance behavior of the perturbed γ-ray angular distribution. The resonance occurs if the Larmor frequency of the excited nuclei in an external magnetic field is equal to a multiple of half the pulse frequency.[1] The width of the resonance is limited by the natural linewidth of the nuclear state. Corresponding to Freeman's method of the differential measurement of the Larmor precession,[2] a nuclear reaction excites and aligns the nuclei using a pulsed particle beam. If the emitted γ radiation has an anisotropic angular distribution and if the magnetic field H_0 is perpendicular with respect to beam and detector direction, the intensity observed at a fixed angle is modulated with the Larmor frequency ω_L. This method can be applied as long as the lifetime τ of the excited state is shorter than the pulse repetition time T_0 and the pulse length is $\Delta T \ll 1/\omega_L$.

In our experiment many beam pulses occur within the lifetime of the state, i.e., $\tau > T_0$. In the case of resonance, $T_0 = n\pi/\omega_L$ ($n = 1, 2, \cdots$) (Fig. 1), all nuclei originating from different pulses precess with a constant phase with respect to the beam pulses, i.e., one gets a coherent superposition of the intensity modulation produced by the perturbed angular correlation. Off resonance, Larmor frequency and excitation frequency are out of phase which results in an attenuation of the modulation amplitude.

The intensity of the γ radiation at a time t (0

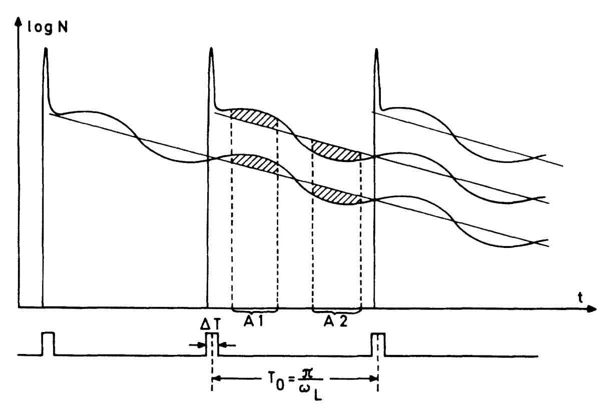

FIG. 1. Schematic illustration of the principle of synchronous Larmor precession. In the resonance, shown here, the counting rates A_1, A_2 are sensitive to the anisotropy of the perturbed angular correlation.

$\leq t < T_0$) is given by

$$I(t, \tau, T_0, \theta, \omega_L) = e^{-t/\tau} \sum_{k \text{ even}} B_k \cos[k(\theta - \omega_L t) - \delta_k], \quad (1)$$

$$B_k = b_k(1 - 2e^{-T_0/\tau} \cos k\omega_L T_0 + e^{-2T_0/\tau})^{-\frac{1}{2}}, \quad (1a)$$

$$\tan\delta_k = e^{-T_0/\tau} \sin k\omega_L T_0 \times (1 - e^{-T_0/\tau} \cos k\omega_L T_0)^{-1}. \quad (1b)$$

The b_k are coefficients defined in perturbed angular correlation work.[3] θ is the angle between beam and detector direction. The coefficients B_k, which depend on the Larmor frequency, i.e., the magnetic field H_0, have a maximum $B_{k\,\max} = b_k/(1-e^{-T_0/\tau})$ at $k\omega_L T_0 = 2n\pi$ ($n = 1, 2, \cdots$) and a minimum $B_{k\,\min} = b_k/(1+e^{-T_0/\tau})$ at $k\omega_L T_0 = (2n-1)\pi$ ($n = 1, 2, \cdots$). The intensity Eq. (1) averaged over a fixed time interval $t_0 - \Delta t < t < t_0$ also shows a resonance behavior if t_0 and Δt are properly chosen. In this case $k = 2, n = 1$ one gets optimum conditions with $\theta = \pm 45°$ or $\pm 135°$ and $t_0 = \frac{1}{4}T_0$ or $\frac{3}{4}T_0$ and $\Delta t < \pi/2\omega_L$. For $\tau \gg T_0$ the full width at half-maximum of the resonance curve is given by

$$\frac{\Delta H_0}{H_0} = \frac{1}{\omega_L \tau} = \frac{T_0}{n\pi\tau}. \quad (2)$$

An experimental proof of this method has been performed on the 398-keV ($\frac{9}{2}^+$) state in ^{69}Ge[4] using the reaction ^{69}Ga$(p,n)^{69}$Ge to populate the excited state. The lifetime of the level is $\tau = 4.0 \pm 0.1$ μsec. The pulsed proton beam has a width $\Delta T = 10$ nsec and a repetition time $T_0 = 1$ μsec. To avoid perturbations by intrinsic fields, a liquid-Ga target was used. The γ radiation was detected by two NaI crystals. Fig. 2 shows the double ratio $(A_1/A_2)(B_2/B_1)$ of counting rates, illustrated in Fig. 1, as a function of the magnetic field. The half-width was determined to be $\Delta H_0 = 245 \pm 5$ G which agrees with the prediction deduced from formula (1). From the measured magnetic field $H_0 = 2955$ G the uncorrected g factor is deduced as $g = -0.222 \pm 0.001$. The sign has been determined by phase measurements of the Larmor precession at different observation angles. According to formula (2) the accuracy of the measurement can be improved by using higher fields to get a multiple of ω_L. An experiment for $n = 5$ was performed and the expected behavior could be confirmed.

