[4]See, for instance, R. Rajaraman and H. A. Bethe, Rev. Mod. Phys. 39, 745 (1967).

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[6]For the Rarita-Schwinger propagator we use the form given by S. Gasiorowicz, Elementary Particle Physics (John Wiley & Sons, Inc., New York, 1966), p. 430.

[7]The values of λ and Λ adjusted to obey Adler's consistency condition reproduce, respectively, $\frac{2}{3}$ and $\frac{1}{2}$ the measured N^* width in the static limit. Thus, the crude narrow N^* dominance is a reasonable approximation.

RE-EXAMINATION OF THE NUCLEAR ISOMER SHIFT AS MEASURED IN MUONIC ATOMS*

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The magnetic hyperfine splitting for rotational levels of deformed, even-even nuclei in muonic atoms is shown to result in an asymmetric γ-ray doublet, whose center of gravity is in general shifted towards lower energies. This pseudoisomer shift is of the order of magnitude of the reported "isomer" shifts.

The cascade of the muon from the high-lying muonic orbits to the muonic ground state of deformed nuclei is often accompanied by the excitation of nuclear rotational levels.[1,2] As the lifetime of the muon in the $1s$ state is long compared with the lifetime of the rotational levels, the deexciting nuclear γ ray is emitted in the presence of the $1s$ muon with the result that the transition energies differ from the respective energies in the absence of the "spectator muon."[3,4] Such γ rays have been observed for transitions in a number of deformed nuclei from ^{150}Nd to ^{186}W.[5-7] The observed energy shifts of the radiation, assumed to be an unsplit line, have been interpreted as arising entirely from the radius difference between the ground and excited states of the host nucleus, i.e., as isomer shifts. In this note we point out that the nuclear transition, in the presence of the muon, is in general an asymmetric doublet whose center of gravity is shifted from that of the unsplit line even in the absence of an isomer shift. The shift of the center of gravity of the decay spectra arises from two effects. First, there is a nonstatistical feeding of the nuclear magnetic hyperfine levels, and second, the $M1$ intradoublet transition enhances the population of the lower hyperfine level. The latter effect generally dominates, resulting in a shift of the center of gravity which is of the same order of magnitude as the observed energy shifts. Nuclear polarization phenomena are considered in the second half of this Letter.

The Hamiltonian of the muonic atom,

$$H=H_N+[T(\mu)+V(r_\mu)]+(H^C+H^M) \tag{1}$$

consists of three parts: (i) the nuclear Hamiltonian H_N in the absence of the muon [eigenstates $|\Psi_\alpha(1,\cdots,A)\rangle$]; (ii) the muonic Hamiltonian, $[T(\mu)+V(r_\mu)]$, with the average Coulomb potential $V(r_\mu)$ due to the charge distribution of the nuclear ground state (eigenstates $|nlj\rangle$); and (iii) the residual Coulomb force, $H^C=-e^2\sum_p|\vec{r}_\mu-\vec{r}_p|^{-1}-V(r_\mu)$, together with the magnetic interaction H^M between the muon and the nucleus. The last part, H^C+H^M, is usually considered to be small and can be treated in perturbation theory, at least for the muon in the $1s$ orbit.

The isomer shift between two nuclear states Ψ_α and Ψ_β is defined as the energy difference

$$\Delta E_{\alpha,\beta}^{\text{isomer}}=\langle\Psi_\alpha,1s_{\frac{1}{2}}|H^C|\Psi_\alpha,1s_{\frac{1}{2}}\rangle-\langle\Psi_\beta,1s_{\frac{1}{2}}|H^C|\Psi_\beta,1s_{\frac{1}{2}}\rangle, \tag{2}$$

where $1s_{1/2}$ denotes the spectator muon. The energy, ΔE^{isomer}, depends essentially on the radius difference between the two nuclear states and is a quantity which contains valuable information about the nucleus. Each nuclear state Ψ_α with spin $I_\alpha\neq 0$ also exhibits a hyperfine splitting which originates from the interaction of the nucleus with the magnetic moment of the $1s$ muon. The splitting is given in first order by

