sion neutrons has little effect on the shape of the distribution. Fine details in the behavior of $\bar{\nu}(A)$ may be obscured by the dispersion in mass. The result of $\bar{\nu}(A)$ for $Pu^{239}+n_{th}$[9] is shown for comparison with the U^{238} results. The fissioning nuclei are similar and the difference in the two curves is presumably mainly due to the difference in excitation energy. The difference in $\bar{\nu}(A)$ seems to be much larger for the heavy fragments than for the light fragments.

Figure 3 shows $\bar{\nu}(A)$ plotted for groups of "reduced" total kinetic energies $\tilde{E}_K$ defined by $\tilde{E}_K = E_K(239/2)^2/(239-A)A$. $\bar{\nu}(A)$ is shown for groups of "reduced" total kinetic energy $\tilde{E}_K$ rather than the uncorrected kinetic energy E_K in order to eliminate to a first approximation the effect of the charge division (as correlated with the mass division) on the variation of the total kinetic energy. Britt, Wegner, and Gursky[10] have found that in medium-excitation fission the average $\tilde{E}_K$ (they use a parameter called "scission distance" which is inversely proportional to $\tilde{E}_K$) is lower for events in the symmetric mass region than for events in the asymmetric region and have suggested that the symmetric fission events appearing in medium-excitation fission are due to a second "fission mode" not influenced by shell structure of the fragments and occurring with increasing yield as the excitation energy increases.

The graph for high values of $\tilde{E}_K$ [Fig. 3(a)] shows very strong variations in number of neutrons as a function of mass and very asymmetric mass distribution, whereas the sawtooth behavior becomes less pronounced as $\tilde{E}_K$ decreases and the symmetric mass yield increases [Figs. 3(b) and 3(c)]. For very low $\tilde{E}_K$ there is no minimum of $\bar{\nu}(A)$ at $A=130$; this function increases smoothly in the symmetric mass region between $A=115$ and $A=140$ as also observed by Schmitt and Konecny[3] in the symmetric mass region of Ra^{226}.

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STRUCTURE OF O^{16}†

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An exact shell-model calculation is presented that gives a good description of the properties of O^{16} and neighboring nuclei.

We wish to present a model which gives a good description of the energy spectra of O^{16} and neighboring nuclei. An outstanding property of O^{16} is the coexistence of low-lying states with so widely divergent characteristics that they have hitherto been described by unrelated models, e.g., developments of the spherical shell model using the particle-hole picture for negative parities[1] and deformed basis calculations for the rotational band beginning at 6 MeV.[2-4] The new feature of the present approach is to give a unified account of the coexisting states within the framework of an exact shell-model calculation.[5] The basis we selected includes all possible states of four particles moving in the $1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$ orbitals. The $1p_{3/2}$ and $1d_{3/2}$ levels were ignored in

the hope that they were of lesser importance in determining the structure of the lower states. This choice keeps the problem within reasonable limits while still allowing for a large number of the configurations of up to four particles and four holes that are thought to be necessary.

Even this simplified scheme requires specification of two single-particle level spacings and thirty antisymmetrized two-body matrix elements (to be denoted by $\langle l_1 l_2 | l_3 l_4 : JT \rangle$, the j values being understood since there is no possibility of confusion). Progress was possible only if these parameters could, to a large extent, be fixed a priori. This was feasible for the matrix elements because of the extensive work done recently by various authors; but the values to be adopted for the single-particle energies were highly uncertain. The choice of configurations suggested the idea of four particles moving outside a C^{12} core and therefore the use of a single-particle spectrum close to that of C^{13} (or N^{13}) in which the $2s_{1/2}$ level is 3.1 (2.4) MeV above the $1p_{1/2}$ level and 0.76 (1.2) MeV below the $1d_{5/2}$ level. On the other hand, particle-hole calculations in O^{16} have always used a much larger separation between the p and sd shells. We decided, therefore, in the first instance to leave both numbers as free parameters. In the following we discuss two choices for the set of numbers defining the interaction and the corresponding level spacings (cases I and II).

