when

$$\lambda = e^{3\pi i/2}\{6\pi N + 3\pi n - \tfrac{3}{4}\pi - 3i \times [\ln(2n)! + 2^{-1}\ln\tfrac{1}{2}\pi - \ln\ln N - (2n+\tfrac{1}{2})\ln(24\pi N) - 1]\}^{-1}$$

for even parity;

$$E = 2n + \tfrac{3}{2} + \eta$$

$$\eta = \left\{\ln[24\pi(N+1)] + \gamma - \sum_{k=1}^{2n+1} k^{-1}\right\}^{-1},$$

when

$$\lambda = e^{3\pi i/2}\{6\pi(N+1) + 3\pi n + \tfrac{3}{4}\pi - 3i \times [\ln(2n+1)! + 2^{-1}\ln\tfrac{1}{2}\pi - \ln\ln N - (2n+\tfrac{3}{2})\ln[24\pi(N+1)] - 1]\}^{-1}$$

for odd parity. γ is Euler's constant and N is a positive integer.

(2) The resolvent $(z-H)^{-1}$ has poles at

$$\lambda = e^{3\pi i/2}\left\{-3i\ln\frac{\Gamma(\frac{1}{4}+\frac{1}{2}z)}{\Gamma(\frac{1}{4}-\frac{1}{2}z)} + 6\pi N + \tfrac{1}{4}\pi\right\}^{-1} \tag{11a}$$

provided that $z \neq 2N + \frac{1}{2}$; and at

$$\lambda = e^{3\pi i/2}\left\{-3i\ln\frac{\Gamma(\frac{3}{4}+\frac{1}{2}z)}{\Gamma(\frac{3}{4}-\frac{1}{2}z)} + 6\pi N - \tfrac{1}{4}\pi\right\}^{-1} \tag{11b}$$

provided that $z \neq 2N + \frac{3}{2}$. In Eq. (11) N is any sufficiently large positive integer.

That the resolvent has poles is a surprise to us and as far as we know this has not even been conjectured.[5]

$E(\lambda)$ has some interesting symmetry properties. Since Eqs. (2) and (2a) are real in their respective variables, we have

$$E(\lambda) = E^*(\lambda^*) = -E^*(e^{3\pi i}\lambda^*) = -E(e^{3\pi i}\lambda).$$

As a result of this symmetry the branch cuts are very "short."

It is tempting to conjecture that many of the qualitative features of this model are also present in a realistic field theory.

We wish to thank Professor A. M. Jaffe for many interesting discussions.

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[1]We believe that the question of convergence of perturbation series was first discussed by F. J. Dyson [Phys Rev. 85, 631 (1952)].

[2]T. T. Wu, Phys. Rev. 125, 1436 (1962).

[3]If the Hamiltonian is Wick ordered, there are fewer diagrams of order n because no internal line may have both ends connected to the same vertex. Thus the terms in the Wick-ordered perturbation series are slightly smaller than those of the non-Wick-ordered series. However this estimate holds for either perturbation series.

[4]T. T. Wu, Phys. Rev. 143, 1110 (1966). Note that WKB techniques are used here solely to locate singularities approximately and not to define the analytic continuation of $E(\lambda)$.

[5]A. M. Jaffe [thesis, Princeton University, 1965 (unpublished)] has proved that for negative z, the resolvent is analytic in the cut λ plane with a cut along the negative real axis extending from the origin to $-\infty$. This is entirely consistent with our results which maintain that nothing interesting happens until the phase of the coupling constant reaches nearly 270°.

MODEL FOR THE VIOLATION OF *CP* INVARIANCE

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An explicit model for *CP* nonconservation is constructed within the framework of the current-current form of weak interactions, which has $\Delta I = \frac{1}{2}$ for the *CP*-invariant part of (nonleptonic) H_w with $|\Delta S| = 1$, violates the $\Delta I = \frac{1}{2}$ rule for the *CP*-nonconserving part, and has no observable effects of T nonconservation in the leptonic decay modes and the electric dipole moment of the neutron.

Since the time the violation of *CP* invariance was first noticed[1] through the decay of the long-lived component K_L of the neutral kaon complex into $\pi^+\pi^-$, there have been considerable experimental as well as theoretical investigations about the nature and the structure of the *CP*-nonconserving interactions.[2] On the theoretical side, because of the elegance of the current-current theory of weak interactions with the $V-A$ structure of the currents, one naturally wishes to see whether CP nonconservation can be simply incorporated into the current-current form of the weak-interaction Hamiltonian. Thus, in a model such as Glashow's,[3] one starts with the usual charged currents and introduces CP nonconservation by inserting suitable relative phase factors between the vector and the axial-vector hadronic currents. If in such a model one further invoked the octet-enhancement hypothesis[4] to have an appropriate $\Delta I = \frac{1}{2}$ rule for the CP-conserving, weak nonleptonic Hamiltonian, one could have a relatively large magnitude for the CP-nonconservation parameter, which would lead to an observable violation of T invariance in some hadron semileptonic decays and might even yield a relatively large magnitude for the electric dipole moment of the neutron.[5,6] The present paper serves to show a possible alternative way by which to introduce CP-nonconservation through the current-current form of weak interactions while retaining the V–A structure of the currents and discusses its consequences.

