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# TEST OF THE $\Delta S = \Delta Q$ RULE IN LEPTONIC DECAYS OF NEUTRAL $K$ MESONS*

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(Received 1 April 1968)

We have studied the time distributions of 242 leptonic decays of neutral $K$ mesons produced in the reaction $K^- + p \to \bar{K}^0 + n$. For the $\Delta S = -\Delta Q$ parameter $x$ we find $\mathrm{Re}(x) = 0.22_{-0.09}^{+0.07}$, $\mathrm{Im}(x) = -0.88 \pm 0.08$.

Theory. – A violation of the $\Delta S = \Delta Q$ rule in the decays

$$\text{neutral } K \to \pi^{\pm} + e^{\mp} + \nu, \tag{1}$$

$$\text{neutral } K \to \pi^{\pm} + \mu^{\mp} + \nu, \tag{2}$$

would be manifest in their time distributions. If $f$ is the amplitude for $K^0 \to \pi^- + e^+ + \nu$ ($\Delta S = \Delta Q$) and $g$ is that for $\bar{K}^0 \to \pi^- + e^+ + \nu$ ($\Delta S = -\Delta Q$), and if these amplitudes have approximately the same energy dependence in the kinematically allowed region, then the time distribution of electronic decays of an initial $\bar{K}^0$ state is give by[1]

$$N^{\pm}(t) \propto |1+x|^2 \exp(-\lambda_S t) + |1-x|^2 \exp(-\lambda_L t) -2[2\,\mathrm{Im}(x)\sin\delta t \pm (1-|x|^2)\cos\delta t] \times \exp[-\tfrac{1}{2}(\lambda_S+\lambda_L)t], \tag{3}$$

where the $\pm$ corresponds to the electron charge, $x = g/f$, $\lambda_S$ and $\lambda_L$ are the $K_S$ and $K_L$ total decay rates, and $\delta$ is the mass difference $m(K_S) - m(K_L)$.[2] Conservation of $CP$ in the decay process implies that $x$ is real; the $\Delta S = \Delta Q$ rule requires $x = 0$.

After further assumptions,[3] the time distribution of muonic decays may also be expressed by Eq. (3), with the same value of $x$.

Scanning and selection of candidates. – We obtained $\bar{K}^0$ mesons by exposing the 25-in. Lawrence Radiation Laboratory hydrogen bubble chamber to a $K^-$ beam with momenta in the range 340-425 MeV/$c$. The data on which we report here are based on $10^6$ pictures, in which 45 000 $\bar{K}^0$ were produced inside the fiducial volume in the reaction

$$K^- + p \to \bar{K}^0 + n. \tag{4}$$

These data represent about 70% of the expected total.

The pictures are scanned for $V$'s; if both a $V$ and a 0-prong are found, they are measured and four-constant (4C) fits to $\bar{K}^0$ and $\Lambda$ production and two-body decay are attempted. If the confidence level for the $K^0$ fit is less than $5\times10^{-4}$ and the $V$ is not identified as a $\Lambda$ during scanning, fits to all three-body $K^0$ decay hypotheses are tried. If the confidence level for any of these is greater than 0.02, the event is called a three-body $K^0$ decay candidate and is remeasured. Pictures containing more than one 0-prong are rejected. Pictures in which the scanner recorded a $K^0$ but no 0-prong are carefully rescanned for 0-prongs and measured if one is found. In this way we obtain those $K^0$'s which are associated with a unique production vertex but fit only the 1C $\bar{K}^0$ production and three-body decay hypotheses. From a second scan of 20% of the film, we estimate the scanning efficiency for three-body $K^0$ decays to be 95%. Those events missed in the first scan show no bias in the distribution of the length of the neutral track.

Our choice of a fiducial volume and a maximum dip angle for charged tracks is governed by our desire to identify lepton tracks by comparison of bubble densities. To be free of time-dependent biases, the probability of identification must be independent of the position of the decay vertex in the chamber. However, we find that this probability depends on track length if the lepton track appears shorter than 3 cm in two or more views on a scanning table. From simulated events,[4] we find that this bias is eliminated by making a dip-angle cut at 63 deg and by placing the boundaries of the fiducial volume 8 cm from the top and bottom windows of the chamber, and at least 5 cm from the other limits of the visible region.

