THREE-BODY FORCES IN NUCLEAR MATTER*

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The pionic three-body potential in nuclear matter is shown to be comparable with the two-body potential.

There have been many attempts to calculate three-body forces in nuclei from meson theory.[1] It was early realized that the long-range part of the three-body force comes from Feynman diagrams in which a virtual pion emitted by one nucleon is scattered by the second and then absorbed by the third (Fig. 1). Of course, the part of the nucleon pole term in the π-N scattering amplitude which arises from a single positive-energy nucleon intermediate state [Fig. 1(a)] is already included in the iterated two-body potential. Only the two-nucleon, one-antinucleon state contribution from the nucleon pole term [Fig. 1(b)] applies to three-body forces. Historically, this was one of the first three-body potentials calculated and found to be large. However, the corresponding approximation to π-N scattering gives a large S-wave scattering length contrary to experiment. Therefore, the pole term was scaled down by hand to correspond to the correct S-wave scattering length, and then the three-body force from Fig. 1(b) was estimated. Subsequently, the contributions of πN and $\pi\pi$ resonances [Figs. 1(c), 1(d), 1(e), etc.] were added.

Recently, Brown, Green, and Gerace[2] (BGG) have treated the sum of all these contributions to the three-body force in a unified and consistent way, for cases where the momentum transfers of the nucleons are small. They base their analysis on the Adler partially conserved axial-vector current consistency condition,[3] which requires that the π-N scattering amplitude vanish when one or both of the pion four-momenta are zero. This implies that the diagrams in Fig. 1 cancel each other, when the virtual pions are soft. BGG then point out that the two-body force contribution in Fig. 1(a) vanishes by itself in the soft-pion limit, so that the sum of the remaining terms in Fig. 1 which contribute to the three-body potential also vanishes in this limit. They further show that this three-body potential extrapolates slowly in q^2 (the square of the virtual pion mass) and q_0 (the pion lab energy) for $q^2/m^2 \ll 1$ and $q_0/m \ll 1$, where m is the nucleon mass. Finally, if Fig. 1 is considered as a Goldstone diagram for the energy of nuclear matter, or of the triton, then the momenta of the nucleons 1, 2, and 3 are small, implying $q^2 \simeq \mu^2$ and $q_0 \simeq \pm\mu$, where μ is the physical pion mass. Thus, the three-body potential contribution to the energy, arising from these diagrams, would be small. From this, BGG proceed to conclude that the pionic three-body forces in nuclear matter and nuclei are small.

Such a general conclusion is not justified when drawn from the diagram in Fig. 1, which is only to first order in the three-body potential. Indeed, BGG have shown in a convincing fashion that the small-momentum-transfer components of their three-body potential is small. However, any property of a nuclear system, such as the energy or the wave function, will involve in general terms of all orders in both the three- and the two-body potentials. Even if the three-body potential were to be treated in first order, it would still appear in diagrams in conjunction with arbitrary orders of the strong two-body reaction matrix g. Now, it is well known[4] that because of the hard core in the two-nucleon force, the g matrices excite nucleons strongly to states typically of momenta about 4 F^{-1}. Consider, for example, the Goldstone diagrams in Fig. 2, which must necessarily occur. The intermediate-state momenta k_1, k_2, k_3, and k_4 are typically about 4 F^{-1} or about 5μ as a result of the action of the two-body g matrices on the initial or final states 1, 2, and 3. The corresponding virtual pions

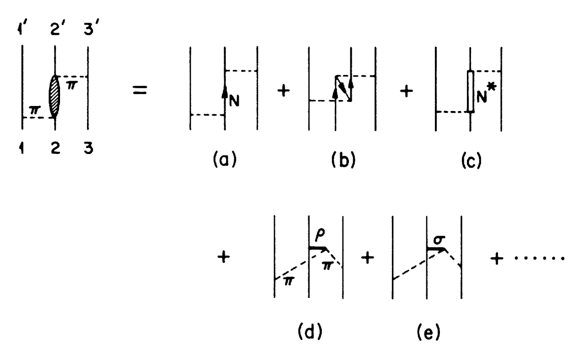

FIG. 1. Diagrams related to the long-range (pionic) three-body force. All diagrams except (a) contribute to this force.

