# IMPLICATIONS OF UNEQUAL CHARGE ASYMMETRY IN $K_L{}^0 \to \pi^\pm e^\mp \nu$ AND $K_L{}^0 \to \pi^\pm \mu^\mp \nu$

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The possibility is considered that the charge asymmetries $\delta_e$ and $\delta_\mu$ in the decays $K_L{}^0 \to \pi^\pm e^\mp \nu$ and $K_L{}^0 \to \pi^\pm \mu^\mp \nu$ might turn out to be different. Some implications, including a test of $CPT$, are pointed out.

In two recent experiments,[1,2] it was found that in the three-body leptonic decay of $K_L{}^0$ the mode with the positive lepton has a rate different from the mode with the negative lepton. The charge asymmetry is defined as

$$\delta = (\Gamma_+ - \Gamma_-)/(\Gamma_+ + \Gamma_-), \tag{1}$$

where $\Gamma_+$ and $\Gamma_-$ denote the decay rates into the $l^+$ and $l^-$ modes, respectively. The values quoted for the asymmetry in the $e$[1] and $\mu$[2] modes are

$$\delta_e = +(2.24 \pm 0.36) \times 10^{-3},$$

$$\delta_\mu = +(4.05 \pm 1.35) \times 10^{-3}.$$

Considering the experimental errors, the above numbers are compatible with $\delta_e = \delta_\mu$, in accord with most expectations. It is possible, however, that improved precision will reveal a genuine difference between $\delta_e$ and $\delta_\mu$. Such a difference will have implications to which we wish to draw attention.

The expression usually given for the charge asymmetry, assuming $CPT$ invariance, is[3]

$$\delta = 2\,\mathrm{Re}\,\epsilon \frac{1 - |x|^2}{|1 + x|^2}, \tag{2}$$

where $x$ is defined as the ratio of $\Delta Q = -\Delta S$ and $\Delta Q = \Delta S$ amplitudes in the following way:

$$\text{Amplitude } (K^0 \to \pi^- l^+ \nu) = f \quad (\Delta Q = \Delta S),$$
$$\text{Amplitude } (\bar{K}^0 \to \pi^- l^+ \nu) = g \quad (\Delta Q = -\Delta S), \tag{3}$$
$$x = g/f.$$

If $x$ is identically zero, the charge asymmetry is

$$\delta = 2\,\mathrm{Re}\,\epsilon. \tag{4}$$

Therefore, in the absence of $\Delta Q = -\Delta S$ amplitudes, the asymmetry is independent of the dynamics of the decay and must be the same for the $e$ and $\mu$ modes. (This conclusion is independent of any assumption about $\mu$-$e$ symmetry.) Conversely, if it is established that $\delta_e$ and $\delta_\mu$ are different, that will be evidence for the existence of $\Delta Q = -\Delta S$ amplitudes.

In order to analyze the situation $\delta_e \neq \delta_\mu$ we must allow the ratio $x$ to be not simply a constant but, in general, a function of the variables which specify the state of the $\pi l \nu$ system, for example, the spins of the leptons and the position in the Dalitz plot. For this purpose, we define amplitude matrices $f$ and $g$ in the following way:

$$\langle \alpha, ij; \pi^- l^+ \nu | T | K^0 \rangle = f_{ij}(\alpha),$$
$$\langle \alpha, ij; \pi^- l^+ \nu | T | \bar{K}^0 \rangle = g_{ij}(\alpha) \tag{5}$$

Here $\alpha$ denotes the variables which specify the point in the Dalitz plot at which the amplitude is defined,[4] while $i$ and $j$ refer to the helicities of the (charged) lepton and the neutrino. (We will take $i, j$ to be the integers 1 or 2, denoting positive and negative helicity, respectively.) $CPT$ invariance gives

$$\langle \alpha, ij; \pi^+ l^- \nu | T | \bar{K}^0 \rangle = f_{ji}{}^*(\alpha),$$
$$\langle \alpha, ij; \pi^+ l^- \nu | T | K^0 \rangle = g_{ji}{}^*(\alpha), \tag{6}$$

and the partial decay rates are

$$\Gamma_\pm = \textstyle\sum_\alpha \sum_{ij} |\langle \alpha, ij; \pi^\mp l^\pm \nu | T | K_L{}^0 \rangle|^2 \tag{7}$$