To remove the scanning bias against events with shorter neutral tracks, we reject a candidate if the decay vertex, projected onto the beam plane, lies inside a rectangular region extending 3.5 mm ahead of the production vertex, 2.5 mm behind it, and 1.75 mm to each side.

Elimination of background. —We make three cuts before careful examination of candidates.

(a) We eliminate two-pion decays with a Coulomb scattering or $\pi\rightarrow\mu\nu$ decay of one pion by means of 1C fits to $\bar{K}^0$ production and two-pion decay, in which first one and then the other pion is considered unmeasured.[5]

(b) We reject events fitting the 1C hypothesis of $\bar{K}^0$ production and radiative two-pion ($\pi^+\pi^-\gamma$) decay when the photon momentum in the $K$ rest-frame, $p_\gamma$, is less than 50 MeV/$c$.

(c) We remove "second-order" background, amounting to about 10 events, by means of a fit in which the errors on the tracks of the $V$ are increased. Principal sources of these events are (1) $\pi^+\pi^-\gamma$ decays in which a Coulomb scattering of one of the pions spoils the fit to the $\pi\pi\gamma$ hypothesis, (2) $\pi^+\pi^-\gamma\gamma$ decays, and (3) events in which the reaction sequence is $K^-+p\rightarrow\bar{K}^0+n+\gamma$, $K_S\rightarrow\pi^++\pi^-$, followed by Coulomb scattering of a pion. The increases in the errors are calculated from the expected photon spectrum and the Rutherford formula.

Having made the above cuts, we examine the track densities of the remaining candidates on a scanning projector. When the appearance agrees with the interpretation as background, we reject an event if any of the following conditions is met:

(d) The confidence level for the 4C fit to $\bar{K}^0$ production and two-pion decay is greater than 5 $\times10^{-5}$. This removes all remaining two-pion decays in which the $K_S$ comes from a normal production vertex.

(e) The confidence level for a 1C fit to the two-body decay of a $K_S$ or $\Lambda$ of unknown origin is greater than $5\times10^{-4}$. This cut eliminates two-body wall $V$'s and events in which a neutral particle is produced abnormally or scatters in flight before decaying via a two-body mode.

(f) On interpretation of the tracks of the $V$ as electrons, the mass of the pair is less than 140 MeV. This removes Dalitz decays of the $\pi^0$.

(g) The $V$ could be the decay of an incoming muon, or the decay or elastic scattering of a charged pion. In 35 events the tracks have beem remeasured in the directions appropriate to these hypotheses; 28 of these had fits or missing mass values that led to their rejection.

After a study of wall $V$'s (that is, $V$'s in pictures containing no 0-prong) in a sample of 1.6 $\times10^4$ pictures, we conclude that this source of background is negligible.

Finally, we find that the three-pion decay mode of the $K^0$ is well separated from the leptonic modes by kinematics, so it does not give rise to any background.

We subjected simulated leptonic decays to cuts similar to those described above and found that 35% of them were rejected. The rejected simulated events showed no biases in their decay time distribution.

Identification of leptons. —We have been able to identify the lepton in 125 of the 242 events remaining after cuts (a)-(g). In the great majority of cases this was done by comparison of track densities, but occasionally decays or $\delta$ rays were used.

We have identified 32 $e^+$, 52 $e^-$, 14 $\mu^+$, and 27 $\mu^-$. The charge of the lepton, but not its type, is determined in nine additional events (five positive and four negative), leaving 108 events whose appearances are ambiguous.

We do not use kinematics to resolve any ambiguities between leptonic-decay hypotheses. Our simulated events revealed that the probability of a leptonic decay fitting an incorrect leptonic hypothesis depends on the length of the neutral track. Events resolved by kinematics would therefore have biased time distributions.

Since we do not use strong interactions of the pions to identify events, our identification procedure is charge symmetric.

