GRAVITATIONAL RADIATION FROM THE PULSARS*

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Estimates are made of the expected gravitational radiation from the pulsars, assuming that they are either oscillating dwarf or neutron stars, or fast rotating binaries. It is concluded that a radiative Riemann tensor of magnitude $>10^{-42}$ cm^{-2} may be expected on earth as a consequence of some of these assumptions. A large detector of the type already developed is likely to succeed in detecting a radiative Riemann tensor exceeding 10^{-42} cm^{-2} at pulsar frequencies.

A number of rapidly pulsating radio sources have recently been observed.[1] These are characterized by radio emission at regular intervals of the order of a second. The unusually great regularity suggests some lightly damped periodic process such as rotation of a small component about a large one, or the oscillations of a very dense star. Either process might result in appreciable amounts of gravitational radiation. The recently developed methods of searching[2] for such radiation would appear capable of detection under favorable conditions if the objects are not too distant and if other sources of noise do not interfere.

Gravitational radiation at pulsar frequencies.—The weak-field approximations of general relativity enable us to estimate the gravitational radiation fields from either rotating or vibrating systems. At large distances the curvature tensor component R_{0i0i} which drives a gravitational wave detector is given approximately by[3]

$$R_{0i0i} \approx \frac{AGI\omega^4}{c^6 r}\cos\omega\left(t-\frac{r}{c}\right). \tag{1}$$

In (1) G is the constant of gravitation, and I is the moment of inertia or quadrupole moment. For vibration, ω is the angular frequency given in terms of the vibrational frequency ν by $\omega = 2\pi\nu$. For a rotating system, ω is given in terms of the angular velocity $d\varphi/dt$ by $\omega = 2d\varphi/dt$. The numerical factor A will depend on the details of the system, but A is ordinarily greater that $\frac{1}{2}$ and less than 10. For a rotating system with components having masses m_1 and m_2 we have

$$I = m_1 r_1^2 + m_2 r_2^2. \tag{2}$$

In (2) r_1 and r_2 are distances from the center of mass. We write

$$r_c = r_1 + r_2, \tag{3}$$

$$M = m_1 + m_2. \tag{4}$$

The reduced mass μ is given by

$$\mu = m_1 m_2/M. \tag{5}$$

Making use of (3)-(5) enables us to write (2) as

$$I = \mu r_c^2. \tag{6}$$

From the Kepler problem it is known that the angular velocity $d\varphi/dt$ is given by

$$\left(\frac{d\varphi}{dt}\right)^2 = \frac{GM}{r_c^2} = \frac{\omega^2}{4}; \tag{7}$$

(6) may be written as

$$I = \mu(4GM/\omega^2)^{2/3}. \tag{8}$$

For a rotating system, therefore, we may expect a radiation-field Riemann tensor

$$R_{0i0i} = \frac{AG^{5/3}\mu M^{2/3}\omega^{8/3}}{c^6 r}(16)^{1/3}\cos\omega\left(t-\frac{r}{c}\right). \tag{9}$$

For a vibrating system an expression similar to (9) is expected with μ, the effective vibrating mass, now defined by (9).

The strains to be expected in a gravitational wave detector by (9) may be calculated using the expressions already given.[3] The expected strain is given by

$$\epsilon \approx c^2 R_{0i0i} Q/\omega^2. \tag{10}$$

For rotating systems a decreasing period is expected as a result of radiation-damping reduction in size of the orbit. The great stability of the[4] observed periods probably rules this out as a powerful gravitational radiator.[5,6] For vibrational motions of a dwarf the effect of damping on the period will be small. For this case, if we assume that $M \approx 10^{33}$ g, $\mu \approx 10^{29}$ g, $r \approx 10^{20}$ cm, and $\omega \approx 2\pi$, (9) is evaluated as

$$R_{i0i0} \approx 5\times10^{-42}\ \text{cm}^{-2}. \tag{11}$$

Several possibilities are suggested by (11). One is to use as a detector the earth itself. A conservative estimate is $Q \approx 1$ in which case the expected strain is given by

$$\epsilon = >10^{-22}. \tag{12}$$

For a sound velocity in earth of 4000 m/sec the peak displacements implied by (11) are given by δ with

$$\delta \approx \epsilon\lambda/2 \approx 2\times10^{-17} \text{ cm}. \tag{13}$$

A study of the power spectrum of the microseisms indicates that for a bandwidth of about one cycle per hour, the ground displacement amplitude exceeds (13) by about 8 orders; therefore, the use of the earth will be difficult. However, the use of the moon is a better possibility.

A more promising alternative is to use a large mass well isolated from earth, which represents an extension, larger in mass and lower in frequency than the apparatus already described.[2] A $Q \approx 10^6$ is considered feasible,[7] in which case (11) gives

$$\epsilon \approx 10^{-16}. \tag{14}$$

Strains as small as (14) have already been observed.[2] Let us further assume that a detector consisting of a metal beam is employed with end loading to reduce its resonant length to manageable proportions. A reasonable length would be $\approx$50 m, with an end loading of 10^8 g. For observations over roughly one month we would expect the thermal fluctuations to be with rms displacement d given by $\frac{1}{2}m\omega^2\langle d^2\rangle = \frac{1}{2}kT$. For $T = 4$°K this gives

$$\langle d^2\rangle^{1/2} = 3\times10^{-13} \text{ cm} \tag{15}$$

with implied strain

$$\epsilon = \frac{3\times10^{13}}{5000} = 0.6\times10^{-16}. \tag{16}$$

The pulsar-induced strain is seen to exceed the thermal fluctuations even for a month's operation. Longer integration times, approaching one year, would yield improvement in signal-to-noise ratio.

The strain given by (12) is also the expected fractional change in distance between almost free bodies. For the earth-moon system an oscillation amplitude $\sim 4\times10^{-12}$ cm would be expected as a pulsar frequency modulation of the distance as measured by the proposed laser corner-reflector experiments.[8]

The estimates given here are very rough and may change substantially as more observational evidence is accumulated on the pulsars. Independent suggestions for search for gravitational radiation at pulsar frequencies have been made by Dyson.[8] I thank H. Zapolsky and C. W. Misner for very helpful discussion.

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[2]See, for example, J. Weber, Phys. Today 21, No. 4, 34 (1968).

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[4]J. D. H. Pilkington, A. Hewish, S. J. Bell, and T. D. Cole, Nature 218, 126 (1968).

[5]F. Pacini and E. E. Salpeter, Nature 218, 733 (1968).

[6]J. Ostriker, Nature 217, 1227 (1968).

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