[2]A. A. Penzias and R. W. Wilson, Astrophys. J. 142, 419 (1965); P. G. Roll and David T. Wilkinson, Phys. Rev. Letters 16, 405 (1966), and Ann. Phys. (N.Y.) 44, 289 (1967); George B. Field and John L. Hitchcock, Phys. Rev. Letters 16, 817 (1966); Patrick Thaddeus and John F. Clauser, Phys. Rev. Letters 16, 819 (1966); T. F. Howell and J. R. Shakeshaft, Nature 210, 1318 (1966), and 216, 753 (1967); W. J. Welch, S. Keachie, D. D. Thornton, and G. Wrixon, Phys. Rev. Letters 18, 1068 (1967); M. S. Ewing, B. F. Burke, and D. H. Staelin, Phys. Rev. Letters 19, 1251 (1967); V. I. Puzanov, A. E. Salomonovitch, and K. S. Stankevich, Astron. Zh. 44, 1128 (1927).

[3]David T. Wilkinson, Phys. Rev. Letters 19, 1195 (1967); R. A. Stokes, R. B. Partridge, and David T. Wilkinson, Phys. Rev. Letters 19, 1199 (1967).

DIPS IN DIFFERENTIAL CROSS SECTIONS*

Norman E. Booth
The Enrico Fermi Institute and Department of Physics, The University of Chicago, Chicago, Illinois 60637
(Received 18 December 1967; revised manuscript received 24 April 1968)

We suggest that a dip at $t=-3$ (GeV/c)^2 which appears in both π^+p and π^-p scattering over a wide range of laboratory momentum is related to the dips in the 180-deg cross sections at 2.1 GeV/c. The energy dependence of the cross sections in the vicinity of the dip is consistent with a Regge trajectory for which $\alpha=-2$ at $t=-3$.

In a recent experiment at the Brookhaven National Laboratory alternating-gradient-synchrotron the angular distribution in π^-p elastic scattering at 5.9 GeV/c was measured with considerable precision out to $t=-6$ (GeV/c)^2.[1] A particularly interesting feature of the data is the sharp dip at about $t=-3$ (GeV/c)^2. The dip was also observed at 7.9 GeV/c. It seems plausible that this dip could correspond to a second diffraction minimum, the first being associated with the kink in the cross section at about $t=-1$ (GeV/c)^2.[2] The structure could also be associated with plural scattering, for example in a quark model.[3]

At present there are no other such detailed measurements at higher momenta. However, data are available at lower momenta and they also show evidence for a dip at about $t=-3$, a fact overlooked until now. In Fig. 1(a) we have plotted some of the results available.[4-6] The curve near the bottom of Fig. 1(a) illustrates the shape of the data at 5.9 GeV/c. The errors are small enough to establish definitely the existence of the dip. The data at 3.5 and 3.0 GeV/c also show evidence for a dip at $t=-3$. Dips at the same value of t are also apparent at 2.5 and 2.28 GeV/c. This is somewhat surprising for several reasons. First, the dip is approaching 180 deg (indicated by the vertical lines and hatched areas of Fig. 1) as the momentum is decreased, and contributions from u-channel baryon exchange are expected to become appreciable. Secondly, there are several resonances in this energy region and we might expect any behavior at fixed t to be modified extensively by contributions from the resonances. The dip at $t=-3$ at the higher momenta could be in the spin-nonflip or the spin-flip amplitude, or both.

If we extrapolate the dip at $t=-3$ to lower momenta, it crosses 180 deg into the nonphysical region at 2 GeV/c.[7] Surprisingly enough, the cross section at 180 deg measured as a function of momentum exhibits a very sharp dip at just about 2 GeV/c.[8] This dip must be in the spin-nonflip amplitude since the spin-flip amplitude is required to vanish at 180 deg. Is this just a coincidence or are the two phenomena related—the dip at $t=-3$ in the angular distribution data, which when extrapolated to lower momenta crosses 180 deg at 2 GeV/c, and the dip in the 180-deg cross-section data at 2 GeV/c?[9] We will attempt to amswer this question later.

