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[11]The observed mean value $\langle(\Delta x)^2\rangle/\Delta z \simeq 5\times 10^{10}$ yields an rms $\Delta\varphi \sim 0.1$ rad in $\Delta z \sim$A.U. Thus, the distribution in longitude should be $\sim\exp[-(\Delta\phi)^2/(0.1)^2\times 2]$ or a half-width at $(1/e)$ times maximum of about 0.15 rad.

EVIDENCE FOR THE KINEMATIC ORIGIN OF THE H ENHANCEMENT*

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The reaction $\pi^+p\rightarrow\pi^+\pi^-\pi^0\Delta^{++}$ has been studied in the range 3-4 GeV/c of incident pion momenta. Results suggest that the origin of the H enhancement is purely a kinematic one that arises from the ρ-band cut in an essentially structureless three-pion Dalitz plot.

The H enhancement, a peak near 990 MeV in the invariant mass of the neutral three-pion system $(\pi^+\pi^-\pi^0)$ in which at least one pion pair forms a ρ meson $[(\rho\pi)^0$ cut$]$, was first reported by the Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration[1] in the reaction

$$\pi^+p\rightarrow\pi^+\pi^-\pi^0\Delta^{++} \quad (1)$$

at 4 GeV/c. Goldhaber et al.[2] observed a similar, but statistically less significant, peak in the same reaction at 3.65 GeV/c. Subsequently, Benson et al.[3] observed a significant signal for H in the reaction

$$\pi^+d\rightarrow pp(\pi\rho)^0 \quad (2)$$

at 3.65 GeV/c. However, the data of the Bari-Bologna-Firenze-Orsay Collaboration[4] for the same reaction at 5.1 GeV/c yield only a broad shoulder in the H region and do not show the same effect. The similar reaction at 3.29 GeV/c was also studied by Cohn et al.,[5] who concluded that H is present. It may be noted that for all these experiments (i) no significant H enhancement is seen in the three-pion invariant-mass plot until a ρ cut has been effected[6]; (ii) the isospin of H was inferred to be zero from a roughly equal yield of $\rho^+\pi^-$, $\rho^-\pi^+$, and $\rho^0\pi^0$ events, and from the observation in the charged $(\rho\pi)^\pm$ invariant-mass plot of reactions such as $\pi^\pm p\rightarrow\pi^\pm\pi^-\pi^+p$ of a broad A_1 peak at around 1080 MeV instead of an H peak.

The result of our investigation suggests that the origin of the H enhancement is purely a kinematic one, arising from the ρ-band cuts in an essentially structureless three-pion Dalitz plot.

The present experiment involves π^+p interactions at five incident pion momenta: 2.95, 3.2, 3.5, 3.75, and 4.1 GeV/c. The experimental details have been described elsewhere.[7] A total of 9303 events of the reaction

$$\pi^+p\rightarrow\pi^+p\pi^+\pi^-\pi^0 \quad (3)$$

were obtained, of which 3159 events were selected as belonging to Reaction (1) with the Δ^{++} defined by the mass interval $M(\pi^+p)=1220\pm 80$

MeV. The 700 additional events for which more than one π^+p combination fell within the Δ^{++} mass interval were left out of this analysis, and all momenta were combined to increase statistics. It was checked that neither of these procedures affects our conclusions.

The invariant-mass distribution of the three pions from Reaction (1) is shown in the upper histogram of Fig. 1(a) and shows no structure in the 1-GeV region. The ω^0 and η^0 bands are cut out for display purposes. However, upon demanding that at least one ρ ($M_{\pi\pi} = 740 \pm 80$ MeV) be formed, a general enhancement in that region appears, as shown in lower histogram in Fig. 1(a). It has been suggested[8] that for spin-parity determination of the resonance it is desirable to avoid interference effects from overlapping bands by separating the events into three mutually exclusive sets: $\rho^-\pi^+$, $\rho^0\pi^0$, and $\rho^+\pi^-$. The resulting invariant-mass plots from our data [Figs. 1(e)-1(g)] all show two prominent peaks centered about 1.0 and 1.3 GeV, respectively, at first glance presumed to correspond to the H and the A_2. However, the A_2, being $I=1$, cannot decay into $\rho^0\pi^0$, and it is difficult to explain this peak as due to overlap from the $\rho^\pm\pi^\mp$ events. Furthermore, no reasonable background could mask the prominent peaks near 1.0 GeV so as to give the structureless 3π mass plot observed [Fig. 1(a)]. We have examined the 3π Dalitz-plot distribution as a function of 3π invariant mass as well as scatter plots of $M(2\pi)$ vs $M(3\pi)$ (not illustrated). Some $\rho^\pm$ enhancement is seen in the A_2 region, but no ρ cluster is evident in the H-mass region. These striking features prompted us to look into the possibility that the ρ-band cut, because of its large width and because of the unusual overlapping of the three ρ bands in the Dalitz plot,[9] could produce spurious peaks in this region.