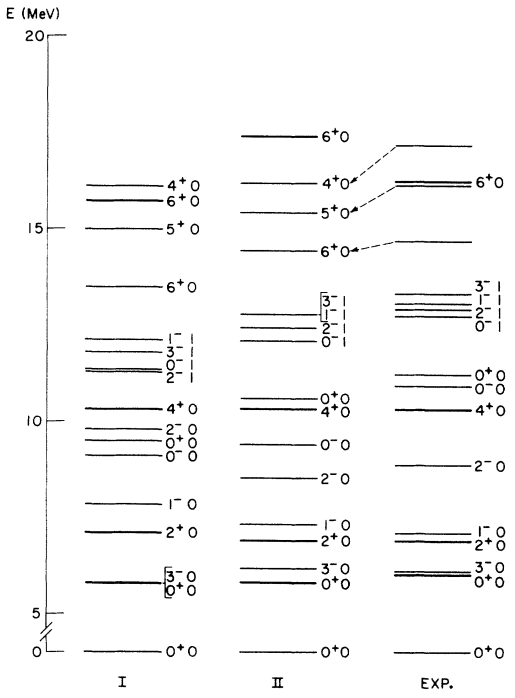

FIG. 1. Energies of some important states in O^{16}. Heavier lines are used for the rotational band and arrows refer to the states of B. G. Harvey, J. Cerny, R. H. Pehl, and E. Rivet, Nucl. Phys. 39, 160 (1962).

The results shown in Fig. 1 (case I)[6] were obtained with the matrix elements and rather strange single-particle spacings given in column I of Table I. It is seen that agreement with experiment is encouraging. The interaction is almost entirely the one obtained in the reaction matrix calculations of Kuo and Brown and given in different sources.[7-9] Only four matrix elements were missing, namely $\langle d^2 | p^2 : 10 \rangle$ and $\langle s^2 | p^2 : 10 \rangle$, which do not play any significant role in the calculations, and the pair $\langle p^2 | p^2 : 01 \rangle, \langle p^2 | p^2 : 10 \rangle$, whose influence we discuss later. The former two were essentially guessed, but we were guided by preliminary realistic calculations of Lee and Scott.[10] The latter two were taken arbitrarily from the work of Talmi and Unna[11] and are therefore the

Table I. Two-particle matrix elements $\langle l_1 l_2 | l_3 l_4 : JT \rangle$ and single-particle energies ϵ (in MeV). Here, $p = (1p_{1/2})$, $s = (2s_{1/2})$, and $d = (1d_{5/2})$. Radial wave functions are taken to be positive near the origin and the coupling order is $\vec{j} = \vec{l} + \vec{s}$.

$(l_1\ l_2)$	$(l_3\ l_4)$	J	T	I	II
dd	dd	0	1	-2.81	-3.41
dd	dd	1	0	-1.30	+0.01
dd	dd	2	1	-0.98	-1.21
dd	dd	3	0	-1.02	+0.38
dd	dd	4	1	+0.12	-0.08
dd	dd	5	0	-3.86	-4.26
dd	ds	2	1	-0.84	-0.88
dd	ds	3	0	-1.69	-3.53
dd	ss	0	1	-1.20	-1.04
dd	ss	1	0	-0.93	-4.27
dd	pp	0	1	+3.37	+3.37
dd	pp	1	0	-1.50	-1.50
ds	ds	2	0	-0.80	-3.70
ds	ds	2	1	-1.15	-1.17
ds	ds	3	0	-3.90	-2.60
ds	ds	3	1	+0.24	+1.16
dp	dp	2	0	-4.65	-4.74
dp	dp	2	1	+0.67	+1.25
dp	dp	3	0	-2.71	-4.14
dp	dp	3	1	-0.95	+0.50
ss	ss	0	1	-2.28	-2.17
ss	ss	1	0	-4.04	-3.67
ss	pp	0	1	+0.73	+0.73
ss	pp	1	0	-0.50	-0.50
sp	sp	0	0	-3.17	-3.57
sp	sp	0	1	+0.35	+1.55
sp	sp	1	0	-3.01	-3.00
sp	sp	1	1	+0.47	+0.95
pp	pp	0	1	-2.37	-0.26
pp	pp	1	0	-4.55	-4.15
		ϵ_d		0.80	3.50
		ϵ_s		0.30	2.75
		ϵ_p		0.00	0.00

only "effective" values in the otherwise "realistic" set of matrix elements. It must be said, however, that they are quite close to the values obtained by Lee and Scott.[10] The above interaction was fairly successful, but we were disturbed by the necessity of using such a small separation between the p and sd levels. Some later work was devoted to resolving this problem and is discussed below.

