[5]We reject events with $p_{fit}\beta_{fit}\theta < 2200$ (MeV/$c$) deg, where $p_{fit}$ and $\beta_{fit}$ are the fitted momentum and velocity of the "unmeasured" pion, and $\theta$ is the space angle between its fitted and measured directions.

[6]The rate and photon spectrum for this process have been calculated by M. Bég, R. Friedberg, and J. Schultz, as quoted by P. Franzini, L. Kirsch, P. Schmidt, J. Steinberger, and J. Plano, Phys. Rev. 140, B127 (1965), and have been previously verified (with 27 events) by E. Bellotti, A. Pullia, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, H. Huzita, F. Mattioli, and A. Sconza, Nuovo Cimento 45A, 737 (1966).

[7]For example, a change in $|\delta|$ of $\pm 0.05\times10^{10}$ sec$^{-1}$ gives a change in $x$ of only $\mp 0.01 \mp 0.02i$.

[8]Rosenfeld, Barash-Schmidt, Barbaro-Galtieri, Price, Söding, Wohl, Roos, and Willis, Ref. 2.

[9]In a preliminary report on these data [B. R. Webber et al., University of California Radiation Laboratory Report No. UCRL-18135, 1968 (unpublished)], we gave item 3 (overall $\chi^2$ test) more weight than it deserves. Also, we had not then incorporated the charge ratio into our likelihood function.

[10]D. G. Hill, D. Luers, D. K. Robinson, M. Sakitt, O. Skjeggestad, J. Canter, Y. Cho, A. Dralle, A. Engler, H. E. Fisk, R. W. Kraemer, and C. M. Meltzer, Phys. Rev. Letters 19, 668 (1967). We change the sign of Im($x$) given in that paper, since the value they used for $\delta$ was positive, not negative as stated in their footnote 10 (A. Engler and H. E. Fisk, private communication).

[11]R. P. Ely, W. M. Powell, H. White, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, O. Fabbri, F. Farini, C. Filippi, H. Huzita, G. Miari, U. Camerini, W. F. Fry, and S. Natali, Phys. Rev. Letters 8, 132 (1962); and G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., ibid. 9, 69 (1962); B. Aubert, L. Behr, F. L. Canavan, L. M. Chounet, J. P. Lowys, P. Mittner, and C. Pascaud, Phys. Letters 17, 59 (1965); M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, C. Filippi-Filosofo, H. Huzita, F. Mattioli, and G. Miari, Nuovo Cimento 38, 684 (1965); Franzini et al., Ref. 5; L. Feldman, S. Frankel, V. L. Highland, T. Sloan, O. B. Van Dyck, W. D. Wales, R. Winston, and D. M. Wolfe, Phys. Rev. 155, 1611 (1967).

# NOTE ON THE CHIRAL-DYNAMIC-LAGRANGIAN CALCULATION OF THE $\chi^0$ DECAY

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The decay rate and the energy asymmetry of the $\eta$ particle observed in the strong decay $\chi \to \eta + 2\pi$ are discussed in the context of SU(3)$\otimes$SU(3) chiral dynamic Lagrangian.

The purpose of this note is to discuss the sources of discrepancy in the chiral dynamic prediction of the slope of the Dalitz plot distribution for the strong decay $\chi^0 \to \eta\pi\pi$ and to develop a Lagrangian which might explain the energy asymmetry for this decay. This process is of particular interest, since it provides us with a direct test of chiral dynamics in the domain of strong interaction physics.

The basic idea in chiral dynamics[1] is to construct a Lagrangian in which there is a chiral-invariant part and a part in which the chiral symmetry is broken in a definite way. Usually the chiral invariance is broken in such a way that the hypothesis of the partial conservation of axial-vector current (PCAC) is satisfied. Because the status of scalar mesons is presently quite ambiguous, we wish to adopt a formalism which does not require the presence of these fields. As a result of this formalism the Lagrangian becomes a highly nonlinear function of the fields. In a sense, one assigns fields to nonlinear realizations of the chiral group under consideration.