We calculated the fraction of the area of the 3π Dalitz plot for which at least one pion pair falls in the ρ band, as a function of the three-pion mass. This is shown as the upper curve in Fig. 1(b). This type of curve will hereafter be referred to as the Dalitz-plot fraction (DPF) curve. It shows a sharp peak near 1 GeV followed by a dip and then a broader enhancement near 1.3 GeV. In the lower plot of Fig. 1(b) we show the histogram obtained from multiplying the DPF curve by our experimental 3π invariant-mass distribution [Fig. 1(a)].[10] This is then the distribution expected from making the $(\rho\pi)^0$ cut on an isotropic Dalitz plot. A broad enhancement in the 1-GeV region appears which com-

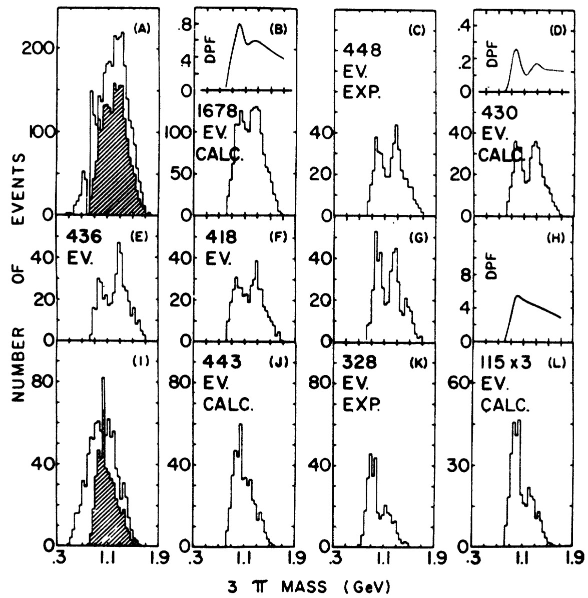

FIG. 1. (a) Uncut 3π distribution. Shaded histogram, $(\rho\pi)^0$ cut. (b) Upper curve is DPF curve for $(\rho\pi)^0$ cut; ρ band is 740 ± 80 MeV. Histogram is the uncut curve in (a) multiplied by upper curve. (c) Average of experimental ME histograms for $\rho^-\pi^+$, $\rho^0\pi^0$, $\rho^+\pi^-$. (d) Upper curve is DPF curve for ME cut; ρ band is 740 ± 80 MeV. Histogram is uncut curve in (a) multiplied by upper curve. (e)-(g) $\rho^-\pi^+$, $\rho^0\pi^0$, and $\rho^+\pi^-$ ME histograms. (h) DPF curve for $\pi^+\pi^+\pi^\pm\pi^-$ with either $\pi^+\pi^-$ pair in ρ band. (i)-(l) equivalent of (a)-(d), respectively, for the reaction $\pi^+p \to \Delta^-\pi^+\pi^+\pi^+$.

pares well with the distribution obtained by making the same $(\rho\pi)^0$ cut on the actual data [lower histogram in Fig. 1(a)]. Similarly, a DPF curve for the mutually exclusive cut (ME cut) is shown in the upper curve of Fig. 1(d). Multiplying this curve by our uncut 3π distribution we produce peaks in the H and A_2 regions [Fig. 1(d)]. This is to be compared with the average $(\rho^{\pm,0}\pi^{\mp,0})$ distribution obtained by making the same experimental ME cut on the actual events [Fig. 1(c)]. The agreement is excellent. We can now look separately at the $\rho^-\pi^+$, $\rho^0\pi^0$, and $\rho^+\pi^-$ distributions in Figs. 1(e)-1(g). The larger $(\rho^\pm\pi^\mp)$ peaks around 1.3 GeV are certainly due to the genuine A_2 production already evident in the uncut 3π distribution. The absence of structure in the 3π Dalitz plot would produce equal numbers of $\rho^+\pi^-$, $\rho^0\pi^0$, and $\rho^-\pi^+$. The larger $\rho^+\pi^-$ peak at 1.0 GeV can come from the reaction

$$\pi^+p \to \rho^+N^{*+}$$
$$\quad\quad\hookrightarrow \pi^- + \Delta^{++}.$$

This production of higher N^* resonances and their subsequent cascade decay has been observed in our data. A Deck-type mechanism would also favor the ρ^+ production.