$$\Delta E_\alpha^M=\langle\Psi_\alpha,1s_{\frac{1}{2}};\ F=I_\alpha+\tfrac{1}{2}|H^M|\Psi_\alpha,1s_{\frac{1}{2}};\ F=I_\alpha+\tfrac{1}{2}\rangle-\langle\Psi_\alpha,1s_{\frac{1}{2}};\ F=I_\alpha-\tfrac{1}{2}|H^M|\Psi_\alpha,1s_{\frac{1}{2}};\ F=I_\alpha-\tfrac{1}{2}\rangle, \tag{3}$$

Table I. Hyperfine center-of-gravity shift for $E2$ transitions in several even-even deformed nuclei.

Isotope (2^+ Energy) (keV)	ΔE^M (eV)	Feeding $R_F(\frac{3}{2}:\frac{5}{2})$	Electron Binding Energy[f] (eV)	M1-Int. Conv. Coeff. $\times 10^{-4}$	Branch. Ratio $R_T(\frac{M1}{E2})$	$\frac{I(\frac{3}{2}\to g.s.)}{I(\frac{5}{2}\to g.s.)}$	Cent.-of-Grav. Shift (eV)	Experimental Shifts (eV)
^{152}Sm (122)	740[a)] 490[b)]	0.18	333(N_I)	1.7 5.4	7 7	8.5 8.5	-360 -240	+560 ± 50[g)]
^{166}Er (81)	570[c)]	0.54	435(N_I)	6	15	24	-320	-350 ± 100[h)]
^{170}Yb (84)	615[d)]	0.55	471(N_I)	6	16	26	-350	
^{182}W (100)	495[e)]	0.48	565(N_I) 71(O_I)	24 4	30 5	45 8	-290 -240	-320 ± 100[h)]
^{184}W (111)	550[e)]	0.38	565(N_I) 71(O_I)	16 3	24 5	34 7	-315 -250	-340 ± 100[h)]
^{186}W (122)	580[e)]	0.31	565(N_I) 71(O_I)	13 2.5	20 4	25 5	-340 -250	-360 ± 100[h)]

[a]Using $g_{2^+}(^{152}Sm) = 0.416 \pm 0.025$: U. Atzmony, E. R. Bauminger, D. Froindlich, and S. Ofer, Phys. Letters 26B, 81 (1968).

[b]Using $g_{2^+}(^{152}Sm) = 0.277 \pm 0.028$: P. J. Wolfe and R. P. Scharenberg, Phys. Rev. 160, 866 (1967).

[c]Using $g_{2^+}(^{166}Er) = 0.312 \pm 0.006$: H. Dobler, G. Petrich, S. Hufner, P. Kienle, W. Wiedemann, and H. Eicher, Phys. Letters 10, 319 (1964).

[d]Using $g_{2^+}(^{170}Yb) = 0.334 \pm 0.005$: A. Hüller, W. Wiedemann, P. Kienly, and S. Hufner, Phys. Letters 15, 269 (1965).

[e]Using $g_{2^+}(^{182}W) = 0.266 \pm 0.009$; $g_{2^+}(^{184}W) = 0.295 \pm 0.010$; $g_{2^+}(^{186}W) = 0.312 \pm 0.011$: B. Persson, H. Blumberg, and D. Agresti, in Hyperfine Structure and Nuclear Radiations, edited by E. Matthias and D. A. Shirley (North-Holland Publishing Company, Amsterdam, The Netherlands, 1968), p. 268

[f]Taken from S. Hagström, C. Nordling, and K. Siegbahn, Alpha-, Beta-, and Gamma-Ray Spectroscopy, edited by K. Siegbahn (North-Hollan Publishing Company, Amsterdam, The Netherlands, 1965), p. 845.

[g]Ref. 5.