Correction for remaining background. —The principal remaining background consists of $K_S\rightarrow\pi^+\pi^-\gamma$ with $p_\gamma>50$ MeV/$c$. To correct for these events, we assume that radiative decay takes place only through inner bremsstrahlung of the two-pion mode.[6] This leads to a prediction of 29 events remaining after our cuts; ten of these should be kinematically unambiguous.[4] In fact, we find 12 kinematically unambiguous events. Their time distribution shows that they are due to $K_S$ decay. Their photon spectrum is in good agreement with inner bremsstrahlung.

We find that the probability of a $\pi\pi\gamma$ decay fitting the hypothesis of $\pi\mu\nu$ decay depends on the length of the neutral track. We therefore retain even the 12 unambiguous $\pi^+\pi^-\gamma$ events, and include a correction term with $K_S$ time dependence in the time distribution of the 108 events in which the lepton is not identified.

We estimate that the other sources of background contribute about one event with the $K_S$ lifetime and about one with the $K_L$ lifetime. We have not corrected for these events.

Maximum-likelihood analysis.—The time distributions of the 242 events are shown in Fig. 1, where the $\Delta S=\Delta Q$ prediction is indicated by solid curves. The distributions predicted by Eq. (3) are multiplied by the geometrical detection efficiency, as determined from the leptonic events themselves. Our methods of lepton identification are charge symmetric. Therefore (a) the relative normalization of the theoretical positive and negative lepton distributions is fixed by the charge ratio predicted by Eq. (3) and the geometrical efficiency, and (b) the predicted distribution of ambiguous events is the sum of the theoretical positive and negative lepton distributions, corrected for the $\pi^+\pi^-\gamma$ contribution. For every event, the uncertainty in the decay time is less than 12%, and we neglect it. We use the maximum-likelihood method to estimate the value of the parameter $x$ in Eq. (3). For events in which the lepton charge is known, we use a likelihood contribution given by Eq. (3), normalized to the time interval in which the event was detectable, and multiplied by the time-integrated probability of the appropriate lepton charge. Thus we fit the observed charge ratio in addition to the shape of the time distribution. For ambiguous events we use the corrected distributions discussed above.

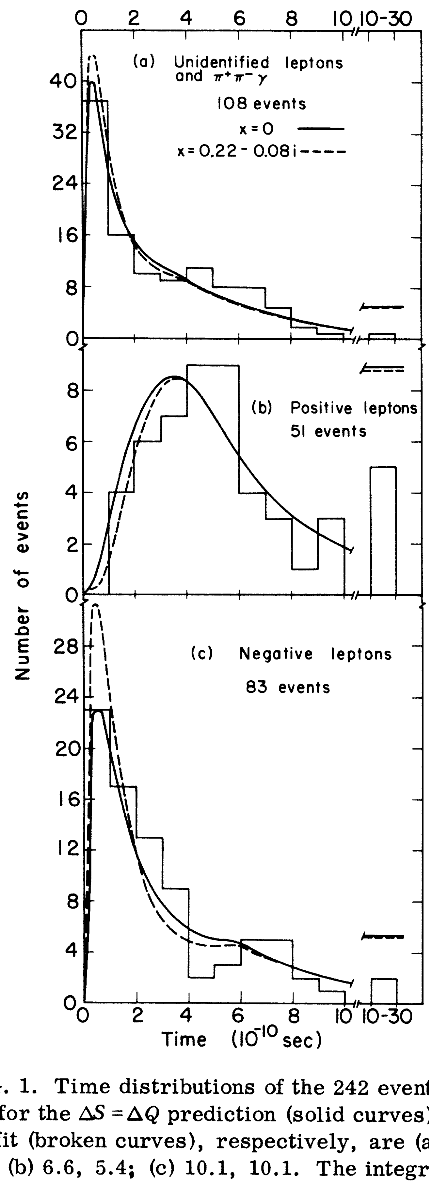


FIG. 1. Time distributions of the 242 events. Values of $\chi^2$ for the $\Delta S=\Delta Q$ prediction (solid curves) and the best fit (broken curves), respectively, are (a) 11.6, 10.5; (b) 6.6, 5.4; (c) 10.1, 10.1. The integrals over the first time bin of the observed counts, the $\Delta S=\Delta Q$ prediction, and the best fit, respectively, are (a) 37, 29.7, 33.1; (b) 0, 1.1, 0.5; (c) 23, 17.8, 23.9.

