QUANTUM TRANSPORT THEORY IN STRONG MAGNETIC FIELDS

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We present a theory of transport in strong magnetic fields which is not limited to linear response in the electric field. The steady-state occupation probabilities of the eigenstates of the crossed-field Hamiltonian are determined by a master equation and are shown to be independent of the absolute coupling constants to the scattering system, but dependent on the form of the interaction. A generalized orbit-jump formula is derived for the dissipative current.

While considerable progress has been made in establishing a quantum theory of galvanomagnetic phenomena for Ohmic conductivity[1] (linear response in the electric field), relatively little has been done with the non-Ohmic problem,[2] which we treat in this Letter.[3] We present a theory which is valid for strong magnetic fields $\omega_c\tau \gg 1$, where ω_c and $1/\tau$ are the cyclotron and mean collision frequencies, respectively, but is not limited to linear response in the electric field. We merely outline the theory here and show how it reduces simply to the usual results for Ohmic conductivity. The details will be published elsewhere.

We take a simple effective-mass Hamiltonian for an electron in the presence of an electric field E in the x direction and a magnetic field B in the z direction. In the Landau gauge we have

$$H_{\text{eff}} = \frac{p_x^2 + (p_y - eBx)^2 + p_z^2}{2m} - eEx. \tag{1}$$

The associated eigenfunctions and eigenvalues are[4]

$$\Psi_{n,k} = \Phi_n(x-\bar{x})\exp[i(k_y y + k_z z)],$$

$$\bar{x} = \frac{\hbar k_y}{eB} + \frac{mE}{eB^2};$$

$$\epsilon_{n,k} = (n+\tfrac{1}{2})\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m} - eE\bar{x} + \frac{m}{2}\left(\frac{E}{B}\right)^2,$$

$$\omega_c = \frac{eB}{m}. \tag{2}$$

The Φ_n are the usual Hermite functions associated with the harmonic oscillator. We denote by H_S and V the Hamiltonian of the scattering system (SS) and the electron-SS interaction, respectively. The kinetic equation for the density matrix ρ describing the total system is

$$\frac{\partial\rho}{\partial t} = \frac{i}{\hbar}[\rho, H_e + H_S + V]. \tag{3}$$

The derivation of the "master equation" describing the evolution of the occupation probabilities (diagonal elements of ρ) is well known by now,[5,6] and we merely state the result:

$$\frac{\partial\rho_{\alpha S}}{\partial t} = \frac{2\pi}{\hbar}\sum_{\beta,S'}(\rho_{\beta S'} - \rho_{\alpha S})\,|V_{\alpha S,\beta S'}|^2 \times \delta(E_{\alpha S} - E_{\beta S'}), \tag{4}$$

where α, β and S, S' are the set of quantum numbers defining the eigenstates of H_e and H_S, respectively. $E_{\alpha S}$ and $\rho_{\alpha S}$ are, respectively, the energy and occupation probability for the total system in the state αS. Neglecting any correlation between electrons and SS we set $\rho = fP$, where f and P are the electron and SS density matrices, respectively, and take the trace over the SS variables in Eq. (4):

$$\frac{\partial f_\alpha}{\partial t} = \sum_\beta [f_\beta T_{\beta\alpha} - f_\alpha T_{\alpha\beta}] \tag{5}$$

with

$$T_{\alpha\beta} = \frac{2\pi}{\hbar}\sum_{S,S'} P(S)\,|V_{\alpha S,\beta S'}|^2 \delta(E_{\alpha S} - E_{\beta S'}).$$

We take $P(S)$ to be given: the thermal-equilibrium distribution for phonons or an appropriate random ensemble[7] for impurity scattering. If we continue to neglect electron-electron interactions, but allow for Fermi statistics, the $T_{\alpha\beta}$ of Eq. (5) are to be multiplied by the usual exclusion factor $1 - f_\beta$.

We see from Eq. (5) that the steady-state f_α are independent of the absolute coupling constants to the SS, just as they are in the weak-coupling theory of thermal equilibrium; they do however depend on the form of the interaction. Thus the diagonal elements of ρ are of zeroth order in V, but depend on the form of V. These diagonal elements are not in general simply related to the thermal-equilibrium distribution as

has been asserted by Adams and Holstein[4] and by Kahn and Frederikse,[8] although the methods employed by these authors are valid for the linear-response theory. This is a fundamental difference between the nonlinear and linear theories. The zero-order probabilities are not given, but must be determined by solving the irreversible kinetic equation (5) in the steady state. It is only in the linear case that they may be determined without recourse to Eq. (5). The linear problem will be discussed below.

The f_α we seek describe an almost spatially uniform electron distribution which is quite different from the thermal-equilibrium solutions in an electric field, which correspond to highly nonuniform distributions. More specifically we require that our f_α be k_y independent, whereas the thermal-equilibrium solutions are functions of the electron energy, Eq. (2), and are thus k_y dependent.

Since the average velocity for the eigenstate (2) is $\bar{V}_x = 0$, $\bar{V}_y = -E/B$, we shall need the off-diagonal elements of ρ in order to calculate the dissipative (x-directed) current. We simplify this calculation by multiplying Eq. (4) by V_y and taking the trace. This yields in the steady state

$$\frac{d}{dt}\langle V_y\rangle = \frac{i}{\hbar}\mathrm{Tr}\{\rho[H, V_y] + \rho[V, V_y]\} = 0, \tag{6}$$

$$\mathrm{Tr}\rho V_x = \langle V_x\rangle = \frac{i}{\hbar\omega_c}\mathrm{Tr}\rho[V, V_y] = \frac{i}{\hbar eB}\mathrm{Tr}\rho[V, p_y].$$

The off-diagonal elements of ρ are expressed in terms of the diagonal elements, by use of the Laplace-transform procedure of Kohn and Luttinger.[7] To lowest order in V Eq. (6) then becomes

$$\langle V_x\rangle = \sum_{\alpha,\beta}\left[\frac{\hbar k_y(\beta) - \hbar k_y(\alpha)}{2eB}\right][f_\alpha T_{\alpha\beta} - f_\beta T_{\beta\alpha}]$$

$$= \sum_{\alpha,\beta}\left[\frac{\bar{x}(\beta) - \bar{x}(\alpha)}{2}\right][f_\alpha T_{\alpha\beta} - f_\beta T_{\beta\alpha}] \tag{7}$$

which is the generalization of the usual orbit-jump formula,[4] which we derive below. Equation (7) thus provides us with a relation between the dissipative current and the occupancy probabilities f_α.[9,10]

In considering the Ohmic limit of Eqs. (5) and (7), we assume for simplicity that the scattering system is isotropic. It is then readily verified that to first order in the electric field

$$f_\alpha = f_0(\epsilon_\alpha^0), \tag{8}$$

where f_0 is the thermal-equilibrium distribution and ϵ_α^0 is the electron energy in the absence of the electric field: Eq. (2) with $E = 0$. Inserting Eq. (8) in Eq. (7) and keeping only linear terms in E in the $T_{\alpha\beta}$, we obtain

$$\langle V_x\rangle = \frac{eE}{kT}\sum_{\alpha,\beta}\frac{[\bar{x}(\alpha) - \bar{x}(\beta)]^2}{2} f_0(\epsilon_\alpha^0) T_{\alpha\beta}^0, \tag{9}$$

where $T_{\alpha\beta}^0 = T_{\alpha\beta}(E = 0)$. Here again, if Fermi statistics are applicable, the $T_{\alpha\beta}$ appearing in this equation and similarly in Eq. (7) are to be multiplied by $1 - f_\beta$.

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