# S-WAVE SHAPE-DEPENDENT SCATTERING PARAMETERS OF THE PROTON-PROTON INTERACTION*

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The shape-dependent parameters $P$ and $Q$ of the effective-range expansion for the $S$-wave $p$-$p$ interaction have been obtained from experimental data between 0 and 10 MeV, including recent results at 6.141, 8.097, and 9.918 MeV. The preferred best values are $P=0.072\pm0.005$ and $Q=0.034\pm0.004$.

The $S$-wave nucleon-nucleon interaction between 0 and 10 MeV can be parametrized by a convergent power series,[1] and thus can be approximated by a polynomial

$$K=\sum_0^n A_n E^n, \tag{1}$$

where $E$ is the energy, usually expressed in MeV. The relation of (1) with the $^1S_0$ $p$-$p$ phase shifts and more currently used scattering parameters is obtained through the equation $K=RF$, where

$$F=C^2k\cot\delta_0+(1/R)h(\eta)=-1/a_p+\tfrac{1}{2}r_ek^2-Pr_e^3k^4+Qr_e^5k^6-\cdots, \tag{2}$$

where

$$C^2=\frac{2\pi\eta-1}{e^{2\pi\eta}-1},\quad R=\frac{\hbar^2}{M_p\epsilon^2},\quad h(\eta)=\mathrm{Re}\frac{\Gamma'(-i\eta)}{\Gamma(-i\eta)}-\ln\eta$$

$k$ is the relative momentum in units of $\hbar$, $\eta=\epsilon^2/\hbar v$ (Coulomb parameter), $\epsilon$ is the proton charge, $v$ is the relative velocity, and $a$ is the proton-proton scattering length, $r_e$ is the effective range. $P, Q, \cdots$ are known as shape-dependent parameters; i.e., their values and sign depend on the detailed shape of the potential well in a Hamiltonian formulation or on model characteristics, in general. Conversely, an empirical determination of such parameters would prescribe a shape for the potential of interaction or determine a model. Calculations of the parameters $P$ and $Q$ for different well shapes or models are available in the literature.[2-4] Noyes[3] attempted first a determination of the shape parameter $P$ for the $^1S$ proton-proton interaction. This work was based on five accurate phase shifts at 0.382 43,[5] 1.397, 1.855, 2.425, and 3.037 MeV.[6] A summary of difficulties associated with a determination based on these five phase shifts may be found in the work of Slobodrian.[7] However, the ambiguity is reduced[7] if the effective-range expansion analysis includes the higher energy data of Worthington, McGruer, and Findley.[8] Heller[9] has recently added the phase shift from data[10] at 9.69 MeV to the low-energy phase shifts of Refs. 5 and 6, and performed fits up to and including the parameter $Q$. Heller recognized that the radius of convergence of (1) or (2) is approximately 10 MeV, and therefore the number of terms necessary may extend beyond the assumed polynomial. The errors of the parameters $P$ and $Q$ turn out to be large. From a practical point of view, the energy gap between 3.037 and 9.69 MeV is very large and conclusions drawn from such a set of data should be viewed diffidently. Noyes and Lipinski[11] have recently reanalyzed the data at 9.69 MeV extrapolating the recent information on spin-correlation parameters.[12] They conclude that at 9.69 MeV there is modest evidence for shape dependence consistent with one-pion exchange (OPE). However, the cross section measured at 9.69 MeV may be systematically high as noted by several authors,[11,13,14] and thus conclusions drawn from these data at a single energy may be subject to revision. Noyes and Lipinski[11] nevertheless also conclude that the shape correction is established beyond reasonable doubt if the results below 3 MeV and near 27 MeV are added to the result at 9.69 MeV. In the opinion of the present author the evidence drawn from data between 0 and 3 MeV is questionable as explained in Ref. 7, and therefore another attack to the problem is very desirable. The advent of new cross-section results[14] at 6.141, 8.097, and 9.918 MeV accurate to less than 1% and their phase shifts has made possible a determination of the shape-dependent parameters $P$ and $Q$, with a reanalysis of the existing experimental cross sections of Refs. 5, 6, 8, and 10.