Let us adopt a tensor notation in which the upper (lower) indices transform cogrediently (contragrediently) and the undotted (dotted) indices refer to the transformations generated by $Q_\lambda + Q_\lambda^5$ $(Q_\lambda - Q_\lambda^5)$ where[2]

$$Q_\lambda = \int d^3x\, V_{\mu=0}^{\lambda}(\vec{x}, t)$$

and

$$Q_\lambda^5 = \int d^3x\, A_{\mu=0}^{\lambda}(\vec{x}, t). \tag{1}$$

We denote by $M$ the pseudoscalar complex

$$M_{\dot\beta}^{\alpha} = (M_{\alpha}^{\dot\beta})^\dagger = (\Sigma + i\Pi)_{\alpha\beta}, \quad \alpha, \beta = 1, 2, 3, \tag{2}$$

where $\Pi$ is the usual $3\times3$ pseudoscalar meson matrix and $\Sigma$ is to be determined[3] from the constraint

$$\Sigma^2 + \Pi^2 = f^2, \tag{3}$$

where $f^2$ is a $c$-number constant.

Let us first analyze Cronin's model. In this model one gets a constant matrix element for the decay $\chi \to \eta\pi\pi$ and a decay rate of $(6.8 \pm 1.5)$ MeV, which is too large. From the experimental data available at present it seems that this decay matrix element has a small energy dependence.[4] We would like to show that if one constructs a Lagrangian similar to Cronin's model—that is, one which has a chiral-invariant kinetic energy term (like $\mathrm{Tr}\,\partial_\mu M \partial^\mu M^+$)—plus a few nonderivative breaking terms—there will be no slope in the Dalitz-plot distribution of the decay. Consider a Lagrangian

$$L = \tfrac{1}{2}\partial_\mu M_{\dot\beta}{}^{\alpha}\partial^\mu M_\alpha{}^{\dot\beta} + \text{nonderivative breaking term.} \tag{4}$$

Because of mixing, the $\eta$ and $\chi_0$ are given by

$$\Pi_8 = \eta\cos\theta - \chi_0\sin\theta,$$
$$\Pi_0 = \chi_0\cos\theta + \eta\sin\theta. \tag{5}$$

The invariant part of the Lagrangian contributes to the $\varphi^2\chi\eta$ interaction a term

$$L_{\varphi^2\chi\eta} = \frac{1}{3\sqrt{2}f^2}\left(\cos2\theta + \frac{\sin2\theta}{2\sqrt{2}}\right)[\partial_\mu\varphi^2\partial^\mu(\chi\eta) + (\partial_\mu\varphi)^2\chi\eta + \varphi^2\partial_\mu\chi\partial^\mu\eta]. \tag{6}$$

The contribution of the breaking term, being purely nonderivative, can only be of the form of a constant times $\varphi^2\eta\chi$. Thus in this nonderivative breaking $U(3)\otimes U(3)$ model we will get

$$L_{\varphi^2\chi\eta} = \text{const}[\partial_\mu\varphi^2\partial^\mu(\chi\eta) + (\partial_\mu\varphi)^2\chi\eta + \varphi^2\partial^\mu\chi\partial_\mu\eta + K\varphi^2\chi\eta], \tag{7}$$

where $K$ is an arbitrary constant. On the mass shell this is equivalent to

$$L_{\varphi^2\chi\eta} = \text{const}\left(1 + \frac{2K + m_\eta^2 + m_\chi^2}{2m_\pi^2}\right)[\partial_\mu\varphi^2\partial^\mu(\chi\eta) - m_\pi^2\varphi^2\chi\eta + 2(\partial_\mu\varphi)^2\chi\eta]. \tag{8}$$