Specifically, we study the possibility that in addition to the usual charged hadron currents and the charged lepton currents, the current-current form of the weak Hamiltonian also contains neutral hadron currents of the following structure[7,8]:

$$K_\mu = \cos\theta[(V_\mu^{\;3} + 3^{-\frac{1}{2}}V_\mu^{\;8}) + (A_\mu^{\;3} + 3^{-\frac{1}{2}}A_\mu^{\;8})] + e^{i\varphi}\sin\theta[(V_\mu^{\;6} + iV_\mu^{\;7}) + (A_\mu^{\;6} + iA_\mu^{\;7})], \tag{1}$$

where θ denotes the Cabibbo angle[9] and φ is a real phase, whose magnitude is required to be fixed. The current K_μ has the V–A structure for either the $\Delta S = 0$ or the $|\Delta S| = 1$ part; the relative magnitude of the strength of the $|\Delta S| = 1$ current to the strength of the $\Delta S = 0$ current in K_μ has the same absolute value as in the usual charged current J_μ. The total weak Hamiltonian H_w which is CPT invariant is then given by[8]

$$H_w = \frac{G}{\sqrt{2}}([J_\mu^{\;+}, J_\mu]_+ + [K_\mu^{\;+}, K_\mu]_+), \tag{2}$$

where J_μ denotes the usual Cabibbo current[9] plus the charged lepton currents L_μ, being given by

$$J_\mu = \cos\theta[(V_\mu^{\;1} + iV_\mu^{\;2}) + (A_\mu^{\;1} + iA_\mu^{\;2})] + \sin\theta[(V_\mu^{\;4} + iV_\mu^{\;5}) + (A_\mu^{\;4} + iA_\mu^{\;5})] + L_\mu. \tag{3}$$

We note the following features:

(a) If φ is close to 180°, the CP-conserving part of H_w (nonleptonic) with $|\Delta S| = 1$ satisfies an (almost) exact $\Delta I = \frac{1}{2}$ rule and more particularly an octet rule.[10]

(b) H_w with $|\Delta S| = 1$ has a CP-nonconserving (nonleptonic) part with a coupling strength of the order of $G\sin\varphi\cos\theta\sin\theta$. It contains both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ terms which have relative strengths of the same order of magnitude. From the experimental results on $K_L \to 2\pi$ decays,[1,11] we expect $|\varphi - \pi| \approx 10^{-3}$.

(c) The $\Delta S = 0$ part of H_w (nonleptonic) is exactly T invariant for both parity-conserving and parity-nonconserving interactions.

The weak-interaction Hamiltonian H_w of Eq. (2) then has the following further consequences:

(d) Because of the smallness of the magnitude $|\varphi - \pi|$, the decays $K^\pm \to 2\pi$, assuming the present model to be correct, must proceed via a combination of weak and electromagnetic interactions. In general, all measurable violations of the $\Delta I = \frac{1}{2}$ rule in the usual nonleptonic decays should be due to electromagnetic corrections.

(e) There should be no observable violation of T invariance in leptonic and semileptonic decays. This feature is consistent with the present experimental analysis of the β decays[12] of neutron and Ne^{19} and the $K_{\mu 3}$ decays.[13]

(f) Since the magnitude of $|\sin\varphi| \approx 10^{-3}$, the effects of T nonconservation in the hyperon nonleptonic decays should be negligibly small, a result which is consistent with the analysis of the $\Delta \to p\pi^-$ decay.[14]

(g) There should be no observable electric dipole moment of the neutron, which is in accordance with the most recent experiments.[15]

(h) Using from now on the standard notation and the phase convention introduced by Wu and Yang[16] and because of the absence of $\Delta Q = -\Delta S$ currents in the present model, we expect the charge asymmetry δ in the $K_L \to \pi l \nu$ decays to be given by

$$\delta \equiv \frac{\Gamma(K_L \to l^+\pi^-\nu) - \Gamma(K_L \to l^-\pi^+\bar{\nu})}{\Gamma(K_L \to l^+\pi^-\nu) + \Gamma(K_L \to l^-\pi^+\bar{\nu})} = \mathrm{Re}\,\epsilon, \tag{4}$$

for $l = \mu$ as well as e. This result is not inconsistent with the presently available experimental data.[17] Further, since in this model $y_l = 0$ and $y_{3\pi}$ is expected to add a negligible contribution[18] to ϵ, one can obtain the following sum rule[19] to the lowest order in the CP-nonconservation parameters:

$$2\eta_{+-} + \eta_{00} = \tfrac{3}{2}\delta(1 - 2i\Delta/\Gamma_S), \tag{5}$$

which involves only experimentally knowable parameters (provided, of course, there are no $\Delta Q = -\Delta S$ currents); η_{+-} and η_{00} are the standard CP-nonconservation parameters involving $K_{L,S} \to 2\pi$, $\Delta = m_S - m_L$, and Γ_S is the width of K_S and δ is defined through Eq. (4). In general, because of the presence of CP-nonconserving $\Delta I = \frac{3}{2}$ terms in Eq. (2) we expect $\eta_{+-} \neq \eta_{00}$. The initially reported values[11] of $|\eta_{00}|$ indeed seem to indicate $\eta_{+-} \neq \eta_{00}$.