($\sum_\alpha$ means an integration over the phase space). This yields for the charge asymmetry the expression (neglecting terms of order $\epsilon$ compared with unity)

$$\delta = 2\,\mathrm{Re}\,\epsilon \frac{\sum_\alpha \mathrm{Tr} f^\dagger(\alpha) f(\alpha) - \sum_\alpha \mathrm{Tr} g^\dagger(\alpha) g(\alpha)}{\sum_\alpha \mathrm{Tr} f^\dagger(\alpha) f(\alpha) + \sum_\alpha \mathrm{Tr} g^\dagger(\alpha) g(\alpha) + 2\,\mathrm{Re} \sum_\alpha \mathrm{Tr} f^\dagger(\alpha) g(\alpha)}. \tag{8}$$

In the circumstance that the $\Delta Q = -\Delta S$ amplitude is a constant multiple of the $\Delta Q = \Delta S$ amplitude, the matrices $f$ and $g$ become proportional, i.e., $g(\alpha) = x f(\alpha)$, where $x$ is a constant, and the above expression reduces to (2). To simplify the discussion, we assume that the $\Delta Q = -\Delta S$ amplitudes are small compared with the $\Delta Q = \Delta S$, in the sense that

$$\sum_\alpha \mathrm{Tr}g^\dagger(\alpha)g(\alpha) \ll \sum_\alpha \mathrm{Tr}f^\dagger(\alpha)f(\alpha). \tag{9}$$

The available evidence is not conclusive,[5] but it is commonly assumed that this approximation is good to a few percent. Defining the matrix $x(\alpha)$ through the matrix relation

$$g(\alpha) = x(\alpha)f(\alpha), \tag{10}$$

we get

$$\delta = 2\,\mathrm{Re}\epsilon[1 - 2\,\mathrm{Re}\langle x\rangle], \tag{11}$$

where $\langle x\rangle$ is defined as

$$\langle x\rangle = \frac{\sum_\alpha \mathrm{Tr}f^\dagger(\alpha)x(\alpha)f(\alpha)}{\sum_\alpha \mathrm{Tr}f^\dagger(\alpha)f^\dagger(\alpha)}. \tag{12}$$

We now introduce explicit reference to the nature of the lepton (whether $\mu$ or $e$) by means of a subscript. Then the charge asymmetry in $K_L{}^0 \to \pi^\pm l^\mp \nu$ is

$$\delta_l = 2\,\mathrm{Re}\epsilon[1 - 2\,\mathrm{Re}\langle x\rangle_l], \tag{13}$$

where $\langle x\rangle_l$ is

$$\langle x\rangle_l = \frac{\sum_{\alpha\in l} \mathrm{Tr}f_l{}^\dagger(\alpha)x_l(\alpha)f_l(\alpha)}{\sum_{\alpha\in l} \mathrm{Tr}f_l{}^\dagger(\alpha)f_l(\alpha)}. \tag{14}$$

By $\sum_{\alpha\in l}$ we mean integration over the phase space of the mode $l$. Defining as a measure of the inequality of $\delta_e$ and $\delta_\mu$ the parameter

$$\Delta = (\delta_e - \delta_\mu)/(\delta_e + \delta_\mu), \tag{15}$$

we obtain

$$\Delta = \mathrm{Re}[\langle x\rangle_\mu - \langle x\rangle_e]. \tag{16}$$

Equation (16) represents a relation between three measurable quantities. While $\Delta$ is determined by a measurement of the charge asymmetries $\delta_e$ and $\delta_\mu$, $\langle x\rangle_e$ and $\langle x\rangle_\mu$ are determined by studying the time dependence of the decays $K^0 \to \pi^\pm e^\mp \nu$ and $K^0 \to \pi^\pm \mu^\mp \nu$. Assuming (9), the time distribution of $l^\pm$ in the decay $K^0 \to \pi l \nu$ is[6]

$$N^\pm(t) = C[(1 + 2\,\mathrm{Re}\langle x\rangle_l)e^{-\gamma_1 t} + (1 - 2\,\mathrm{Re}\langle x\rangle_l)e^{-\gamma_2 t} + 2e^{-\frac{1}{2}(\gamma_1+\gamma_2)t}(\pm\cos\Delta m t + 2\,\mathrm{Im}\langle x\rangle_l \sin\Delta m t)]. \tag{17}$$

Since the only assumption [aside from (9)] employed in deriving (16) is $CPT$ invariance, the relation serves as a test of $CPT$.