First we estimate $x$ independently for electrons and muons, using the events in which the lepton has been identified. Contours of equal likelihood in the $x$ complex plane are shown in Figs. 2(a) and 2(b). We find from 84 electrons,

$$\mathrm{Re}(x)=0.27_{-0.13}^{+0.09},\ \mathrm{Im}(x)=0.02_{-0.09}^{+0.11}; \tag{5}$$

from 41 muons,

$$\mathrm{Re}(x)=0.16_{-0.18}^{+0.13},\ \mathrm{Im}(x)=-0.17_{-0.19}^{+0.22}, \tag{6}$$

where the errors refer to the location of the $e^{-0.5}$ relative likelihood contour. Our result (6) is the first published measurement of $x$ for a pure sample of muonic decays.

Results (5) and (6) are consistent with a common value of $x$ for both types of leptons. Therefore we use all our events to estimate this common value. The likelihood contours are shown in Fig. 2(c). We find from all the events

$$\mathrm{Re}(x)=0.22_{-0.09}^{+0.07},\ \mathrm{Im}(x)=-0.08\pm0.08. \tag{7}$$

According to Fig. 2(c) these two results are essentially independent of one another; i.e., they are uncorrelated. They are also sensitive to our choice for $|\delta|$.[7]

The time distributions corresponding to our result (7) are indicated by broken curves in Fig. 1. Our positive result for Re($x$) is due mainly to an excess of about 10 negative leptons in the first $3\times10^{-10}$ sec, and partly to an excess of about 7 unidentified events in the first $10^{-10}$ sec, as would result if we had not eliminated all $K_S\rightarrow2\pi$ background. Accordingly we have increased the severity of cuts (a) to (f) one at a time so as to

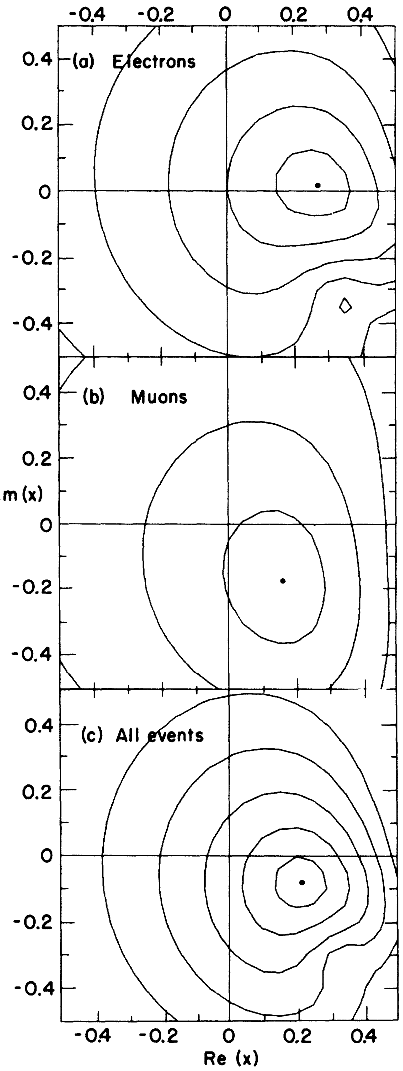


FIG. 2. Contours of equal likelihood in the $x$ plane. In each plot, the solid circle marks the likelihood peak and the contours indicate relative likelihood $e^{-(\frac{1}{2})n^2}$, where $n = 1, 2, 3, 4, 5$.

remove each time about 20 additional events from our sample. These removals have no significant effect on our results, and we are convinced that we have negligible $K_S \to 2\pi$ background.

