RANDOM WALK OF MAGNETIC LINES OF FORCE IN ASTROPHYSICS*

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We point out the importance of the random fluctuations in astrophysical magnetic fields, which lead among other things to rapid diffusion of energetic particles across the average magnetic field. Application to the interplanetary magnetic field leads to a theory in which the field-line random walk, measured by the observed power spectrum of field fluctuations, is in agreement with that deduced from the observed turbulent motions in the solar photosphere. The two determinations agree well with the recently observed spread in longitude of low-energy particles released at the sun.

To a first approximation, the lines of force of a large-scale ambient magnetic field $\vec{B}(\vec{r},t)$ represent the trajectories of fast charged particles (cosmic rays). The lines of force are defined by the instantaneous family of solutions to the two first-order equations

$$\frac{dx}{B_x}=\frac{dy}{B_y}=\frac{dz}{B_z}. \qquad (1)$$

Because of unavoidable random fluctuations, adjacent field lines random-walk away from each other, exhibiting a fundamentally stochastic and ergodic nature. It is the purpose of this Letter to point out these properties of magnetic field lines in the astrophysical universe and to illustrate some of the novel and important consequences for the propagation of cosmic rays. Detailed results will appear in later papers.

The observed fact that $\nabla\cdot\vec{B}=0$ insures that magnetic lines of force do not end. In the idealized magnetic fields (usually multipoles) used in conventional theory, the symmetry is perfect, with the result that the lines of force close exactly on themselves. But such perfect closure is a priori improbable: It does not occur in nature. Because the lines of force in astrophysics are tied to the ambient fluid, which is usually turbulent, the lines partake in the random displacements of the fluids. Hence, even if the lines were highly symmetric at some time in the past, they are now stochastic.

In order to discuss these stochastic or random properties of the magnetic field quantitatively, we introduce the concept of a random field $\vec{B}(\vec{r})$ which is defined on a statistical ensemble of systems. A statistically complete specification of $\vec{B}(\vec{r})$ is given by the set of all n-point correlations

$$\langle B_i(\vec{r}_1)\cdots B_k(\vec{r}_n)\rangle, \qquad (2)$$

where $\langle\,\rangle$ denotes an ensemble average.[1] Clearly, any practical analysis must involve only a small number of this infinite family of correlation functions.

The conventional analysis, which neglects the inevitable random fluctuations, is equivalent to considering only the average field $\langle\vec{B}\rangle$ and neglecting all higher order correlations. The present discussion is concerned with effects associated with the second-order correlation; neglect of the higher order terms is justified if the fluctuating field is small compared with the average field. Let $\vec{B}(r)$ be a homogeneous, random function of position which is essentially time independent over the times scales of interest. This neglect of time variations is justified in the present context if the Alfvén speed is sufficiently small compared with the energetic particle velocities. Define $\langle\vec{B}(\vec{r})\rangle=B_0\vec{e}_z$ and $\vec{B}_1(\vec{r})=\vec{B}(\vec{r})-B_0\vec{e}_z$, and assume $\langle B_1^2\rangle \ll B_0^2$. Then Eq. (1) for the field lines becomes, to first order in B_1/B_0,

$$\Delta x=x-x_0=\frac{1}{B_0}\int_0^{\Delta z}B_{1x}(x_0,y_0,z')dz', \qquad (3)$$

$$\Delta y=y-y_0=\frac{1}{B_0}\int_0^{\Delta z}B_{1y}(x_0,y_0,z')dz', \qquad (4)$$

where z' denotes distance along the mean field B_0. Clearly, $\langle\Delta x\rangle=\langle\Delta y\rangle=0$ because $\langle\vec{B}_1\rangle=0$. In terms of the two-point correlation tensor of $\vec{B}_1$ defined as $C_{ij}(\vec{\xi})=\langle B_{1j}(\vec{r})B_{1j}(\vec{r}+\vec{\xi})\rangle$, we have for Δz greater than L, the correlation length of the field,[2]

$$\frac{\langle(\Delta x)^2\rangle}{\Delta z}\simeq\frac{1}{B_0^2}\int_{-\infty}^{\infty}C_{xx}(0,0,\rho)d\rho, \qquad (5)$$

$$\frac{\langle(\Delta y)^2\rangle}{\Delta z}\simeq\frac{1}{B_0^2}\int_{-\infty}^{\infty}C_{yy}(0,0,\rho)d\rho, \qquad (6)$$

etc. The lines of force random walk among each other as a function of distance along the average field; the rate of random walk is simply related to the correlation tensor. The distribution of a certain, identifiable bundle of field lines passing through x_0, y_0, z_0 is then a Gaussian with widths $2[\langle(\Delta x)^2\rangle/\Delta z](z-z_0)$ and $2[\langle(\Delta y)^2\rangle/\Delta z](z-z_0)$ in the x and y directions. As a particle (with average cyclotron radius $r_c < L$) moves along the z axis at velocity $V_{\parallel}$, its guiding center moves perpendicular to the average field with a coefficient $\langle(\Delta x)^2\rangle/\Delta t = V_{\parallel}\langle(\Delta x)^2\rangle/\Delta z$, etc., which may be much faster than the perpendicular motion due to resonant scattering by small scale irregularities.[3] This conclusion has important applications, both in the solar wind and in the galaxy.