To differentiate this kinematic effect from a true resonance two tests were made: (1) For a true $(\rho\pi)$ resonance, narrowing the ρ-width cut will include less background and the resonance should stand out more. On the other hand, if our assumption is true and the H peak arises from the large ρ-width cut, then the wider the cut, the more the peaking. Since for our energy region the H peaking in the $(\rho\pi)^0$ system occurs on a steeply rising portion of the mass distribution, we make an equivalent, but more effective, display by plotting the 3π events where no ρ is formed (referred to as no-ρ cut). The DPF curve for this is the inverse of the DPF curve for the $(\rho\pi)^0$ cut, i.e., a peaking in the $(\rho\pi)^0$-cut distribution would correspond to a dip in the no-ρ distribution. In Fig. 2(a) we show the DPF curves for the no-ρ cut as a function of the ρ width while holding the ρ mass fixed at (a) 740 ± 120, (b) 740 ± 80, and (c) 740 ± 40 MeV. The equivalent DPF curves for the ME cuts are shown in Fig. 2(b). As can be seen, our corresponding experimental curves in Fig. 2(e) for the no-ρ cut show a deeper dip as the ρ width is increased, going almost to zero at 120 MeV as predicted.[11] For the ME set the experimental results in Fig. 2(f) (only $\rho^0\pi^0$ is illustrated) show increased peaks when wider ρ widths are used. (2) If the H peaking arises from the ρ cut, then by shifting the "ρ"-band center, the dips and the peaks should move accordingly. In Figs. 2(c) and 2(d) we display the DPF curves for the no-ρ cut and the ME cut, respectively, holding the ρ width constant (80 MeV), but shifting its center of mass [(a) 650, (b) 740, and (c) 850 MeV]. Our corresponding experimental results for these cuts are shown in Figs. 2(g) and 2(h), and the dips and the peakings indeed move with the cuts.

As a further check, we have selected 940 events of the type $\pi^+p \rightarrow \pi^+\pi^+\pi^+\pi^-n$ from the same experiment in which the π^-n system lies in the Δ mass band. We have carried out the same analysis on these unrelated three π^+'s. The three-pion $(3\pi^+)$ mass distribution is structureless but peaks at lower masses than in Reaction (1). Again an enhancement is produced by our hypothetical $(\rho\pi)$ cuts; the corresponding peaks and dips are again generated by the various cuts [Figs. 1(i)-1(l), and 2(i)-(l)].

Since the A_1 is not observed in the neutral 3π system and since it has been speculated that the