[h]Ref. 7.

where F denotes the total spin. The magnetic splittings for several first 2^+ states in the region from ^{152}Sm to ^{186}W are given in Table I, column 2; the $F=\frac{5}{2}$ member of the doublet lies higher in energy. The values were calculated according to the prescription of Ehrlich et al.,[8] using Z_{eff} values calculated by Ford and Wills.[9] The g factors of the nuclear states have been taken from references cited in column 2, Table I.

Contrary to what one might expect, the components of the hyperfine doublet built on the nuclear rotational state are not populated statistically, i.e., proportionally to $2F+1$. Their feeding depends on the mechanism by which these rotational states are excited and varies from one nucleus to another. A calculation of this feeding has been performed taking into account the "dynamical" quadrupole interaction between the muon and the nucleus[1,2] for the mixing of nuclear 0^+ and 2^+ states and muonic p and d states. The muonic wave functions of Pustovalov[10] were used. The calculated ratios of feedings to the two hyperfine states, $R_F(\frac{3}{2}:\frac{5}{2})$ given in Table I, column 3, are less than the statistical ratio, 2:3, implying a center-of-gravity shift towards higher energy. That conclusion can be reversed, however, by the effect of the $M1$ hyperfine transition which depopulates the upper state. (A similar effect of the strong intradoublet transition has been shown to be important for μ^- capture by Winston and Telegdi.[11]) The ratio of the $M1$ transition rate to the $E2$ de-excitation rate, $R_T(M1{:}E2)$, from the $F=\frac{5}{2}$ level is given by the product of gamma-ray probabilities times internal-conversion probabilities; that is, $R_T(M1{:}E2)=[\Gamma_\gamma(M1)/\Gamma_\gamma(E2)][1+\alpha(M1)]/[1+\alpha(E2)]$. The ratios $\Gamma_\gamma(M1)/\Gamma_\gamma(E2)$ which range from $(3 \text{ to } 13)\times 10^{-4}$ have little uncertainty since the $M1$ rates can be reliably calculated while the $E2$ rates are experimentally known. The $E2$ conversion coefficients can also be reliably evaluated even in the presence of the muon. The principal uncertainty in $R_T(M1{:}E2)$ is the internal conversion coefficient for the 400- to 800-eV $M1$ transition. We have estimated N_I- and O_I-shell coefficients by extrapolating the low-energy K-, L-, and M-shell values calculated recently by Hager and Seltzer.[12] Effects due to the finite size of the muon orbit were ignored; shielding effects were estimated by the procedures of O'Connell and Carroll.[13] The internal conversion coefficients are given in column 5 of Table I. The extrapolation procedures are expected to be accurate to within a factor of 2 for the N_I coefficients and to within a factor of 3 for the O_I coefficients. There are, however, further sources of ambiguity. The values of ΔE^M are close to the values of the N_I threshold energies so that effects of the nuclear magnetic form factor, which can produce 10% changes in ΔE^M, and the changes in the electron binding energies due to the presence of the muon and to the atomic environment assume an amplified importance, particularly for the states in tungsten. The resulting branching ratios, $R_T(M1{:}E2)$, are given in column 6.

The final intensity ratios $I(\frac{3}{2}\to\text{g.s.})/I(\frac{5}{2}\to\text{g.s.})$ from the decay of the doublet to the ground state are given in column 7. In general, this ratio is much greater than 1, so that in first approximation only the lower member of the transition doublet is observed. The calculated center-of-gravity shift of the hyperfine doublet, given in column 8, is of the order of the reported energy shifts, column 9. The uncertainties in the values of these pseudoisomer shifts for the Sm, Er, and Yb cases are due mainly to uncertainties in ΔE^M, while in the W cases one has additional large uncertainties due to ambiguities in the internal conversion values.