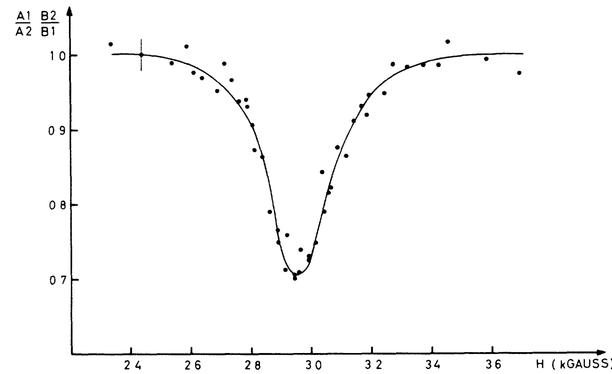

FIG. 2. Resonance behavior of the perturbed angular correlation of the 398-keV γ rays following the reaction ^{69}Ga$(p,n)^{69}$Ge as a function of the applied magnetic field. The pulse repetition rate was 1 MHz. A_1, A_2, B_1, B_2 are the counting rates of the counters A ($\theta = +45°$) and B ($\theta = -45°$), respectively, in the time intervals $\frac{1}{4}T_0 \pm \Delta t$ and $\frac{3}{4}T_0 \pm \Delta t$, respectively, $\Delta t = 100$ nsec.

In contrast to the known perturbed angular correlation techniques this method extends the range of application to lifetimes longer than 10^{-6} sec without principal intensity limitations, due to the fact that the nuclei are frequently excited during the lifetime. Similar to NMR detection by perturbed angular correlation,[5] this method combines high precision with the sensitivity of nuclear detection techniques. The main difference between these two methods is that for the detection of NMR one gets a perturbation of the Larmor precession by the rf field while in this method the Larmor precession remains undisturbed. Therefore, according to formula (2) the accuracy of hyperfine interaction measurements is only limited by the natural linewidth. Information about relaxation phenomena and intrinsic fields in the target material can be deduced from the broadening and from the shifts of the resonance line. Quadrupole interactions in solid state material can be measured in an analogous manner by variation of the pulse repetition frequency.

We thank Professor K. H. Lindenberger for helpful discussions and U. Morfeld and G. Schatz for their assistance during the experiments.

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DYNAMICAL FORMATION OF A SMALL-SCALE FILAMENT

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(Received 25 August 1968)

We report a numerical analysis of the electromagnetic wave equation with a saturable, intensity-dependent, refractive index. The solutions show the dynamical self-focusing of an intense optical beam through the focus. The transverse intensity profile develops a complex ringed structure, the central maximum of which has many of the properties of the small-scale filaments observed experimentally.

Experimental studies of self-focusing of intense optical beams in liquids possessing intensity-dependent refractive indices have indicated two apparently distinct regions of importance.[1] In large-scale focusing, the beam contracts uniformly toward a point at some well-defined distance. Near this point however the contraction stops and one or more rather stable filaments of light are formed, the number of filaments depending on the power. These filaments are said to arise from a small-scale self-trapping mechanism. While large-scale self-focusing seems rather well understood theoretically, there has been considerable discussion about the origin of the small scale filaments.

In this paper we report a numerical analysis of the nonlinear wave equation which shows the dynamical formation of a small-scale "filament." We stress that it has not been possible to include in this analysis the plethora of physical phenomena known to be associated with small-scale self-trapping. The significance of this work is that filament formation is predicted by a model with very little physical structure: We include only the effects of saturation of the nonlinear index appearing in the wave equation.

Previous theoretical work has shown the existence of transverse intensity profiles which propagate in a medium with saturable index without changing shape.[2-6] These steady-state solutions, with characteristic radius of about 0.1 μ for CS_2, have been associated with the small-scale trapping, presumably because of their persistence. However the process of trapping is inherently dynamical and any theory of filament formation based upon the stationary profile solutions must explain how the beam evolves to such a profile from an arbitrary input shape. Moreover, approximate analyses of the dynamical self-focusing problem suggest that the beam radius oscillates producing periodic foci, rather than approaching a steady-state value.[2,7]

Dynamical focusing solutions of the nonlinear wave equation were found numerically by Kelley[8] for a nonsaturable, nonlinear refractive index. To reduce the number of independent variables in the problem, Kelley sought time-periodic, cylindrically symmetric solutions. To solve the resulting equation numerically with initial conditions (open boundary), he ignored the second derivative of the slowly varying amplitude E' with respect to longitudinal distance z (see below for notation). Unfortunately the approximations made in this procedure are not valid when E' changes rapidly within a wavelength, for example near the self-focus. This is a serious impediment to quantitative studies of filament formation.

When the nonlinear index is saturable however, an analysis based upon the paraxial-ray approximation[9] indicates that for "easy" saturation the first focus may be reached before the beam shrinks to wavelength dimensions. Thus for a medium which saturates rapidly, we may expect Kelley's equation (with saturation) to hold up to and beyond the first focus. Numerical computation is vastly simplified for this case because the criteria for a stable computing scheme, which require smaller step sizes for larger intensities, are easily satisfied even in the region of the focus.