The reanalysis of previous experimental data was advisable in order to avoid possible systematic differences in the central values of the phase shifts, related to criteria employed in the analysis, values of fundamental constants, approximations employed for relativistic effects, etc. A program due to Knecht[6] was used for the phase-shift analysis. Another program was written for the effective-range expansion analysis. Both programs were used with CDC 6600 machines of the Lawrence Radiation Laboratory computing center. The aim of this work has been to obtain the shape-dependent coefficients $P$ and $Q$ of the effective-range expansion on a basis as empirical as possible and to ascertain their stability. The reanalysis of experimental differential cross sections was carried out consistently as described in Ref. 14. Two different $P$-wave splittings were employed, one consistent with the OPE signature $(+-+)$, the other appropriate to spin-orbit effects producing positive polarizations at small angles $(++-)$, spin-orbit (SO) signature. The strength was extrapolated from 10 MeV down as prescribed by the low-energy limit of phase shifts, valid when $\sin\delta_l \cong \delta_l$, and by the possible absolute value of polarizations.[14] The value for the phase shift at 0.382 43 MeV was taken in common for both sets of phase shifts, as determined by Noyes.[3] The justification for this is that both sets of phase shifts converge to the same low-energy limit. Table I contains a summary of phase shifts. To reduce the size of the table only the $^1S_0$ phase shift and $\delta_{1\,\mathrm{eff}} = \delta_{1,0} + 3\delta_{1,1} + 5\delta_{1,2}$ are transcribed (full split $P$- and $^1D_2$-wave phase shifts are available upon request). The analysis in terms of expansions (1) and (2) was carried out up to and including a term in $k^8$ (shape parameter $R$). Vacuum polarization effects in the $S$-wave phase shifts were corrected following Foldy and Eriksen.[15] Effects due to the electromagnetic structure of nucleons were explored in terms of the approach of Slobodrian.[16] A summary of results is contained in Table II.[17] The redundancy of the term in $k^8$ is apparent in it. The preferred values of proton-proton scattering parameters (giving $\Phi$

Table I. $^1S_0$ phase shifts $\delta_0$ and $J$-weighted $P$ phase shift calculated as $\delta_{1\,\mathrm{eff}} = \delta_{1,0} + 3\delta_{1,1} + 5\delta_{1,2}$ determined by a least-squares fit to experimental differential cross sections, using $S$, split $P$, and $D$ phases, correcting for vacuum polarization in $l \geq 1$ according to Durand (Ref. 18).

| Lab Energy | OPE type phases | | SO type phases | |
|---|---|---|---|---|
| MeV | $\delta_0$ | $\delta_{1\mathrm{eff}}$ | $\delta_0$ | $\delta_{1\mathrm{eff}}$ |
| 1.397[b] | 39.231±.018 | -.148±.018 | 39.229±.034 | -.150±.040 |
| 1.855[a] | 44.286±.055 | .149±.079 | 44.281±.035 | -.143±.062 |
| 1.855[b] | 44.279±.021 | -.058±.030 | 44.274±.052 | -.064±.030 |
| 1.858[a] | 44.376±.040 | .180±.073 | 44.371±.040 | .174±.073 |
| 2.425[a] | 48.388±.039 | -.068±.065 | 48.377±.039 | -.083±.112 |
| 2.425[b] | 48.314±.020 | .009±.047 | 48.303±.020 | -.123±.255 |
| 3.037[a] | 51.016±.064 | .071±.082 | 50.975±.065 | -.064±.055 |
| 3.037[b] | 50.999±.025 | -.180±.033 | 50.978±.025 | -.001±.041 |
| 3.527[a] | 52.572±.055 | -.142±.071 | 52.539±.055 | -.196±.056 |
| 3.899[a] | 53.339±.061 | -.286±.071 | 53.267±.061 | -.381±.071 |
| 4.203[a] | 53.893±.060 | -.079±.062 | 53.833±.061 | -.176±.064 |
| 6.141[c] | 55.676±.109 | -.745±.168 | 55.492±.112 | -1.76±.166 |
| 8.097[c] | 55.915±.114 | -.584±.271 | 55.398±.133 | -1.372±.271 |
| 9.69[d] | 55.835±.110 | .525±.157 | 54.908±.116 | -.834±.167 |
| 9.918[c] | 55.087±.159 | -1.563±.053 | 54.053±.108 | -3.017±.601 |