This gives no slope in the Dalitz plot distribution whatever $K$ might be. To explain the experimental data, therefore, one must consider a Lagrangian with at least one additional term containing derivatives of fields.[5]

We will now digress a little from the chiral dynamics and consider the form of the $\varphi^2\chi\eta$ Lagrangian from a phenomenological point of view. One can easily prove that, for this decay, the most general Lagrangian with not more than two derivatives of the fields can have only two arbitrary parameters, and can be written as[6]

$$\begin{aligned} L_{\varphi^2\chi\eta} &= \text{const}[\partial_\mu\varphi^2\partial^\mu(\chi\eta) \\ &\quad - m_\pi^2\varphi^2\chi\eta + g(\partial_\mu\varphi)^2\chi\eta], \\ &= L_{\text{Sch}} + \text{const}(\partial_\mu\varphi)^2\chi\eta, \end{aligned} \tag{9}$$

where $L_{\text{Sch}}$ is the Lagrangian derived by Schwinger[7] from his "minimal" method. The second term could be interpreted as an anomalous term in Schwinger's model. This gives us an amplitude

$$A_{\chi\to\eta\varphi^\alpha\varphi^\beta} = \text{const}[1 - (h/m_\pi^2)S]\delta_{\alpha\beta}, \tag{10}$$

where

$$S = [(p_\pi)_\alpha + (p_\pi)_\beta]^2 \text{ and } h = \frac{1}{2}\frac{2-g}{1-g}. \tag{11}$$

From the available experimental data we see that a wide range of values for $h$ and hence for $g$ is allowed. In particular, $g=0$ (Schwinger's choice) seems to fit the present experimental data.[8]

Let us now consider the case of a $SU(3)\otimes SU(3)$ Lagrangian. We have said before that we must have other derivative terms in the Lagrangian besides the kinetic energy term. We would also like to have a minimum number of breaking terms which will produce all the masses exactly and which, at the same time, will satisfy the hypothesis of PCAC. Subject to all these conditions we have as our Lagrangian[9]

$$L=\tfrac{1}{2}(1-\delta-\epsilon)\partial_{\mu}M_{\dot{\nu}}{}^{\lambda}\partial^{\mu}M_{\lambda}{}^{\dot{\nu}}+\gamma(\epsilon_{\lambda\tau\nu}{}^{\dot{\lambda}\dot{\tau}\dot{\nu}}M_{\dot{\lambda}}{}^{\lambda}M_{\dot{\tau}}{}^{\tau}M_{\dot{\nu}}{}^{\nu}+\text{H.c.})$$
$$+\frac{\delta}{4f}(\epsilon_{\lambda\tau\nu}{}^{\dot{\lambda}\dot{\tau}\dot{\nu}}\partial_{\mu}M_{\dot{\lambda}}{}^{\lambda}\partial^{\mu}M_{\dot{\tau}}{}^{\tau}M_{\dot{\nu}}{}^{\nu}+\text{H.c.})+\frac{\epsilon}{4f^{3}}(\partial_{\mu}M_{\dot{\nu}}{}^{\lambda}\partial^{\mu}M_{\lambda}{}^{\dot{\nu}}\det M+\text{H.c.})+\alpha\Sigma_{0}+\beta\Sigma_{8}. \quad (12)$$

Here[10] $\alpha$, $\beta$, $\gamma$, $\delta$ and $\epsilon$ are arbitrary parameters of the theory, of which the first four will be fixed by the observed $0^-$ meson mass spectrum. There are only two breaking terms in the Lagrangian which transform as members of $(\underline{3},\underline{3}^*)\otimes(\underline{3}^*,\underline{3})$ representation, and are such that the chiral SU(2)⊗SU(2) is broken only by the finite pion mass. The difference between this and Cronin's model should be noted: In Cronin's model (i) $\delta$ and $\epsilon$ terms are not present, (ii) the SU(3) ⊗ SU(3)-invariant $\gamma$ term[11] is represented by a $\varphi_0{}^2$ term, and (iii) an additional breaking term $\Pi_0\Pi_8$ is present.