In Fig. 1(b) we have plotted the angular distributions for π^+p scattering at several momenta.[4,6,10] Some data are available at 8 GeV/c (not plotted here) which suggest a dip at $t=-3$.[11] The data at 4 GeV/c [see Fig. 1(b)] give only a slight indication of a dip at $t=-3$.[12] However, there are definite dips at 3.7, 3.5, and 3.0 GeV/c. At lower momenta, although there are valleys in the vicinity of $t=-3$, we would again expect a fixed-t behavior to be modified by other effects. As for π^-p, an effect at $t=-3$ coincides with 180 deg at 2 GeV/c. In a recent experiment the angular distributions for π^+p scattering in the backward direction were measured at several momenta. The 180-deg cross section as a function of mo-

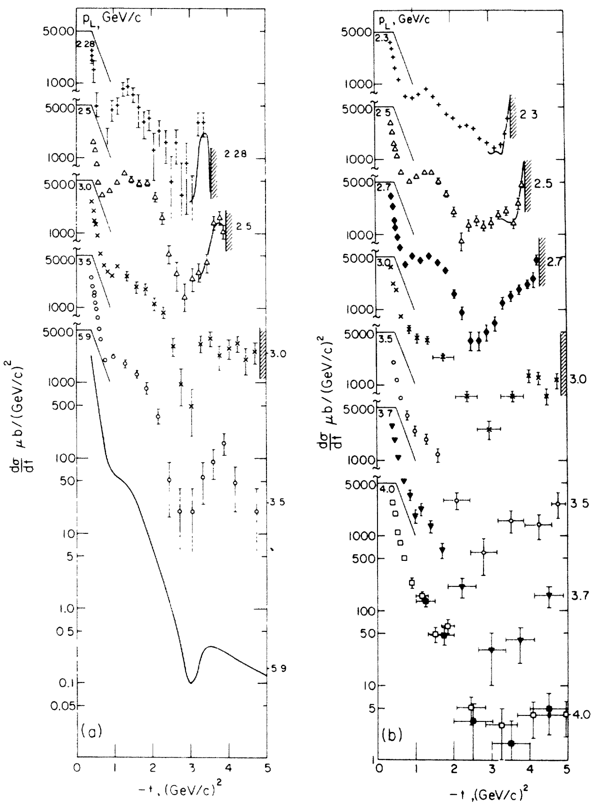

FIG. 1. Differential cross sections for $\pi^{\pm}p$ elastic scattering exhibiting the dip at $t=-3$ $(\mathrm{GeV}/c)^2$. (a) π^-p. Data represented by the solid curve at 5.9 GeV/c are from Ref. 1. Data points at 3.5, 3.0, and 2.5 GeV/c, from Ref. 5. The curves at 2.5 and 2.28 GeV/c show the trend of the data of Ref. 6. (b) π^+p. All points are from Ref. 4 except the solid squares at 4.0 GeV/c which are from Ref. 10. The solid curves at 2.5 and 2.3 GeV/c show the trend of data of Ref. 6.

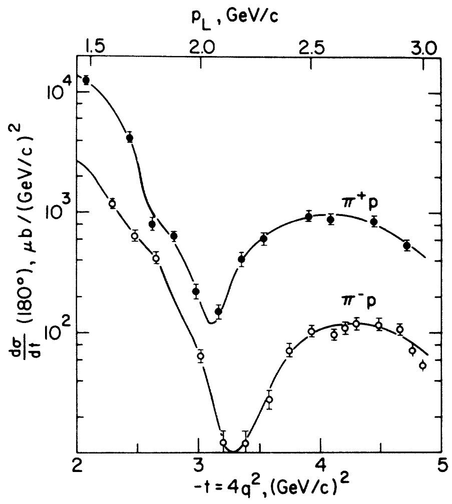

FIG. 2. Differential cross sections for $\pi^{\pm}p$ scattering at 180 deg. plotted versus t. The data for π^-p are from Ref. 8; for π^+p, from Ref. 6.

mentum exhibits a striking dip at 2 GeV/c, similar in shape to the π^-p dip and centered at almost the same momentum.[6]

Thus, there are dips at $t=-3$ in the angular distributions for both π^-p and π^+p scattering at high momenta. Also, there are dips in the 180-deg data for both π^-p and π^+p at about 2 GeV/c. Finally, 2 GeV/c at 180 deg corresponds to $t=-3$. All the dips occur at the same value of t. For reference, the behavior of the 180-deg cross sections is shown in Fig. 2.