In conclusion, we would like to mention that in the present paper we have explicitly constructed a model in which the CP-invariant part of the nonleptonic weak Hamiltonian with $|\Delta S| = 1$ satisfies a $\Delta I = \frac{1}{2}$ rule,[20] while the CP-nonconserving part of H_w violates the $\Delta I = \frac{1}{2}$ rule (strengths of the $\Delta I = \frac{1}{2}$ and the $\Delta I = \frac{3}{2}$ terms are of the same order of magnitude). The violation of the $\Delta I = \frac{1}{2}$ rule is directly associated with the CP nonconservation. Further, the model predicts directly observable effects of T or CP nonconservation only in $K_L \to 2\pi$ and through the charge asymmetry in the $K_L \to \pi l \nu$ decays and little or no observable effects of T or CP nonconservation elsewhere.

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[2] CPT invariance will be assumed throughout our analysis and, consequently, the violations of CP and T invariance will be treated as equivalent.

[3] S. L. Glashow, Phys. Rev. Letters 14, 35 (1965).

[4] R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara, California Institute of Technology Report, 1964 (unpublished); S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964); R. F. Dashen and S. Frautschi, Phys. Rev. Letters 13, 497 (1964).

[5] For example, on the basis of a simple dimensional analysis the electric dipole moment of neutron is expected to be of the order of $\approx 10^{-21} e$ cm in Glashow's model, which seems to be considerably higher than the present experimental upper limit (see Ref. 15). Although this argument is purely dimensional and far from anything conclusive, it may be suggestive of some problems in introducing CP nonconservation through the charged currents alone, while having an octet enhancement mechanism to ensure a $\Delta I = \frac{1}{2}$ rule for the usual CP-preserving, weak nonleptonic Hamiltonian.

[6] In a recent article, Pati [J. C. Pati, Phys. Rev. Letters 20, 812 (1968)] has reconsidered the model in Ref. 3, although only in the context of the soft-pion limits.

[7] The superscripts on the currents are the standard SU(3) indices; see, for example, M. Gell-Mann, California Institute of Technology Report No. CTSL-20, 1961 (unpublished).

[8] We have supressed all space-time dependence.

[9] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[10] The idea of introducing neutral hadron currents so as to have a desired transformation property of H_w has been brought forward many times before; see, for example, M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964); footnote 8, in particular. However, the full context in which the current K_μ is being proposed here is new.

[11] For a summary of the present experimental situation, see, for example, J. W. Cronin, in Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester, New York, 1967, edited by C. R. Hagen et al. (Interscience Publishers, Inc., New York, 1968).

[12] M. T. Burgy et al., Phys. Rev. 120, 1829 (1960); F. P. Calaprice et al., Phys. Rev. Letters 18, 918 (1967).

[13] K. K. Young, M. J. Longo, and J. A. Helland, Phys. Rev. Letters 18, 806 (1967).

[14] O. E. Overseth and R. F. Roth, Phys. Rev. Letters 19, 391 (1967).

[15] P. D. Miller, W. B. Dress, J. K. Baird, and N. F. Ramsey, Phys. Rev. Letters 19, 381 (1967); C. G. Shull and R. Nathans, Phys. Rev. Letters 19, 384 (1967).

[16] T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).

[17] S. Bennett et al., Phys. Rev. Letters 19, 993 (1967); D. Dorfan et al., Phys Rev. Letters 19, 987 (1967).

[18] From the experiments on the charge asymmetry in $K_L \to \pi l \nu$, we expect $|\epsilon| \sim 10^{-3}$, while for the present model $|Y_{3\pi}/A_0{}^2|$ is expected to be of the order $\ll 10^{-3}$. Thus, $Y_{3\pi}$ is expected to have little effect on ϵ. See also M. Gaillard, Nuovo Cimento 35, 1225 (1965); L. Wolfenstein, ibid. 42A, 17 (1966).

[19] Unfortunately, the sum rule in Eq. (5) is not exclusively true only in the present model; but any model with $Y_l \approx 0$, $Y_{3\pi} \approx 0$ will reproduce the sum rule (see, e.g. Ref. 16).

[20] In particular it satisfies an octet rule. It also leaves unaffected all the well-known sum rules in the $K \to 3\pi$ and the $Y \to N\pi$ decays derived by using current algebra, soft-pion limits, and the current-current form of weak interactions; M. Suzuki, Phys. Rev. 144, 1154 (1966); Y. Hara and Y. Nambu, Phys. Rev. Letters 16, 875 (1966); H. Sugawara, Phys. Rev. Letters 15, 870, 997 (1965); M. Suzuki, Phys. Rev. Letters 15, 986 (1965).