We make three checks on the internal consistency of our data:

(1) We compute the absolute leptonic decay rate of $K_L$. Assuming $x = 0$ we find $\Gamma(K_L \to \pi l \nu) = 12.2 \pm 1.0$ in units of $10^6$ sec$^{-1}$. Assuming our most likely value of $x$, given by (7), we find $11.5 \pm 0.9$. These may be compared with the current world average value $11.77 \pm 0.40$, or with the value $13.83 \pm 0.35$ predicted by the $|\Delta I| = \frac{1}{2}$ rule from data on $K^+$ decays.[8]

(2) We make a maximum-likelihood estimate of the magnitude of the mass difference. Assuming $x = 0$, we find $|\delta| = (0.41^{+0.13}_{-0.17}) \times 10^{10}$ sec$^{-1}$. Assuming our result (7) we find $|\delta| = (0.49^{+0.08}_{-0.09}) \times 10^{10}$ sec$^{-1}$. The current world average is $(0.55 \pm 0.02) \times 10^{10}$ sec$^{-1}$.

(3) We make an overall $\chi^2$ test of the time distributions shown in Fig. 1. Assuming $x = 0$ we find $\chi^2 = 28.3$, with $\langle \chi^2 \rangle = 31$ (corresponding to 33 bins minus 2 normalization constants). Assuming our result (7) we fine $\chi^2 = 26.0$, with $\langle \chi^2 \rangle \simeq 29$.

We find satisfactory consistency of our data with the "expected" results for these three tests. None of these tests is sensitive to $x$ and we make no use of them in our determination of $x$.[9]

Our conclusions are: Im($x$) is consistent with zero (one standard deviation); thus we find no evidence for a $CP$-nonconserving contribution to $x$. Re($x$) differs from zero by about 2.4 standard deviations; thus we find evidence for a $CP$-conserving violation of the $\Delta S = \Delta Q$ rule.[9] However, we do not regard 2.4 standard deviations as statistically conclusive.

Our result (7) may be compared with those of earlier experiments.[10,11] In the statistically most significant of these, Hill et al.[10] found from a maximum-likelihood analysis of 335 events the result Re($x$) $= 0.17 \pm 0.10$ and Im($x$) $= -0.20 \pm 0.10$. Our result (7) is consistent with that result.

It is a pleasure to acknowledge the interest and support of Luis W. Alvarez. We also thank the 25-in. bubble chamber crew and our scanners and measurers for their help.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]A complete review of the phenomenology of neutral $K$ leptonic decays is given by T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 16, 511 (1966).

[2]We use the values $\lambda_S^{-1} = 0.87 \times 10^{-10}$ sec, $\lambda_L^{-1} = 5.31 \times 10^{-8}$ sec, $\delta = -0.552 \times 10^{10}$ sec$^{-1}$ from the compilation by A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. 40, 77 (1968).

[3]These assumptions are that the form factors $f_-$ and $g_-$, defined in Ref. 1, do not dominate the hadron current.

[4]For all our computer simulation, we use the new Lawrence Radiation Laboratory program PHONY: D. Drijard and E. R. Burns, Jr., private communication. This program reduces Monte Carlo events to sets of coordinate points, simulating the output from a measuring machine. This enables us to study the effects of Coulomb scattering and measurement errors on the fitting of events.

[5]We reject events with $p_{\rm fit}\beta_{\rm fit}\theta < 2200$ (MeV/$c$) deg, where $p_{\rm fit}$ and $\beta_{\rm fit}$ are the fitted momentum and velocity of the "unmeasured" pion, and $\theta$ is the space angle between its fitted and measured directions.

[6]The rate and photon spectrum for this process have been calculated by M. Bég, R. Friedberg, and J. Schultz, as quoted by P. Franzini, L. Kirsch, P. Schmidt, J. Steinberger, and J. Plano, Phys. Rev. 140, B127 (1965), and have been previously verified (with 27 events) by E. Bellotti, A. Pullia, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, H. Huzita, F. Mattioli, and A. Sconza, Nuovo Cimento 45A, 737 (1966).

[7]For example, a change in $|\delta|$ of $\pm 0.05\times 10^{10}$ sec$^{-1}$ gives a change in $x$ of only $\mp 0.01 \mp 0.02i$.

[8]Rosenfeld, Barash-Schmidt, Barbaro-Galtieri, Price, Söding, Wohl, Roos, and Willis, Ref. 2.

[9]In a preliminary report on these data [B. R. Webber et al., University of California Radiation Laboratory Report No. UCRL-18135, 1968 (unpublished)], we gave item 3 (overall $\chi^2$ test) more weight than it deserves. Also, we had not then incorporated the charge ratio into our likelihood function.