The interplanetary magnetic field, accessible to direct measurement and traversed by both solar and galactic cosmic rays, provides an ideal laboratory for testing these ideas. We have both direct measurements of the interplanetary field from spacecraft and direct measurements of the random velocities of the fluid in which the field is embedded. Further, we have also direct measurements of the spread of energetic particles across the average magnetic field.

Consider first the direct measurements of the magnetic field. The velocity V_W of the solar wind is some five or ten times the Alfvén speed in the wind; so to a first approximation the field is carried rigidly past the spacecraft. If z' is the direction of the wind, then the spatial structure along z' is given by $\vec{B}(z' = V_W t) = \vec{B}(t)$. Taking the field irregularities to be statistically isotropic,[4] we have from (5) and (6) in terms of the power spectrum observed at the spacecraft,

$$P_{ij}(f) = \int_{-\infty}^{\infty} dt' \exp(2\pi i f t')\langle B_i(t) B_j(t+t')\rangle, \qquad (7)$$

that

$$\frac{\langle(\Delta x)^2\rangle}{\Delta z} = \frac{\langle(\Delta y)^2\rangle}{\Delta z} \simeq \frac{V_W}{B_0^2} P_{xx}(f=0). \qquad (8)$$

This relates the random walk of the field lines to the power at zero frequency.

Recent observations of Coleman[5] give $P_{ij}(f)$ at frequencies from 3×10^{-7} to $\sim10^{-2}$ Hz. Extrapolating the power to zero gives $P_{xx}(0) \sim 1.25\times10^3 \times B_0^2$ sec at quiet times in late 1964, from which it follows that

$$\langle(\Delta x)^2\rangle/\Delta z \simeq 5\times10^{10}\text{ cm}. \qquad (9)$$

Observational uncertainties, together with the extrapolation to zero frequency and the assumption of statistical isotropy, suggest that this observational estimate is correct to perhaps a factor of 2. Since the correlation length is observed[5] to be of the order of 0.01 A.U., Δz in Eq. (9) must be greater than 0.01 A.U. for the result to be valid. Equation (9) implies that in a distance $\Delta z = 1$ A.U. the rms displacement of a field line relative to the mean field is

$$\langle(\Delta x)^2\rangle^{1/2} \simeq 1.5\times10^{12}\text{ cm} \simeq 0.1\text{ A.U.}, \qquad (10)$$

corresponding to an rms angular displacement of 0.1 rad.

Most of the random walk in the interplanetary field appears to be introduced by the granule and supergranule motion in the photosphere, which displace the feet of the lines of force during the time that the lines are carried by the solar wind to the orbit of Earth. The general configuration is illustrated in Fig. 1. Leighton[6] estimates that the granules (correlation length $L_g \simeq 10^3$ km and time $t_g \simeq 4\times10^2$ sec) contribute about as much as the supergranules (correlation length $L_S \simeq 1.5\times10^4$ km and time $t_S \simeq 3\times10^4$ sec) to mixing in the photosphere. We use the figures for supergranulation here to obtain an estimate—a lower limit—on the random walk of the field lines. Then the mean-square displacement in the photosphere is, in order of magnitude,

$$\frac{\langle(\Delta x)^2\rangle}{\Delta t} = \frac{L_S^2}{t_S} = 0.75\times10^{14}\text{ cm}^2/\text{sec}. \qquad (11)$$

Hence in the period t ($\cong 4\times10^5$ sec) that a given point on a line is carried to the orbit of Earth, the rms displacement of the foot of the line at the sun is

$$\langle(\Delta x)^2\rangle^{1/2} = L_S(t/t_S)^{1/2} = 0.55\times10^{10}\text{ cm}, \qquad (12)$$