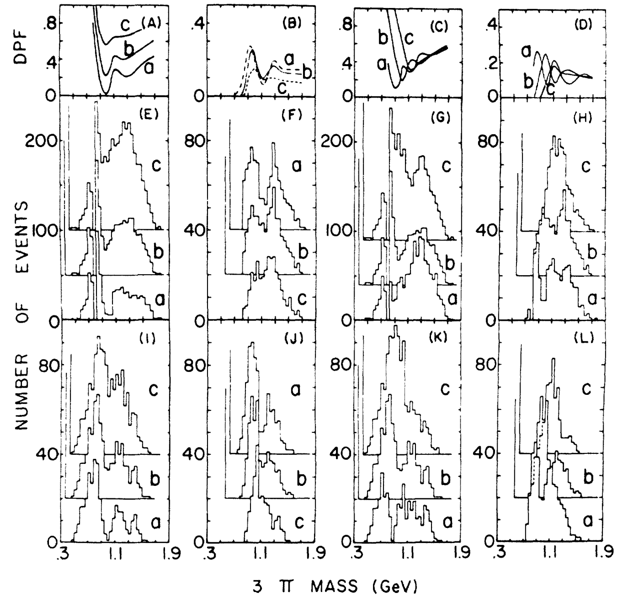

FIG. 2. (a) DPF curves for no-ρ cuts with center of ρ band at 740 MeV and widths a, ± 120; b, ± 80; and c, ± 40 MeV, respectively. (b) DPF curves for ME cuts; a, b, and c, same masses and widths as in (a). (c) DPF curves for no-ρ cuts with width of ρ cut fixed at ± 80 MeV and center of bands at a, 650; b, 740; and c, 850 MeV, respectively. (d) DPF curves for ME cuts; a, b, and c, same masses and widths as in (c). (e) and (i) Experimental no-ρ histograms for $\pi^+\pi^-\pi^0$ and $\pi^+\pi^+\pi^+$ events, respectively. ρ mass and widths chosen as in (a). (f) and (j) Experimental ME histograms for $\pi^+\pi^-\pi^0$ (only $\rho^0\pi^0$ shown) and $\pi^+\pi^+\pi^+$ events, respectively. ρ mass and widths chosen as in (b). (g) and (k) Experimental no-ρ histograms for $\pi^+\pi^-\pi^0$ and $\pi^+\pi^+\pi^+$ events, respectively. ρ masses and widths chosen as in (c). (h) and (l) Experimental ME histograms for $\pi^+\pi^-\pi^0$ (only $\rho^0\pi^0$ shown) and $\pi^+\pi^+\pi^+$ events, respectively. ρ masses and width chosen as in (d).

H may be the A_1 meson displaced, we calculated the DPF curve for a 2-ρ-band cut, which corresponds to the usual charged-A_1 cut. This is shown in Fig. 1(h). Interestingly it shows a slower decline on higher 3π invariant masses, and the peak of the DPF curve moves to around 1040. In the past, the Deck effect and other similar mechanisms have been suggested for the production of the A_1 effect.[12] Calculations on these effects often yielded a broad enhancement which could not completely explain the observed sharper A_1 peaks. In view of our discussions on the origin of the H enhancement, one is tempted to speculate that a combination of a Deck-type effect plus a ρ-cut effect could give the experimental A_1 peak in our range of beam momenta.

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[10]Benson et al. (Ref. 3 above) multiplied the DPF curve by a three-pion phase-space distribution. This distribution peaked towards higher masses than the experimental 3π spectrum, diluting the effects observed here. Such processes as higher N^* cascading into Δ^{++} and the general peripheral characteristics of production processes in this energy region lead us to believe the phase space to be a poor approximation of the background.

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OBSERVATION OF A $\rho^-\rho^0$ ENHANCEMENT AT 1710 MeV †

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In this Letter we present evidence for a strong $\rho^-\rho^0$ enhancement in the reaction $\pi^-p\to p\rho^-\rho^0$ at 8 GeV/c. We interpret the enhancement as a resonance with a mass of 1710 ± 23 MeV and a width of 162^{+58}_{-40} MeV. Branching ratios for different decay modes of the resonance are studied. The relation of this resonance to the previously reported[1-8] $g(1650)$ is discussed.

In this experiment we have studied the following reactions:

$$\pi^-p\to p\pi^+\pi^-\pi^-\pi^0 \text{ (1970 events)}, \tag{1}$$

$$\to n\pi^+\pi^+\pi^-\pi^- \text{ (783 events)}, \tag{2}$$

$$\to n\pi^+\pi^- \text{ (1321 events)}, \tag{3}$$

and

$$\to p\pi^-\pi^0 \text{ (708 events)}, \tag{4}$$

at 8 GeV/c. The interactions occurred in the Brookhaven 80-in. hydrogen bubble chamber and were separated from samples of about 20 000 two-prong and 20 000 four-prong interactions on the basis of GRIND fits and requirements of ionization consistency. Details of the event separation procedure will be published elsewhere.[9]

The $\pi^+\pi^-$ effective-mass distribution for events of Reaction (1) which are not in the $N^{*++}(1236)$ region is shown in Fig. 1(a). The curve shown is a maximum likelihood fit to the data of the form $PS(1+a\text{BW})$. The parameter a is proportional to the intensity of BW, a constant width Breit-Wigner resonance function. PS is a phase space for Reaction (1) modified by the factor e^{bt}, where t is the momentum transfer from the target to the final-state proton. To fit the distribution of this momentum transfer, a value of 3.5 was chosen for b. Production of the ρ^0 is evident. Our fit indicates that 219 ± 29 events involve ρ^0 production. An identical analysis of the $\pi^-\pi^0$ mass distribution indicates the presence of 116 ± 33 events involving ρ^- production. Since these fits were performed independently, they do not indicate the presence or absence of the reaction

$$\pi^-p\to p\rho^-\rho^0 \tag{5}$$

in our data.