[a]Data of Ref. 8.
[b]Data of Ref. 6.
[c]Data of Ref. 14.
[d]Data of Ref. 10.

Table II. Sample of scattering parameters obtained under various assumptions concerning the phase shift solutions and applicable corrections. Fits to 14 points exclude the phases at 9.69 and 9.918 MeV. 12 point fits exclude additional phases at 0.382 43 and 1.397 MeV. The column labeled Φ contains the ratio $\chi^2$ to the number of degrees of freedom. Diagonal errors producing an increase of 1 in Φ are quoted for preferred fits.

| -a fm | r fm | P | Q | R | Φ | No. of Points |
|---|---|---|---|---|---|---|
| 7.8322 | 2.807 | .027 | -.080 | -.081 | 1.60 | 16[a] |
| 7.8431 | 2.867 | .084 | .045 | 0 | 1.55 | 16[a] |
| 7.8399±.0094 | 2.854±.009 | .077±.006 | .038±.011 | 0 | .867 | 14[a] |
| 7.8240 | 2.794 | .040 | 0 | 0 | .956 | 14[a] |
| 7.9029 | 3.003 | .126 | .070 | 0 | .932 | 12[a] |
| 7.8381±.0078 | 2.849±.008 | .081±.005 | .063±.09 | 0 | .721 | 14[b] |
| 7.8139 | 2.892 | .095 | .052 | 0 | .843 | 12[c] |
| 7.7564 | 2.746 | .032 | 0 | 0 | .829 | 12[c] |
| 7.7856±.0078 | 2.840±.009 | .072±.005 | .034±.004 | 0 | .698 | 14[d] |
| 7.7870±.0063 | 2.846±.011 | .080±.003 | .062±.007 | 0 | .752 | 14[e] |

[a]With vacuum polarization corrections (VPC) and no electromagnetic corrections (EMC), OPE phases.
[b]Same as a but with SO phases.
[c]No VPC and no EMC, OPE phases.
[d]With VPC and EMC appropriate to a model with dynamic core, OPE phases.
[e]Same as d but with SO phases.

= minimum) are

$$a = -7.7856 \pm 0.0078 \text{ fm}, \quad r = 2.840 \pm 0.009 \text{ fm},$$

$$P = 0.072 \pm 0.005, \quad Q = 0.034 \pm 0.004.$$

They correspond to a calculation correcting for electromagnetic effects as appropriate in the absence of a core (or when it is velocity dependent and negligible at low energies). However, there are uncertainties in the electromagnetic form factors, and thus, these corrections may have produced a minimum in Φ fortuitously. Figure 1 shows a plot of the preferred fit.

The values obtained from phase shifts assuming a splitting of $P$ waves giving a positive polarization at small angles are

$$a = -7.7870 \pm 0.0063 \text{ fm}, \quad r = 2.846 \pm 0.011 \text{ fm},$$

$$P = 0.080 \pm 0.003, \quad Q = 0.062 \pm 0.007.$$

The shape dependence in the range from 0 to 10 MeV is established in the $S$ wave independently from the accuracy of the VPC, because the exclusion of the points at 0.382 43 and 1.397 MeV does not affect the signature of the parameters $P$ and $Q$. It is also established independently of the assumed splitting of $P$ waves, as long as polarization effects are kept small, in agreement with experiment.