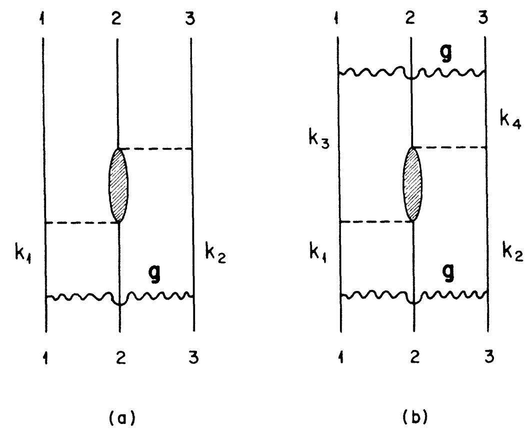

FIG. 2. Two Goldstone diagrams, involving the two-body reaction matrices g and the three-body potential of Fig. 1. The wiggly lines are the g matrices.

are highly spacelike, with $q^2 \sim -20\mu^2$ to $-25\mu^2$.

In general, therefore, the three-body potential of Fig. 1 would occur not between nucleons of small momenta, but between nucleons which typically differ in momentum by about 5μ. Its effect will depend on its large-momentum-transfer components, i.e., on π-N scattering with hard spacelike pions. The small result of BGG, arising from delicate cancellations in the π-N amplitudes when $q \simeq 0$, cannot be expected to hold when the pions are so hard.

To estimate the magnitude of the three-body potential, one clearly has to extrapolate the π-N scattering amplitude to $q^2 \sim -20\mu^2$. This is a fairly bold extrapolation and there is no unambiguous way of doing it. However, we present below two reasonable models for the extrapolation, both of which indicate that the three-body forces in nuclei are about as strong as the two-body forces. We will extrapolate in both pion masses, keeping them equal, and also restrict ourselves for simplicity to forward πN scattering. This is what occurs in Fig. 2, and should give us a reasonable estimate. We will also deal with the amplitude which is symmetric in the isospin variables of the two pions. Only this contributes in nuclear matter. Similar considerations are actually also valid for the antisymmetric amplitude.

The invariant isosymmetric π-N scattering amplitude T consists of a nucleon pole term and the continuum. We have to remove from the pole term T^{Born} the part T_2^{B} which corresponds to a positive-energy nucleon intermediate state [Fig. 1(a)], since this is contained in the iterated two-body potential contribution. We calculate T_2^{B} by usual second-order perturbation theory with a vertex $H_1 = igK(q^2)\bar{\psi}\gamma_5\psi\varphi$, where $K(q^2)$ is the πNN form factor and $g^2/4\pi \simeq 14.4$. Straightforward algebra leads to a contribution T_3^{B} from the pole term [corresponding to Fig. 1(b)] relevant to the three-body force, given by

$$T_3^{\mathrm{B}} = T^{\mathrm{Born}} - T_2^{\mathrm{B}}$$
$$= \frac{g^2}{m}\frac{m}{(m^2+q_0^2-q^2)^{1/2}}\left[\frac{q_0^2}{\{m+(m^2+q_0^2-q^2)^{1/2}\}^2-q_0^2} - \frac{q^4}{4m^2q_0^2-q^4}\right]. \quad (1)$$

This can be directly extrapolated in q and q_0.

To get the continuum contribution, we use two models. The first is the one introduced by Adler[3] and used by BGG in their work. Forward dispersion relations are written for A and B amplitudes in the usual s variable, where $T = -A + i\not{q}B$. The absorptive parts in the dispersion relation are assumed to be dominated by a narrow (3, 3) resonance, which is extrapolated in the square of the pion mass q^2 by

$$\mathrm{Im} f_{33}(s, q^2) = K(q^2)\left(\frac{q_0^2-q^2}{q_0^2-\mu^2}\right)_{\mathrm{c.m.}} \mathrm{Im} f_{33}(s, \mu^2). \quad (2)$$

This method of extrapolating just the threshold factors is found to be satisfactory by Adler in his work on electroproduction,[5] for the photon mass squared up to $-20\mu^2$. In Eq. (2), $f_{33}(s, \mu^2)$ is the Chew-Goldberger-Low-Nambu partial-wave amplitude and is given in the narrow-resonance limit by

$$\mathrm{Im} f_{33}(s, \mu^2) = \pi\lambda\delta(\sqrt{s} - M^*).$$

M^* is the N^* resonance mass, and the coupling constant λ is adjusted to satisfy the Adler consistency condition, i.e., $T(q_0 = 0, q^2 = 0) = 0$. Upon inserting Eq. (2) into the dispersion relation, one obtains a unique extrapolation of the continuum contribution, as a function of q_0 and q^2.