A nonzero value of $\Delta$ implies that $\mathrm{Re}\langle x\rangle_e$ and $\mathrm{Re}\langle x\rangle_\mu$ are different.[7] The available data on $\langle x\rangle_e$ and $\langle x\rangle_\mu$, however, do not permit any conclusion.[8] If we assume tentatively that $|\mathrm{Re}\langle x\rangle_e|$ and $|\mathrm{Re}\langle x\rangle_\mu|$ are each not more than 30%, we predict that $|\Delta|$ should not exceed 60%. It also follows from (16) that the charge asymmetries $\delta_e$ and $\delta_\mu$ should be equal in any of the following situations: (i) if the $\Delta Q = -\Delta S$ amplitude is absent; (ii) if the $\Delta Q = -\Delta S$ amplitude is a constant multiple of the $\Delta Q = \Delta S$, and $\mu$-$e$ symmetry holds[9] (thus ensuring that the constant is the same for the $\mu$ and $e$ modes); and (iii) if the $\Delta Q = -\Delta S$ amplitude is everywhere pure imaginary relative to the $\Delta Q = \Delta S$ (as, for example, in Sachs' model[10] of $CP$ nonconservation).

A more precise measurement of the charge asymmetries $\delta_e$ and $\delta_\mu$, as well as information on $\langle x\rangle_e$ and $\langle x\rangle_\mu$ from the time dependence of $K^0 \to \pi l \nu$, should throw interesting light on the $\Delta Q = -\Delta S$ interaction.

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[1]S. Bennett, D. Nygren, H. Saal, J. Steinberger, and J. Sunderland, Phys. Rev. Letters 19, 993 (1967).

[2]D. Dorfan, J. Enstrom, D. Raymond, M. Schwartz, S. Wojcicki, D. H. Miller, and M. Paciotti, Phys. Rev. Letters 19, 987 (1967).

[3]L. Wolfenstein, Nuovo Cimento 62A, 17 (1966).

[4]The usual choice for $\alpha$ is $q^2$ and $\cos\theta$, $q^2$ being the invariant (four-momentum)$^2$ of the lepton pair, and $\theta$ the angle between the pion and the (charged) lepton in the center-of-mass frame of the lepton pair.

[5]For a summary, see M. Vivargent, CERN Report No. 67-24, 1967 (unpublished), Vol. IV.

[6]Vivargent, Ref. 5. $\gamma_1$ and $\gamma_2$ are the widths of $K_S{}^0$ and $K_L{}^0$, and $\Delta m$ is the mass difference $m_L - m_S$.

[7]It is desirable for this reason, in studying the time dependence of $K^0 \to \pi l \nu$, to analyze the electron and muon events separately.

[8]$\langle x\rangle_e$ has been measured in three recent experiments and the values quoted are the following: (i) $\mathrm{Re}\langle x\rangle_e = (0.035^{+0.11}_{-0.13})$, $\mathrm{Im}\langle x\rangle_e = (0.21^{+0.15}_{-0.11})$ [B. Aubert et al., Phys. Letters 17, 59 (1965)]; (ii) $\mathrm{Re}\langle x\rangle_e = \pm(0.06 \pm 0.25)$, $\mathrm{Im}\langle x\rangle_e = \pm(0.43 \pm 0.25)$ [M. Baldo-Ceolin et al., Nuovo Cimento 38, 684 (1965)]; (iii) $\mathrm{Re}\langle x\rangle_e = (0.17^{+0.16}_{-0.35})$, $\mathrm{Im}\langle x\rangle_e = (0.0 \pm 0.25)$ [L. Feldman et al., Phys. Rev. 155, 1611

(1967)]. No clear information on $\langle x\rangle_\mu$ is available. However, in two experiments, $e$ and $\mu$ events were treated together, and so the measured parameter is some weighted average of $\langle x\rangle_e$ and $\langle x\rangle_\mu$. The values quoted are the following: (i) $\mathrm{Re}\langle x\rangle = -(0.08\pm0.15)$, $\mathrm{Im}\langle x\rangle = -(0.24\pm0.35)$ [P. Franzini et al., Phys. Rev. 140, B127 (1965)], and (ii) $\mathrm{Re}\langle x\rangle = (0.17\pm0.10)$, $\mathrm{Im}\langle x\rangle = (0.20\pm0.10)$ [D. G. Hill et al., Phys. Rev. Letters 19, 668 (1967)].