The central values of $P$ and $Q$ differ from estimates made in the past assuming a Yukawa potential, but are not really inconsistent with it.

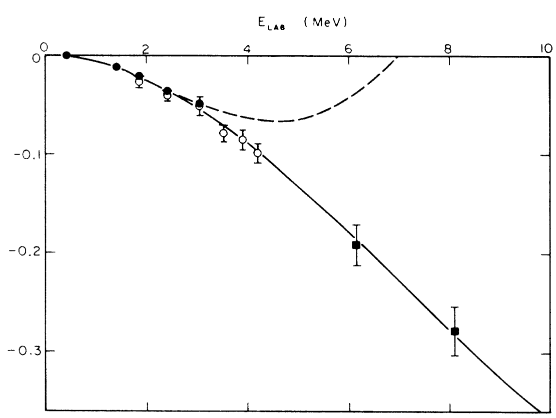


FIG. 1. Plot of the nonlinear part of the function $K$, $\Delta K = K - (A_0 + A_1 E)$. The solid line corresponds to a four-parameter fit to 14 experimental points. The dashed line is obtained with the interference-minimum datum and the results of Ref. 6. The circles correspond to Refs. 5, 6, and 8. The squares correspond to Ref. 14.

The parameters $P$ and $Q$ are strongly correlated and if $Q$ is assumed at the values calculated in Ref. 2 ($Q = 0.019$), $P$ would also fall very close to the value calculated there ($P = 0.055$), as can be seen interpolating the values contained in Table II.

Summarizing, the shape dependence is established in the range from 0 to 10 MeV from the context of a large amount of experimental data, and quite independently from effects attributable to corrections or assumptions made in their analysis.

It is hoped that nuclear calculations based on the detailed proton-proton interaction will abandon the use of potential shapes (or models) inconsistent with the results reported here.

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## CORE EXCITATION IN $Ir^{193}$

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In a recent paper[1] some essential points concerning the level scheme of $Ir^{193}$ (populated by the beta decay of $Os^{193}$) were clarified. The decay scheme is presented in Fig. 1.

The spin assignments for the 139-, 180-, 361-, 460-, and 712-keV states were determined as $\frac{5}{2}^+$, $\frac{3}{2}^+$, $\frac{3}{2}^+$, $\frac{3}{2}^+$, and $\frac{3}{2}^+$, respectively. The $\frac{3}{2}^+$ spin assignment for the 361-keV level showed that this level could not be identified as the $\frac{7}{2}^+$ 357-keV level. The latter level has been found to be populated by Coulomb excitation.[2] Furthermore, it was found that the 181-keV line is composed of two gamma rays and that only about one-third of the total intensity of the line de-excites the 180-keV state. It follows then that the ratio of the $M1$ reduced transition probability of the 107-keV line to that of the 180-keV transition is equal to 20. This behavior of the nucleus $Ir^{193}$ resembles very closely that of $Au^{197}$. This resemblance is further displayed by the sequence of excited levels and the static moments of the states in both nuclei. The mode of de-excitation of the levels and their static moments were accounted for in $Au^{197}$ by describing the states with particle-core wave functions.[3,4]

It was tempting to see whether a similar description of the states in $Ir^{193}$ would account for the mode of de-excitation of the states and for their static moments. $Ir^{193}$ lies in the intermediate region between the deformed and the spherical nuclei, and it is not clear, a priori, whether the principle of "weak coupling" is applicable for this nucleus.

In order to make a quantitative comparison between theory and experiment, we carried out lifetime measurements by the self-comparison method[5] for the most prominent states of $Ir^{193}$ which had not been yet determined. The mean lifetimes of the 139-keV and the 460-keV states were found to be $(1.27 \pm 0.13)\times 10^{-10}$ and (2.7