We note that in this Lagrangian $\gamma$ and $\delta$ terms serve to produce a mass difference between the singlet and the octet of mesons. These terms do not contribute to the scattering amplitudes of particles within the octet. Similarly, the $\epsilon$ term contributes only to the $\chi,\eta$ system. Thus for the octet of $0^-$ mesons the properties of this Lagrangian are similar to those of Cronin's model. To reproduce all the meson masses exactly we must have

$$\alpha=(f/\sqrt{3})(2m_K{}^2+m_\pi{}^2),$$
$$\beta=-2(\sqrt{2}/\sqrt{3})(m_K{}^2-m_\pi{}^2),$$
$$\gamma f=4.5m_\pi{}^2,\quad\text{and }\delta=-0.84. \quad (13)$$

The $\Pi_0$ field in this model has been renormalized so that the kinetic energy part of the Lagrangian is

$$\tfrac{1}{2}\sum_{i=0}^{8}(\partial_\mu\varphi_i)^2.$$

$\eta$ and $\chi$ are then given by the combination

$$\eta=\Pi_8\cos\theta+\Pi_S\sin\theta,$$
$$\chi=\Pi_S\cos\theta-\Pi_8\sin\theta, \quad (14)$$

where $\Pi_S=(1-3\delta)^{1/2}\Pi_0$ is the renormalized singlet field. The mixing angle is found to be $\theta=10°38'$. PCAC is obtained in the form

$$\partial_\mu A_\mu{}^\alpha=i[Q_5{}^\alpha,L]=\sqrt{2}fm_\alpha{}^2\varphi^\alpha$$
$$\text{for }\alpha=1,2,\cdots,7.$$

If we insist that $\partial_\mu A_\mu{}^i=f_\pi m_\pi{}^2\varphi^i$ for $i=1,2,3$ then $f=f_\pi/\sqrt{2}$.

We are now left with only one parameter. To obtain any interaction term, we expand the Lagrangian in powers of the fields. This gives us an interaction Lagrangian[12] for the strong decay $\chi\to\eta\pi\pi$

$$L_{\varphi^2\chi\eta}=\frac{7.7}{f^2}[\partial_\mu\varphi^2\partial_\mu(\chi\eta)$$
$$-m_\pi{}^2\varphi^2\chi\eta+(1.97-0.01\epsilon)(\partial_\mu\varphi)^2\chi\eta] \quad (15)$$

which implies $g=1.97-0.01\epsilon$, and therefore

$$h=-\frac{0.03+0.01\epsilon}{1.94-0.02\epsilon}. \quad (16)$$

If $\epsilon=0$, the slope of the matrix element is very small compared with the experimental data indicating thereby that the $\epsilon$ term must be present, although it is not required to fit the masses. Also if $\epsilon=0$, the decay rate is found to be $\Gamma_{\chi\to\eta\pi^+\pi^-}$

Table I. The decay width and the slope in the Dalitz plot distribution of $\chi\to\eta\pi^+\pi^-$ for different values of the parameter $\epsilon$.[a]

| $\epsilon$ | $2\Gamma_{\chi\to\eta\pi^0\pi^0}$ (MeV) | $h$ | $\epsilon$ | $2\Gamma_{\chi\to\eta\pi^0\pi^0}$ (MeV) | $h$ |
|---|---|---|---|---|---|
| −80 | 5.83 | 0.218 | −40 | 0.72 | 0.135 |
| −75 | 4.46 | 0.210 | −35 | 1.03 | 0.121 |
| −70 | 3.30 | 0.201 | −30 | 1.54 | 0.107 |
| −65 | 2.34 | 0.192 | −25 | 2.27 | 0.090 |
| −60 | 1.60 | 0.182 | −20 | 3.20 | 0.073 |
| −55 | 1.07 | 0.171 | −15 | 4.34 | 0.054 |
| −50 | 0.74 | 0.160 | −10 | 5.70 | 0.033 |
| −45 | 0.63 | 0.145 | | | |

[a]I would like to thank Dr. J. Smith for helping me with the computer programming.