Another model for extrapolating the continuum contribution under N^* dominance is simply to evaluate Feynman diagrams corresponding to N^* poles in the s and u channels. The πNN^* vertex

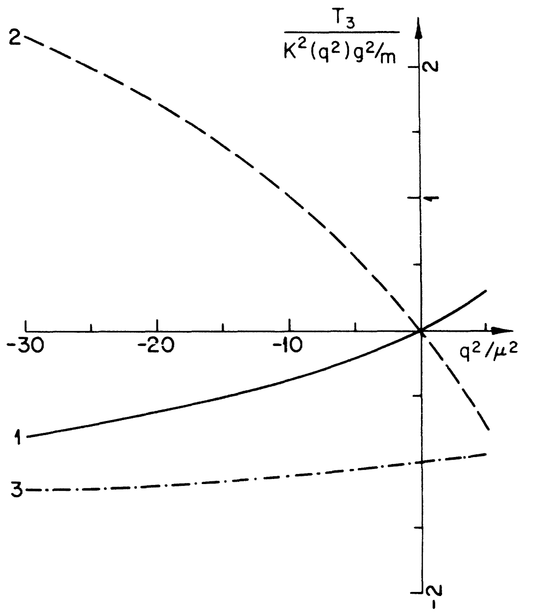

FIG. 3. Curve 1 gives $T_3(q^2)/[K^2(q^2)g^2/m]$ for zero pion laboratory energy. The continuum contribution for this curve is calculated by the Adler extrapolation method. Curve 2 gives the same quantity, but with the continuum now calculated by the N^*-pole Feynman diagram. Curve 3 gives $T_2^{\text{Born}}/[K^2(q^2)g^2/m]$. Comparison of curves 1 and 2 with curve 3 gives an estimate of the strength of the three-body force as compared with the twice iterated two-body force of Fig. 1(a).

is taken to be $K(q^2)(\Lambda/m)\bar{u}_\mu(p_1+q_1)u(p_1)(p_1-q_1)^\mu$, where the u_μ is the Rarita-Schwinger spinor for the N^*.[6] Evaluation of the Feynman diagram is straightforward and yields a result as a function of the q^2, and the pion energy q_0. The coupling constant Λ is once again adjusted[7] to give the Adler consistency condition at $q=0$.

We thus have two models for calculating T(continuum). The three-body force in clearly related to $T_3=T_3^{\text{B}}+T$(continuum). The results, as a function of q^2, for $q_0=0$ are shown in Fig. 3. We can see that $T_3(q^2)$ depends for its sign and precise value on the actual method of extrapolation. However, it is of the order of $K^2(q^2)g^2/m$ and in general large and comparable with T_2^{Born}. Thus, when one considers the large Fourier components of the three-body potential by studying π-N amplitudes for $q^2=-10\mu^2$ to $-25\mu^2$, one concludes that these are in general comparable with the corresponding components of two-body forces, as distinct from the conclusion of BGG. Note that we do not attempt to estimate the effect of these three-body forces on the binding energy or wave functions of nuclear systems. Experience with strong two-body potentials[4] teaches us that these forces should be treated to all orders in perturbation in closed form—a very complicated task when both two- and three-body forces act strongly. We only wish to point out that unless there are some very fortunate cancellations between different cluster diagrams, the effect of long-range three-body forces in nuclear system will be comparable with that of the usual two-body forces.

Finally, unlike electromagnetic form factors, $K_{NN\pi}(q^2)$ cannot be directly measured experimentally for large q^2 by N-N scattering. If one uses a specific theory, such as the pion-pole dominance of the divergence of the axial-vector current, then one can write

$$K(q^2)=1+(q^2-\mu^2)\int_{9\mu^2}^{\infty}\frac{\operatorname{Im}K(q'^2)}{q'^2-q^2}dq'^2.$$

But experiments indicate no strong 3π resonances with the quantum numbers of the pion to saturate the above dispersion integral. An arbitrary choice of a resonance between 3μ and 20μ gives widely different values for $K(-20\mu^2)$. Thus, there is no unambiguous way of estimating the actual strength of the three-body forces in the above models. However, this does not alter our conclusion that the three-body potentials will be comparable with twice iterated two-body potential terms, since the same factor $K^4(q^2)$ occurs in both cases.

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[3]S. L. Adler, Phys. Rev. 137, B1022 (1965), and 140, B736 (1965).

[4]See, for instance, R. Rajaraman and H. A. Bethe, Rev. Mod. Phys. 39, 745 (1967).

[5]S. L. Adler, private communication.