It is tempting to speculate that the dips in the 180-deg data are due (at least partially) to the dip at constant t; as the dip approaches the low-energy region and eventually 180 deg, it forces the partial-wave amplitudes to create resonances to accommodate it. However, this view is too simplistic even though there is evidence through finite-energy sum rules that t-channel exchange amplitudes and direct-channel resonances are very closely related.[13]

It is interesting to look at the energy dependence of the cross sections in the vicinity of the dip at the higher momenta to see if it might be associated with a zero in a Regge-pole amplitude.[14] Recall that the dip at $t=-0.6$ in charge-exchange scattering was explained by the fact that the spin-flip amplitude must vanish when $\alpha_\rho(t)=0$. The trajectory was obtained for $-t \leq 1.6$ from the energy dependence of the cross sections.[15] The elastic cross sections also exhibit a dip at about $t=-0.8$ and a secondary maximum similar in shape to the charge-exchange cross sections.[16] They also fall off rapidly with increasing energy at large values of $-t$. However, the ρ trajectory cannot be invoked to explain the elastic-scattering data, since the charge-exchange cross sections, are, on the average, an order of magnitude smaller than the elastic cross sections; dominance by the ρ trajectory would require them to be a factor of 2 larger. It appears then that the elastic cross sections are dominated by the P' trajectory for values of $-t$ larger than about 0.8.[17]

We have used the expression

$$(d\sigma/dt)_{\pm} = (1/p_L^{\,2})|a\exp(\tfrac{1}{2}i\pi)\nu + C_{\pm}\exp(i\varphi_{\pm})\nu^{\alpha_{\pm}}|^2 \qquad (1)$$