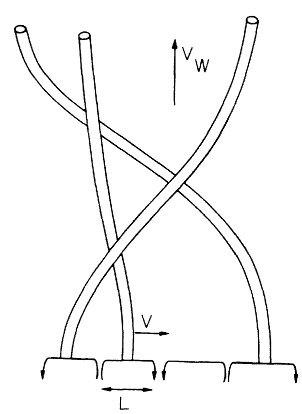

FIG. 1. Schematic illustration of the field-line random walk generated by turbulent motions in the photosphere. A typical element of fluid moves a distance L at velocity V, and a cell lasts a time $\tau \simeq L/V$. The field lines are convected out by the solar wind at velocity V_W, which is several times the Alfvén velocity.

corresponding to an angular displacement of 0.08 rad, or 1.2×10^{12} cm at the orbit of Earth. The ordinary granulation contributes perhaps an equal amount, increasing the total length by the factor $\sqrt{2}$. The observed random walk (10) and the random walk (12) calculated from the supergranulation agree, indicating that the supergranulation is responsible for at least some major part of the observed large-scale randomness. This conclusion is strengthened by the fact that radial projection (via the solar wind) of the correlation length L_S of the supergranulation to the orbit of Earth at 220 solar radii leads to the characteristic length 220 $L_S \simeq 3\times10^{11}$ cm, in rough agreement with the correlation length $L \simeq 2\times10^{11}$ cm observed in the fields at the orbit of Earth.[5]

An independent test of this theory is provided by observations[7-10] of low-energy solar cosmic rays with kinetic energies O(1 to 10 MeV), which serve as "tracers" of the interplanetary magnetic field configuration. It is observed that these low-energy particles often exhibit strong anisotropies, presumably because they are collimated by the diverging interplanetary magnetic field. Bartley et al.[9] and McCracken and Ness[10] suggested a picture in which intertwined tubes of force conduct particles outward from the sun (the so-called "wet spaghetti" model, after the magnetic tubes of force); changing directions of the particle anisotropies correspond to various tubes of force being convected past the spacecraft. This picture is contained within our theory of random-walking field lines (cf. Fig. 3 of Ref. 10), and it is interesting to note that the typical "tube of force" cross section reported by Bartley et al.[9] is 3×10^{11} cm, which again is close to the deduced magnetic-field correlation length L at the orbit of Earth.

Fan et al.[8] observed protons of 0.6- to 13-MeV energy which were emitted continuously by active regions on the sun. During April 1966 there were no significant flare outbursts, so that the steady distribution of particles was observed at the orbit of Earth. Promptly arriving particles from a given active region on the sun were observed over nearly 180° in solar longitude, with a sharp maximum [half-width at $(1/e)$ times maximum of ~1.25 days or 0.29 rad] at the point where the average spiral field line would connect back to the active region. The observed intensity is plotted in Fig. 2. The inference, as pointed out by Fan et al.,[8] is that "the interplanetary magnetic field region in which the protons propagate extends over a longitudinal range of 180°." This result is again an expected consequence of our field-line random walk. Indeed, the half-width at $(1/e)$ times maximum of ~0.15 rad, deduced[11] from the observed power spectrum [Eq. (10)] or the photospheric motions [Eq. (12)], agrees within experimental uncertainties with the observed mean particle spread of ~0.29 rad. The agreement suggests that the random walk of magnetic lines of force carried in the solar wind is responsible for a large part of the observed spread of solar particles. Indeed, the observed broad spread of O(MeV) particles is not readily understandable in any other way.

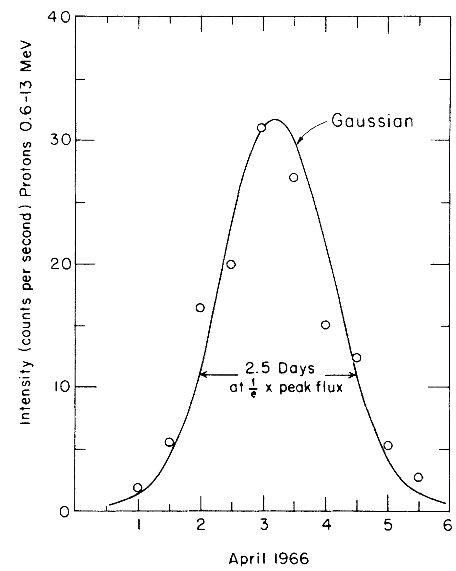

FIG. 2. A typical time history of the intensity of 0.6- to 13-MeV protons observed by Fan et al. (Ref. 8). A noticeable flux was observable for ~10 days or ~140° in solar longitude. The solid line is the Gaussian curve intensity $= 32\exp[-(\Delta t/1.25)^2]$, with Δt in days from maximum.

The above example, exploiting the convenient observational situation presented by the sun, illustrates the fundamental importance of the randomness of the magnetic fields in treating particle propagation. We have considered a number of other problems, which are presented in later papers. We find that the magnitude of the perpendicular diffusion coefficient $K_\perp$ in the interplanetary magnetic field is much larger then previously supposed, with important consequences for the theory of the diurnal variation of galactic cosmic rays. Further, application to escape of galactic cosmic rays from the galaxy indicates that the random walk of field lines plays a large and essential role there, too.