= 9 MeV which is too large. In our Lagrangian as $\epsilon$ is the only free parameter left, we obtain one prediction between the decay rate and the slope in the Dalitz plot distribution, neither of which is, however, known with any accuracy. From the work of London et al.[4] we find that the slope should be negative. The width of this decay is known[13] to be less than 4 MeV. We represent the prediction of this Lagrangian in Table I.[14] We see from Table I that to explain the experimental data presently available, the numerical value of $\epsilon$ could be anywhere between $-20$ and $-70$. But the values of the slope for different values of $\epsilon$ come out to be small; this might really be the case.

It is clear from our development of the phenomenological Lagrangian that it is not possible to predict both the decay rate and the slope of $\chi^0 \to \eta\pi^+\pi^-$ in chiral dynamics without any further ad hoc assumption.[15] If the decay rate and the slope were known accurately and if we can assume that the breaking term is of $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$ type, then we could construct a unique chiral-dynamic Lagrangian for the nonet of pseudoscalar particles. It may also be said that there is no compelling reason why the breaking term should belong to a representation of $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$. This problem of uniqueness of a SU(3) ⊗ SU(3) chiral-dynamic Lagrangian and how far we can go in reproducing the experimental results will be studied elsewhere.[16]

I should like to thank Professor B. W. Lee for suggesting this investigation to me and for his advice during the work. I am also grateful to Dr. W. A. Bardeen for many helpful and clarifying discussions.

[1]References to works on chiral dynamics can be obtained from any one of the following: S. Weinberg, Phys. Rev. Letters 18, 188 (1967); J. Schwinger, Phys. Letters 24B, 473 (1967), and Phys. Rev. 167, 1432 (1968); J. Wess and B. Zumino, Phys. Rev. 163, 1727 (1967); W. A. Bardeen and B. W. Lee, Canadian Summer Institute Lectures, 1967 (W. A. Benjamin, Inc., New York, to be published); P. Chang and F. Gürsey, Phys. Rev. 164, 1752 (1967); J. Cronin, Phys. Rev. 161, 1483 (1967).

[2]$V_\mu{}^\lambda(\vec{x},t)$ and $A_\mu{}^\lambda(\vec{x},t)$ are the vector and the axial-vector currents, respectively. $\lambda$ is the SU(3) index and $\mu$ the Lorentz index. $Q^\alpha$ and $Q^\beta$ satisfy the algebra of SU(3), $[Q^\alpha, Q^\beta] = if_{\alpha\rho\gamma}Q^\gamma$. Also, $[Q^\alpha, Q_5{}^\beta] = if_{\alpha\beta\gamma}Q_5{}^\gamma$ and $[Q_5{}^\alpha, Q_5{}^\beta] = if_{\alpha\beta\gamma}Q^\gamma$.

[3]This is a natural extension of the work of Bardeen and Lee, Ref. 1.

[4]G. London et al., Phys. Rev. 143, 1034 (1966).

[5]When we were finishing our work, we came across a University of Maryland preprint by P. K. Mitter and L. J. Swank on this subject. Their model, however is essentially the same as that of Cronin, Ref. 1.

[6]We could as well write it as $L_{\varphi^2\chi\eta} = \mathrm{const}[\partial_\mu\varphi^2\partial_\mu(\chi\eta) - g' m_\pi{}^2\varphi^2\chi\eta]$, where $g'$ is an arbitrary parameter.