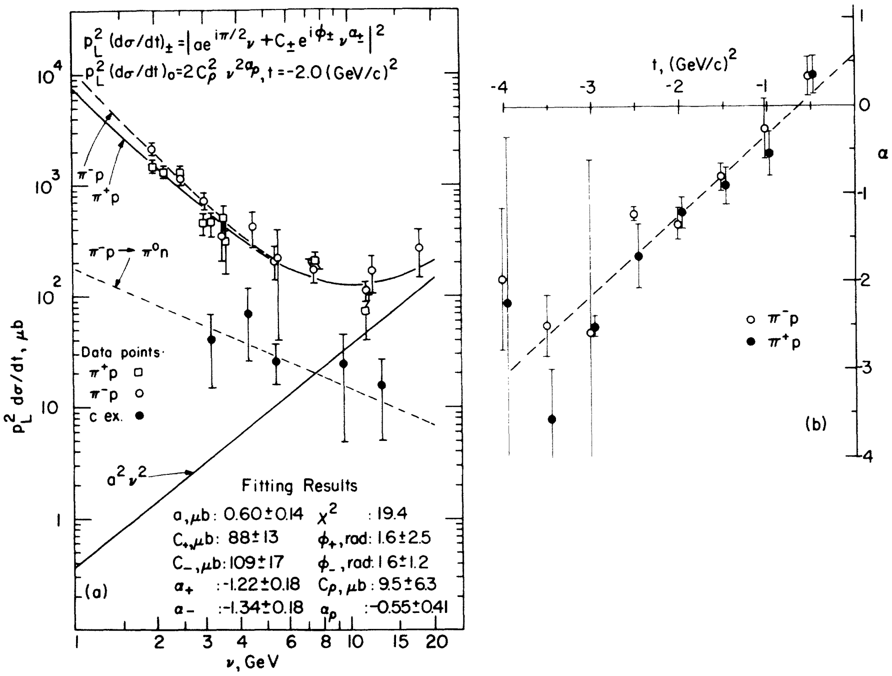

FIG. 3. Results of fitting Eq. (1) to experimental data between 2.5 and 18 GeV/c. (a) Fits at $t=-2.0$ (GeV/c)^2. The laboratory momentum of each datum point may be found by adding $-t/4m = 0.53$ to ν. (b) Values of $\alpha^\pm(t)$. The assigned errors come from the diagonal elements of the error matrix in a seven-parameter simultaneous fit to π^+p and π^-p data at fixed t. The dashed line shows for comparison the ρ trajectory, $\alpha_\rho(t) = 0.57 + 0.91t$, obtained in Ref. 15 and extrapolated to large $|t|$.

to fit the available cross-section data between 2.5 and 18 GeV/c over the range of t from -0.5 to -4.0.[18] Here a is the amplitude corresponding to a flat Pomeranchuk trajectory or to absorption, and $c_\pm$, $\varphi_\pm$, and $\alpha_\pm$ are the parameters of effective Regge trajectories, one for π^+p and one for π^-p. All parameters are functions of t and ν is in units of $\nu_0 = 1$ GeV. The second term of Eq. (1) should be the same for π^+p and π^-p to the extent that the cross sections are dominated by the P' amplitude. However, to be more general and to allow for the ρ-trajectory contribution, we permit this term to be different for π^+p and π^-p.

Good fits to the data are obtained with Eq. (1). An example of the results, namely at $t=-2$, is shown in Fig. 3(a). For large $|t|$, the π^+p and π^-p cross sections in the range of roughly 3-10 GeV/c are mainly associated with the second term. The first term becomes important only at higher momenta. The values of $\alpha_\pm$ obtained, which are shown in Fig. 3(b), are consistent with a linear trajectory for the P'. The slope and intercept at $t=0$ are similar to those of the ρ trajectory.

One interesting feature is that $\alpha_\pm$ pass through -1. In the standard Regge-pole formalism, the amplitude of an even-signature trajectory, such as the P', vanishes at $\alpha=-1$.[19] However, $c_\pm$ do not vanish at $\alpha=-1$ (in fact they tend to peak there) and they are larger than c_ρ deduced from the charge-exchange cross section,[20]

$$(d\sigma/dt)_0 = (2/p_L^2)C_\rho^2\nu^{2\alpha_\rho}.$$

Thus, $c_\pm$ are dominated by the P' and one must conclude that the standard Regge formalism needs some modification. This has also been suggested by Barger and Phillips and our results are in good qualitative agreement with their Ansatz for the P' residue.[21] Our results disagree with some features of the Regge-pole model of Chiu, Chu, and Wang.[19] Their fit forced the P' trajectory to level off above $\alpha=-1$ in order to avoid the zero which was present in their formalism. As a consequence their model does not fit the data very well beyond $t=-2$. Throughout the range of t we find that $c_+ \approx c_-$ except in the vicinity of $t=-3$ where $c_+ > c_-$.[22] Here there appears to be evidence for interference between $I=0$ and $I=1$ exchange amplitudes or for direct-channel effects which make the π^+p and π^-p cross sections different, particularly at lower momenta.

If the dip at $t=-3$ is due to the vanishing of a Regge amplitude (most likely the P') at negative integral α, this occurs at $\alpha=-2$. The dip is not due to the possibility that $\alpha=0$ for the P trajectory at $t=-3$.[23]

It is a pleasure to thank P. G. O. Freund, R. K. Logan, D. I. Meyer, M. Ross, and C. Schmid for helpful discussions. Dr. A. Beretvas assisted in the fitting.

---

*Work supported by the National Science Foundation under Grant No. GP-6135.

[1]A. Ashmore, C. J. S. Dammerell, W. R. Frisken, R. Rubinstein, J. Orear, D. P. Owen, F. C. Petersen, A. L. Read, D. G. Ryan, and D. H. White, Bull. Am. Phys. Soc. 12, 1050 (1967), and Phys. Rev. Letters 21, 387 (1968).

[2]D. H. White (Cornell), talk presented on the work of Ref. 1 at the Conference on π-N Scattering, University of California, Irvine, Calif., December, 1967 (unpublished).