[6]For the Rarita-Schwinger propagator we use the form given by S. Gasiorowicz, Elementary Particle Physics (John Wiley & Sons, Inc., New York, 1966), p. 430.

[7]The values of λ and Λ adjusted to obey Adler's consistency condition reproduce, respectively, $\frac{2}{3}$ and $\frac{1}{2}$ the measured N^* width in the static limit. Thus, the crude narrow N^* dominance is a reasonable approximation.

RE-EXAMINATION OF THE NUCLEAR ISOMER SHIFT AS MEASURED IN MUONIC ATOMS*

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The magnetic hyperfine splitting for rotational levels of deformed, even-even nuclei in muonic atoms is shown to result in an asymmetric γ-ray doublet, whose center of gravity is in general shifted towards lower energies. This pseudoisomer shift is of the order of magnitude of the reported "isomer" shifts.

The cascade of the muon from the high-lying muonic orbits to the muonic ground state of deformed nuclei is often accompanied by the excitation of nuclear rotational levels.[1,2] As the lifetime of the muon in the $1s$ state is long compared with the lifetime of the rotational levels, the deexciting nuclear γ ray is emitted in the presence of the $1s$ muon with the result that the transition energies differ from the respective energies in the absence of the "spectator muon."[3,4] Such γ rays have been observed for transitions in a number of deformed nuclei from ^{150}Nd to ^{186}W.[5-7] The observed energy shifts of the radiation, assumed to be an unsplit line, have been interpreted as arising entirely from the radius difference between the ground and excited states of the host nucleus, i.e., as isomer shifts. In this note we point out that the nuclear transition, in the presence of the muon, is in general an asymmetric doublet whose center of gravity is shifted from that of the unsplit line even in the absence of an isomer shift. The shift of the center of gravity of the decay spectra arises from two effects. First, there is a nonstatistical feeding of the nuclear magnetic hyperfine levels, and second, the $M1$ intradoublet transition enhances the population of the lower hyperfine level. The latter effect generally dominates, resulting in a shift of the center of gravity which is of the same order of magnitude as the observed energy shifts. Nuclear polarization phenomena are considered in the second half of this Letter.

The Hamiltonian of the muonic atom,

$$H = H_N + [T(\mu) + V(r_\mu)] + (H^C + H^M) \tag{1}$$

consists of three parts: (i) the nuclear Hamiltonian H_N in the absence of the muon [eigenstates $|\Psi_\alpha(1, \cdots, A)\rangle$]; (ii) the muonic Hamiltonian, $[T(\mu)+V(r_\mu)]$, with the average Coulomb potential $V(r_\mu)$ due to the charge distribution of the nuclear ground state (eigenstates $|nlj\rangle$); and (iii) the residual Coulomb force, $H^C = -e^2\sum_p |\vec{r}_\mu - \vec{r}_p|^{-1} - V(r_\mu)$, together with the magnetic interaction H^M between the muon and the nucleus. The last part, $H^C + H^M$, is usually considered to be small and can be treated in perturbation theory, at least for the muon in the $1s$ orbit.

The isomer shift between two nuclear states Ψ_α and Ψ_β is defined as the energy difference

$$\Delta E_{\alpha,\beta}^{\text{isomer}} = \langle \Psi_\alpha, 1s_{\frac{1}{2}} | H^C | \Psi_\alpha, 1s_{\frac{1}{2}} \rangle - \langle \Psi_\beta, 1s_{\frac{1}{2}} | H^C | \Psi_\beta, 1s_{\frac{1}{2}} \rangle, \tag{2}$$

where $1s_{1/2}$ denotes the spectator muon. The energy, ΔE^{isomer}, depends essentially on the radius difference between the two nuclear states and is a quantity which contains valuable information about the nucleus. Each nuclear state Ψ_α with spin $I_\alpha \neq 0$ also exhibits a hyperfine splitting which originates from the interaction of the nucleus with the magnetic moment of the $1s$ muon. The splitting is given in first order by

$$\Delta E_\alpha^{M} = \langle \Psi_\alpha, 1s_{\frac{1}{2}};\ F = I_\alpha + \tfrac{1}{2} | H^M | \Psi_\alpha, 1s_{\frac{1}{2}};\ F = I_\alpha + \tfrac{1}{2} \rangle - \langle \Psi_\alpha, 1s_{\frac{1}{2}};\ F = I_\alpha - \tfrac{1}{2} | H^M | \Psi_\alpha, 1s_{\frac{1}{2}};\ F = I_\alpha - \tfrac{1}{2} \rangle, \tag{3}$$