After correcting for the hyperfine center-of-gravity shift, the positive energy shift observed in ^{152}Sm is seen to increase leading to values of $\Delta\langle r^2\rangle$ in closer agreement with published Mössbauer results.[14] The negative energy shifts (which imply that the excited-state radius is smaller than that of the ground state) become smaller and may reverse sign. A positive sign is expected from Mössbauer results.[15]

Energy shifts and splittings of the nuclear levels in a muonic atom can also arise from the term $H^\mathrm{C}+H^\mathrm{M}$ in second order:

$$\Delta E_{\alpha,F}^{(2)}=-\sum_{\substack{\beta,n,l,j\\(\beta,nlj)\neq(\alpha,1s_{1/2})}}\frac{|\langle\Psi_\beta,nlj;F|H^\mathrm{C}+H^\mathrm{M}|\Psi_\alpha,1s_{\frac{1}{2}};F\rangle|^2}{(E_\beta-E_\alpha)+(E_{nlj}-E_{1s_{1/2}})},\tag{4}$$

which is referred to as the nuclear and muonic polarization. In this part of the paper we estimate some of the polarization effects.

The energy shift for a given nuclear level has been evaluated approximately by several authors[16] and they find values in the range of a few keV, the main part of which comes from intermediate nuclear and muonic states of high excitation. The measured quantity is a difference of terms like Eq. (4). It is doubtful whether for the high excitation region an appreciable difference exists between the polarizations of the lowest 0^+ and 2^+ nuclear rotational levels as their internal structures are very similar and the energy denominators in (4) are practically the same for both. Indeed, there is no energy shift of the nuclear γ ray from those terms of (4) where the intermediate states Ψ_β belong to a rotational band, the energy spacings of which may be neglected relative to the muonic excitations. However, appreciable effects may arise from those intermediate states of Eq. (4) where the muon stays in the ground state and the nucleus is not too highly excited. These effects, which require detailed knowledge of $E0$ and $M1$ nuclear excitations, have not been evaluated here.

An additional contribution to the splitting of the two hyperfine lines can arise from the F dependence of the nuclear polarization $\Delta E_{\alpha, F}^{(2)}$. However, this splitting vanishes exactly if the muonic fine structure and terms involving H^{M} are neglected. More specifically, if one replaces the muonic energies E_{nlj} by a spin-averaged value $\bar{E}_{nl}$ and if one neglects the difference in the radial wave functions for the two states with $j=l\pm\frac{1}{2}$, then the sum over j in Eq. (4) is readily performed,

$$\sum_j \frac{|\langle\Psi_\beta, nlj; F|H^{\mathrm{C}}|\Psi_\alpha, 1s_{\frac{1}{2}}; F\rangle|^2}{(E_\beta - E_\alpha)+(\bar{E}_{nl}-E_{1s_{1/2}})} \propto \sum_j \begin{Bmatrix} I_\beta & j & F \\ \frac{1}{2} & I_\alpha & l \end{Bmatrix}^2 (nlj\|Y_l\|1s_{\frac{1}{2}})^2 = (nl\|Y_l\|1s)^2[(2I_\alpha+1)(2l+1)]^{-1}, \quad (5)$$

where factors are omitted which do not depend on j or F. According to Eq. (5), the sum over j leads to terms independent of F and hence there is no hyperfine splitting in this approximation.

The contribution of the muonic fine structure is of the order

$$\left\langle \frac{(E_{nlj}-\bar{E}_{nl})}{(\bar{E}_{nl}-E_{1s_{1/2}})} \right\rangle_{nl} \Delta E_\alpha^{(2)}. \quad (6)$$

As the ratio $(E_{nlj}-E_{nl})/(E_{nl}-E_{1s_{1/2}})$ is already as small as a few percent for $n=2,3$, its average over all n, l is expected to be much less than 1%. The contribution of the muonic fine structure is then negligible compared with ΔE^{M} in Eq. (3), since the total shift $\Delta E_\alpha^{(2)}$ is of the order of few keV. The contribution of H^{M} terms to the polarization leads to a small correction to Eq. (6).

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