[3]D. Harrington and A. Pagnamenta (Rutgers), private communication.

[4]C. T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams, Phys. Rev. 159, 1169 (1967).

[5]W. Busza et al., Nuovo Cimento 52A, 331 (1967).

[6]A. S. Carroll, J. Fischer, A. Lundby, R. H. Phillips, C. W. Wang, F. Lobkowicz, A. C. Melissinos, Y. Nagashima, and S. Tewksbury, Phys. Rev. Letters 20, 607 (1968).

[7]In general $t=-2q^2(1-\cos\theta)$, where $q=mp_L/\sqrt{s}$ is the momentum in the c.m. system. Here m is the nucleon mass, p_L the lab momentum of the pion, and $\sqrt{s}$ the total energy in the c.m. system.

[8]S. W. Kormanyos, A. D. Krisch, J. R. O'Fallon, K. Ruddick, and L. G. Ratner, Phys. Rev. 164, 1661 (1967).

[9]The possibility that the dip in the π^-p 180-deg cross section might be associated with a phenomenon at fixed t was suggested to the author by C. N. Yang at the Stony Brook Conference, April, 1966 (unpublished).

[10]Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Letters 10, 248 (1964).

[11]J. Orear et al., Phys. Rev. 152, 1162 (1966).

[12]Note that the width of the experimental bins is so large as almost to obliterate a possible sharp dip.

[13]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. Letters 19, 402 (1967); C. Schmid, Phys. Rev. Letters 20, 689 (1968).

[14]This was suggested to the author by R. K. Logan.

[15]See, for example, G. Höhler, J. Baake, H. Schaile, and P. Sonderregger, Phys. Letters 20, 79 (1966).

[16]For charge-exchange data see I. Mannelli et al., Phys. Rev. Letters 14, 408 (1965); A. V. Stirling et al., ibid. 14, 763 (1965); P. Sonderegger et al., Phys. Letters 20, 75 (1966).

[17]This has been noticed before. See, for example, C. B. Chiu, S. Y. Chu, and L. L. Wang, Phys. Rev. 161, 1563 (1967), and V. Barger and R. J. N. Phillips, Phys. Rev. Letters 20, 564 (1968).

[18]The data of Refs. 1, 4, 10, and 11 were supplemented by data at 12.4 and 18.4 GeV/c from D. Harting et al., Nuovo Cimento 38, 60 (1965).

[19]Chiu, Chu, and Wong, Ref. 17.

[20]A fit of this expression to the limited charge-exchange data at $t=-2$ is shown in Fig. 3(a).

[21]Barger and Phillips, Ref. 17.

[22]It has been pointed out to us that, to the extent that $\alpha_{P'}=\alpha_\rho$, it follows that $|c_+|=|c_-|$. However, the fact that $\alpha_{P'}\approx\alpha_\rho$ is a result, not an assumption.

[23]C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. 153, 1485 (1967).

MODEL FOR ASTRONOMICAL PULSED RADIO SOURCES*

G. Reading Henry
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California
(Received 24 April 1968)

A very simple model is suggested for the recently reported pulsed radio sources based on the vibrational (rather than compressional) modes of a star.

In order to account for the pulsed radio sources recently discovered,[1] models of varying degrees of complexity have been proposed.[2] While one or more of these proposals may well turn out to be correct, we wish to suggest a "simplest possible" model which appears capable of explaining roughly some of the features observed, and which suggests new properties that may be found in future observation.

A uniform, incompressible liquid drop (polytrope with index $n=0$) acting under gravitational restoring forces may undergo vibrational motion where the surface is deformed in modes corresponding to spherical harmonics of multipolarity l, $l\geq 2$. The frequencies are given by[3]

$$\omega_l^2=\frac{8\pi}{3}\frac{l(l-1)}{(2l+1)}G\rho, \tag{1}$$

where G is the gravitational constant and ρ is the density. Quadrupolar periods are listed in Table I for various densities. Equation (1) is of course only a rough estimate when applied to a neutron