EFFECT OF THE ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON ON SPONTANEOUS PAIR PRODUCTION IN A STRONG MAGNETIC FIELD

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Taking into account the anomalous magnetic moment of the electron by adding the so-called Pauli anomalous interaction term to the usual Dirac Hamiltonian for an electron in a constant magnetic field H, we conclude that spontaneous pair creation may occur for values of H of the order of m^2c^4/e^3, a "critical field" beyond which classical electrodynamics breaks down.

It has been suggested[1] that magnetic fields as large as 10^{14}-10^{16} G may exist in neutron stars, and Hoyle[2] has cited the possibility of a large primordial magnetic field. In a discussion of cosmological models for the expanding universe, Thorne[3] speculates that at magnetic field strengths[4]

$$H \gg H_c = m^2/e = 4.4\times10^{13}\ \text{G}, \tag{1}$$

most of the magnetic energy is converted into electron-positron pairs which in turn annihilate into small-wavelength radiation as the universe expands. However, it has been recently stated that spontaneous pair creation cannot take place in a constant magnetic field.[5] This conclusion is based on an analysis of the energy eigenstates of an electron in a constant magnetic field[6-8] which leads to the result that the separation of positive and negative energy states is always at least $2m$. This analysis however does <u>not</u> take account of the energy of the electron due to its anomalous magnetic moment. It is our purpose in this communication to include this energy and show that, under certain circumstances, spontaneous pair creation is possible. This conclusion has important astrophysical implications.

The Dirac equation for an electron with an anomalous magnetic moment, μ say, in a constant homogeneous magnetic field H takes the form[9,10]

$$i\frac{\partial\psi}{\partial t} = \{\vec{\alpha}\cdot(\vec{p}-e\vec{A}) + \gamma_4 m + \mu\gamma_4\vec{\sigma}\cdot\vec{H}\}\psi, \tag{2}$$

where the term containing μ (the so-called Pauli anomalous interaction term) is an addition to the usual Dirac Hamiltonian. Different values for the energy eigenvalues derived from this equation are quoted in the literature.[9,10] We have rederived the result using standard techniques and we find, in agreement with Ref. 10, that the energy eigenvalues E for an electron in a constant magnetic field H oriented along the z axis are given by

$$E = \pm\{p_z^{\,2} + [m\{1+(H/H_c)(2n+\xi+1)\}^{1/2} + \xi\mu H]^2\}^{1/2}, \tag{3}$$

where $n = 0, 1, 2, \cdots$ is the principal quantum number, $\xi = \pm1$ refers to spin up and spin down, and p_z is the momentum of the particle along the z axis.

Now μ has been calculated[11,12] to order α^2 but for our purposes it is sufficient to consider only the Schwinger[13] result

$$\mu = (\alpha/2\pi)\mu_B, \tag{4}$$

where μ_B is the Bohr magneton. Thus, we can write

$$E = \pm\left\{p_z^{\,2} + m^2\left[\left\{1+\frac{H}{H_c}(2n+\xi+1)\right\}^{1/2} + \xi\frac{\alpha}{4\pi}\frac{H}{H_c}\right]^2\right\}^{1/2}. \tag{5}$$

We would like to emphasize the elegance of this expression for E because it includes the effects of relativity, the Landau diamagnetic contribution due to quantization of the orbits in the plane perpendicular to z, the Pauli paramagnetic contribution due to the "normal" magnetic moment, and the contribution due to the anomalous magnetic moment. For values of $p_z = 0$, $n = 0$, and $\xi = -1$ we find a minimum value for $|E|$ given by

$$|E|_{\min} = m\left(1 - \frac{\alpha}{4\pi}\frac{H}{H_c}\right). \tag{6}$$

Thus, we see that allowance for the anomalous magnetic moment of the electron leads to the conclusion that the minimum separation between positive and negative energy states, ΔE say, is given by

$$\Delta E = 2m\left(1 - \frac{\alpha}{4\pi}\frac{H}{H_c}\right). \quad (7)$$

We can now state the main result of our paper: For values of H equal to $4\pi\alpha^{-1}H_c$, ΔE may be zero and thus spontaneous pair production may occur. It is interesting to note that a value of H of this order represents the well-known maximum value of H beyond which classical electrodynamics breaks down.[14]

It should be emphasized that our expression for $|E|_{\min}$ corresponds to the lowest orbit $n = 0$; for values of $n > 0$ it is clear that $|E|_{\min}$ will be greater than m, particularly for large values of H. Now the total possible number of electrons having values of n equal to zero is given by the level degeneracy number, g say, as follows[15]:

$$g = \frac{V^{2/3}}{2\pi} m^2 \frac{H}{H_c}, \quad (8)$$

where V is the total volume. Thus the large values of H which are necessary to obtain values of $|E|_{\min}$ equal to zero also help to increase the number of particles capable of possessing the quantum numbers appropriate to these zero energy values. The probability that each of the g levels corresponding to $n = 0$ are occupied depends of course on the degeneracy of the gas, and thus the rate of spontaneous pair production will depend on the temperature and density. An important factor also is whether or not the creation takes place in a vacuum. A detailed analysis of these points together with their astrophysical consequences (particularly in respect to the physics of neutron stars and the expanding universe) will be published elsewhere.

As a final remark we note another effect of magnetic fields which has been often ignored in astrophysical investigations, viz., that the rates of all elementary particle processes will be affected. This arises simply because there is a contribution to the energy of a particle due to its interaction with the magnetic field. Thus, for example, the rates of the weak-interaction processes which determine the rate of production of He and other elements in an expanding universe will be affected by the presence of a magnetic field.[16] In addition, certain processes which are forbidden in vacuum (such as neutrino pair production[17]) are now allowed.

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