[10]D. G. Hill, D. Luers, D. K. Robinson, M. Sakitt, O. Skjeggestad, J. Canter, Y. Cho, A. Dralle, A. Engler, H. E. Fisk, R. W. Kraemer, and C. M. Meltzer, Phys. Rev. Letters 19, 668 (1967). We change the sign of Im($x$) given in that paper, since the value they used for $\delta$ was positive, not negative as stated in their footnote 10 (A. Engler and H. E. Fisk, private communication).

[11]R. P. Ely, W. M. Powell, H. White, M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, O. Fabbri, F. Farini, C. Filippi, H. Huzita, G. Miari, U. Camerini, W. F. Fry, and S. Natali, Phys. Rev. Letters 8, 132 (1962); and G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., ibid. 9, 69 (1962); B. Aubert, L. Behr, F. L. Canavan, L. M. Chounet, J. P. Lowys, P. Mittner, and C. Pascaud, Phys. Letters 17, 59 (1965); M. Baldo-Ceolin, E. Calimani, S. Ciampolillo, C. Filippi-Filosofo, H. Huzita, F. Mattioli, and G. Miari, Nuovo Cimento 38, 684 (1965); Franzini et al., Ref. 5; L. Feldman, S. Frankel, V. L. Highland, T. Sloan, O. B. Van Dyck, W. D. Wales, R. Winston, and D. M. Wolfe, Phys. Rev. 155, 1611 (1967).

# NOTE ON THE CHIRAL-DYNAMIC-LAGRANGIAN CALCULATION OF THE $\chi^0$ DECAY

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The decay rate and the energy asymmetry of the $\eta$ particle observed in the strong decay $\chi\to\eta+2\pi$ are discussed in the context of SU(3)$\otimes$SU(3) chiral dynamic Lagrangian.

The purpose of this note is to discuss the sources of discrepancy in the chiral dynamic prediction of the slope of the Dalitz plot distribution for the strong decay $\chi^0\to\eta\pi\pi$ and to develop a Lagrangian which might explain the energy asymmetry for this decay. This process is of particular interest, since it provides us with a direct test of chiral dynamics in the domain of strong interaction physics.

The basic idea in chiral dynamics[1] is to construct a Lagrangian in which there is a chiral-invariant part and a part in which the chiral symmetry is broken in a definite way. Usually the chiral invariance is broken in such a way that the hypothesis of the partial conservation of axial-vector current (PCAC) is satisfied. Because the status of scalar mesons is presently quite ambiguous, we wish to adopt a formalism which does not require the presence of these fields. As a result of this formalism the Lagrangian becomes a highly nonlinear function of the fields. In a sense, one assigns fields to nonlinear realizations of the chiral group under consideration.

Let us adopt a tensor notation in which the upper (lower) indices transform cogrediently (contragrediently) and the undotted (dotted) indices refer to the transformations generated by $Q_\lambda+Q_\lambda{}^5$ $(Q_\lambda-Q_\lambda{}^5)$ where[2]

$$Q_\lambda=\int d^3x\,V_{\mu=0}{}^\lambda(\vec{x},t)$$

and

$$Q_\lambda{}^5=\int d^3x\,A_{\mu=0}{}^\lambda(\vec{x},t). \qquad (1)$$

We denote by $M$ the pseudoscalar complex

$$M_{\dot\beta}{}^\alpha=(M_\alpha{}^{\dot\beta})^\dagger=(\Sigma+i\Pi)_{\alpha\beta},\quad \alpha,\beta=1,2,3, \qquad (2)$$

where $\Pi$ is the usual $3\times 3$ pseudoscalar meson matrix and $\Sigma$ is to be determined[3] from the constraint

$$\Sigma^2+\Pi^2=f^2, \qquad (3)$$

where $f^2$ is a $c$-number constant.

Let us first analyze Cronin's model. In this model one gets a constant matrix element for the decay $\chi\to\eta\pi\pi$ and a decay rate of $(6.8\pm1.5)$ MeV, which is too large. From the experimental data available at present it seems that this