[9]By this we mean that both the $\Delta Q = \Delta S$ and $\Delta Q = -\Delta S$ interaction satisfy $\mu$-$e$ symmetry.

[10]R. G. Sachs, Phys. Rev. Letters 13, 286 (1964).

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# $A_1$-$\pi$ MIXING AND PRODUCTION OF $A_1$ MESONS BY DIFFRACTION DISSOCIATION OF PIONS*

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A mixing model based on current-field identity is used to evaluate the coupling constant for the off-mass-shell transition $A_1 \to \pi$. We find, using either the Weinberg relations and the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relation or the Goldberger-Treiman relation, $g_{A_1\pi} = m_{A_1}$. We then estimate the cross section for production of $A_1$ by pions on heavy nuclei with the assumption that diffraction dissociation is the most important mechanism by analogy to $\rho^0$ photoproduction.

The assumption of vector-meson dominance of form factors and the consequent $\rho^0$-photon mixing has been thoroughly investigated from the point of view of current-field identity by Kroll, Lee, and Zumino.[1] The purpose of the present work is to consider $A_1$-$\pi$ meson mixing[2] in analogy to the above, and show that Weinberg's relations[3] and the Goldberger-Treiman relation are consistent in the sense that both give the same value for the $A_1$-$\pi$ coupling constant. We also suggest that the mixing model can be tested by studying the production of $A_1$ by diffraction dissociation of pions on complex nuclei, in analogy to $\rho^0$ photoproduction.[4]

The fourth component of an axial-vector meson in a virtual state has the same quantum numbers as the pion. The resulting mixing of the two fields may be described by the interaction Lagrangian

$$\mathcal{L} = g_{A_1\pi}\varphi_\mu{}^\alpha \partial_\mu \pi^\alpha, \tag{1}$$

where $\varphi_\mu{}^\alpha$ and $\pi^\alpha$ are the $A_1$ and pion fields, respectively. As is well known, the vector-meson-dominance model for the electromagnetic form factors gives the $\rho^0$-photon coupling constant directly. In contrast, it is necessary to resort to an indirect argument to determine the corresponding $A_1$-$\pi$ coupling constant.

Using (1), the Lagrangian equations of motion give[2]

$$\varphi_\mu{}^\alpha = a_\mu{}^\alpha + g_{A_1\pi} m_{A_1}{}^{-2} \partial_\mu \pi^\alpha, \tag{2}$$

with

$$(\Box - m_{A_1}{}^2) a_\mu{}^\alpha = 0,$$
$$\partial_\mu a^\alpha = 0 \tag{3}$$

In this form the mixing has been made explicit.

Let us now introduce the assumption of field-current identity, and write for the axial-vector current

$$A_\mu{}^\alpha = G_{A_1}\varphi_\mu{}^\alpha$$
$$= G_{A_1} a_\mu{}^\alpha + G_{A_1} g_{A_1\pi} m_{A_1}{}^{-2} \partial_\mu \pi^\alpha, \tag{4}$$

where $G_{A_1}$ is the leptonic decay amplitude of the $A_1$ meson. Since the axial-vector current $A_\mu{}^\alpha$ is assumed to be dominated by the $A_1$ meson and the pion, we may identify the coefficient of $\partial_\mu \pi^\alpha$ in (4) with the leptonic decay amplitude of the pion, namely

$$G_{A_1} g_{A_1\pi} m_{A_1}{}^{-2} = F_\pi. \tag{5}$$

Then, introducing the well-known relations[3,5] $G_\rho{}^2 = 2m_\rho{}^2 F_\pi{}^2$, $m_{A_1}{}^2 = 2m_\rho{}^2$, and either of Weinberg's relations[3]

$$m_\rho{}^{-2} G_\rho{}^2 - m_{A_1}{}^{-2} G_{A_1}{}^2 = F_\pi{}^2,$$
$$G_\rho{}^2 = G_{A_1}{}^2, \tag{6}$$

we obtain

$$g_{A_1\pi} = m_{A_1}. \tag{7}$$