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[1]A. M. Yaglom, An Introduction to the Theory of Stationary Random Functions (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962).

[2]The correlation length L is defined as that distance $|\vec{r}|$ beyond which $C_{ij}(\vec{r})$ approaches zero.

[3]The general treatment of particle propagation, including resonant interation with small-scale irregularities, is presented in J. R. Jokipii, Astrophys. J. 146, 480 (1966). There it is found that the motion perpendicular to the z axis contains both a resonant term and the random-walk term.

[4]For a discussion of the hypothesis of isotropy and its consequences, see J. R. Jokipii, Astrophys. J. 149, 405 (1967).

[5]J. R. Jokipii and Paul J. Coleman, Jr., to be published.

[6]Robert B. Leighton, Astrophys. J. 140, 1547 (1964).

[7]C. Y. Fan, J. E. Lamport, J. A. Simpson, and D. R. Smith, J. Geophys. Res. 71, 3289 (1966).

[8]C. Y. Fan, M. Pick, R. Pyle, J. A. Simpson, and D. R. Smith, J. Geophys. Res. 73, 1555 (1968).

[9]W. C. Bartley, R. P. Bukata, K. G. McCracken, and U. R. Rao, J. Geophys. Res. 71, 3297 (1966).

[10]K. G. McCracken and N. F. Ness, J. Geophys. Res. 71, 3315 (1966).

[11]The observed mean value $\langle(\Delta x)^2\rangle/\Delta z \simeq 5\times10^{10}$ yields an rms $\Delta\varphi \sim 0.1$ rad in $\Delta z \sim$A.U. Thus, the distribution in longitude should be $\sim\exp[-(\Delta\phi)^2/(0.1)^2\times2]$ or a half-width at $(1/e)$ times maximum of about 0.15 rad.

EVIDENCE FOR THE KINEMATIC ORIGIN OF THE H ENHANCEMENT*

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The reaction $\pi^+p\rightarrow\pi^+\pi^-\pi^0\Delta^{++}$ has been studied in the range 3-4 GeV/c of incident pion momenta. Results suggest that the origin of the H enhancement is purely a kinematic one that arises from the ρ-band cut in an essentially structureless three-pion Dalitz plot.

The H enhancement, a peak near 990 MeV in the invariant mass of the neutral three-pion system $(\pi^+\pi^-\pi^0)$ in which at least one pion pair forms a ρ meson $[(\rho\pi)^0$ cut$]$, was first reported by the Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration[1] in the reaction

$$\pi^+p\rightarrow\pi^+\pi^-\pi^0\Delta^{++} \tag{1}$$

at 4 GeV/c. Goldhaber et al.[2] observed a similar, but statistically less significant, peak in the same reaction at 3.65 GeV/c. Subsequently, Benson et al.[3] observed a significant signal for H in the reaction

$$\pi^+d\rightarrow pp(\pi\rho)^0 \tag{2}$$

at 3.65 GeV/c. However, the data of the Bari-Bologna-Firenze-Orsay Collaboration[4] for the same reaction at 5.1 GeV/c yield only a broad shoulder in the H region and do not show the same effect. The similar reaction at 3.29 GeV/c was also studied by Cohn et al.,[5] who concluded that H is present. It may be noted that for all these experiments (i) no significant H enhancement is seen in the three-pion invariant-mass plot until a ρ cut has been effected[6]; (ii) the isospin of H was inferred to be zero from a roughly equal yield of $\rho^+\pi^-$, $\rho^-\pi^+$, and $\rho^0\pi^0$ events, and from the observation in the charged $(\rho\pi)^\pm$ invariant-mass plot of reactions such as $\pi^\pm p\rightarrow\pi^\pm\pi^-\pi^+p$ of a broad A_1 peak at around 1080 MeV instead of an H peak.

The result of our investigation suggests that the origin of the H enhancement is purely a kinematic one, arising from the ρ-band cuts in an essentially structureless three-pion Dalitz plot.

The present experiment involves π^+p interactions at five incident pion momenta: 2.95, 3.2, 3.5, 3.75, and 4.1 GeV/c. The experimental details have been described elsewhere.[7] A total of 9303 events of the reaction

$$\pi^+p\rightarrow\pi^+p\pi^+\pi^-\pi^0 \tag{3}$$

were obtained, of which 3159 events were selected as belonging to Reaction (1) with the Δ^{++} defined by the mass interval $M(\pi^+p)=1220\pm80$