[7]The method which has been developed by Schwinger may be called a "minimal" method of obtaining a chiral-invariant Lagrangian. This method for the pseudoscalar mesons is to keep the infinitesimal response of the Lagrangian to $-m_\pi{}^2\varphi\,\delta\varphi$ while making a most general chiral transformation of the fields. This fixes the interaction Lagrangian completely. But in this model an anomalous term, like the gauge-invariant anomalous magnetic-moment interaction term in electrodynamics, could be introduced which is chiral invariant but nonminimal and arbitrary.

[8]If we consider SU(2) ⊗ SU(2), where $\pi$ and $\sigma$ belong to a $(\frac{1}{2},\frac{1}{2})$ representation and $\eta$ and $\chi$ belong to the (0, 0) representation, then only one chiral invariant Lagrangian is possible, namely $L_{\varphi^2\chi\eta} = \mathrm{const}\,\partial_\mu M_{\dot\delta}{}^{\gamma}\partial^\mu M_\gamma{}^{\dot\delta}\chi\eta = \mathrm{const}[\partial_\mu\varphi^2\partial^\mu(\chi\eta) - 2m_\pi{}^2\varphi^2\chi\eta]$. This gives us $h = 0.5$, which is also consistent with the present experimental data.

[9]S. L. Glashow and S. Weinberg have considered the case of a SU(3) ⊗ SU(3) Lagrangian where the breaking term transforms as a member of the $(\underline{3}^*, \underline{3}) \oplus (\underline{3}, \underline{3}^*)$ representation, Phys. Rev. Letters 20, 224 (1968). But they have not discussed the $\chi^0$ decay.

[10]$\epsilon_{\lambda\mu\nu}{}^{\dot\lambda\dot\mu\dot\nu} = \epsilon^{\dot\lambda\dot\mu\dot\nu}\epsilon_{\lambda\mu\nu}$, where $\epsilon^{\lambda\mu\nu}$ is the Levi-Civita symbol.

[11]It can be easily proved that the second term in Eq. (12) is equal to $\gamma(6g\det M + \mathrm{H.c.})$. In expanding this term we have used an identity that for a $3\times 3$ matrix $M$, $\det(I+M) = 1 + \mathrm{Tr}M - \frac{1}{2}\{\mathrm{Tr}M^2 - (\mathrm{Tr}M)^2\} + \det M$.

[12]Equation (12) gives the following $\pi\pi$, $\pi K$, and $KK$ meson-meson interaction Lagrangian:

$$L_{\pi\pi} = (\tfrac{1}{16}f^2)[(\partial_\mu\textstyle\sum_1{}^3\varphi_i{}^2)^2 - m_\pi{}^2(\sum_1{}^3\varphi_i{}^2)^2],$$

$$L_{\pi K} = (\tfrac{1}{16}f^2)[(\partial_\mu\textstyle\sum_1{}^3\varphi_i{}^2)(\partial^\mu\sum_4{}^7\varphi_j{}^2) + 2\partial_\mu\{\bar K(\tau\varphi)\}\partial^\mu\{(\tau\varphi)K\} - (m_K{}^2 + m_\pi{}^2)(\sum_1{}^3\varphi_i{}^2)(\sum_4{}^7\varphi_j{}^2)],$$

$$L_{KK} = (\tfrac{1}{16}f^2)[(\partial_\mu\textstyle\sum_4{}^7\varphi_i{}^2)^2 - m_K{}^2(\sum_4{}^7\varphi_i{}^2)] + (\tfrac{1}{4}f^2)[\partial_\mu(K^+\bar K^0)\partial^\mu(K^0K^-) - \partial_\mu(K^+K^-)\partial^\mu(K^0\bar K^0)].$$

[13]See A. H. Rosenfeld et al., Rev. Mod Phys. 40, 77 (1968).

[14]I would like to thank Dr. J. Smith for helping me with the computer programing.

[15]It has come to the author's attention that a similar comment has been made by Professor B. Zumino.

[16]W. A. Bardeen and D. P. Majumdar, to be published.