Further exploration of the literature allowed us to put together a completely different interaction (case II) which is derived from Talmi-type fits in the p and sd shells. The new values for the matrix elements $\langle p^2|p^2{:}01\rangle$ and $\langle p^2|p^2{:}10\rangle$ were taken from the work of Cohen and Kurath.[12] One of them (01) differs markedly from results obtained in other fits,[11,13] but we selected the values of Ref. 12 largely because they require the level splittings shown in column II of Table I, which correspond closely to the mass-13 spectra. Furthermore, two realistic calculations[14,15] using different forces and different methods give numbers that are very much in line with the chosen values. However, this selection is not as vital as it seems since we discovered that a significant property of these matrix elements is that they can be approximately absorbed in the central field. This means that other choices lead to essentially the same wave functions and excitation spectra, provided that the single-particle energies are adequately shifted. For the interactions in the sd shell we used the results of Arima et al.,[16] which were derived for particles restricted to the $1d_{5/2}$ and $2s_{1/2}$ orbitals and which therefore seemed particularly suitable for our purposes since they include renormalization effects due to the omission of the $1d_{3/2}$ level. The eight matrix elements of the form $\langle sp|sp{:}JT\rangle$ and $\langle dp|dp{:}JT\rangle$ are those given by Talmi and Unna,[11] modified in a simple way. The prescription is to make the $T=0$ values more attractive by about 1 MeV and the $T=1$ values more repulsive by about 0.75 MeV. This is the only deviation from values quoted in the literature in the whole calculation. The effect of this change was to shift the quartet of $T=1$ states upward by approximately 1 MeV. The modifications of the wave functions and of the rest of the excitation spectrum were negligible. We could not find new information on the four remaining matrix elements $\langle s^2|p^2{:}JT\rangle$ and $\langle d^2|p^2{:}JT\rangle$, and they were taken to be the same as in case I. The complete interaction for case II is given in Table I and the resulting spectrum is shown in the figure. There is good agreement with experiment. In Table II we give the wave functions and energies produced by case II for a number of levels of O^{15}, O^{16}, and O^{17}.

Our model makes possible detailed predictions of many properties in this mass region, but we have room here only for a short summary of some important points directly related to O^{16}.

(i) In spite of the differences between cases I and II, the wave functions have very similar structure.

(ii) The ground state and indicated negative-parity states in O^{16} are quite closely described by the direct coupling of $p_{1/2}$, $d_{5/2}$, or $s_{1/2}$ particles to the ground state of mass 15. The first two levels of mass 17 are predicted in the right order and are again well described by direct coupling of $d_{5/2}$ and $s_{1/2}$ particles to the O^{16} ground state. Hence, in spite of the highly correlated nature of its ground state, O^{16} behaves in many respects as a good closed-shell nucleus.

(iii) The rotational band based on the first excited state of O^{16} is strongly dominated by four-particle, four-hole (4p-4h) components whose structure is strikingly similar to that of the ground-state band of Ne^{20} calculated with the model of Ref. 16.

(iv) The binding energies of O^{15}, O^{16}, and O^{17}, given in Table II, were calculated using a Coulomb energy of 6.5 MeV for the two protons and by assuming an absolute binding energy of 5.67 MeV for the $1p_{1/2}$ level in C^{12}. The agreement with experiment is excellent. This implies that the O^{16} single-particle energies usually assumed for 1p-1h calculations result from the mutual interaction of the four nucleons moving in the field of C^{12}.

(v) There are a number of experimental and theoretical levels above 9 MeV which are not shown; but it is significant that, up to 15-MeV excitation in O^{16}, the calculation gives the right level density for each spin and parity. In fact, the 0^-0 state included here shows one of the worst discrepancies. Most of the deviations could probably be accounted for by reasonable changes in the matrix elements. There are, however, some states that seem to require configurations involving the $1p_{3/2}$ or $1d_{3/2}$ orbitals, e.g., the $\frac{3}{2}^-\frac{1}{2}$ state at 6.2 MeV in mass 15 and the negative-parity rotational band beginning at 9.59 MeV in O^{16}. We believe that this band contains strong 5p-5h components which are likely to exhibit rotational characteristics, and that the final spectrum is distorted by coupling to 3p-3h configurations.

Table II. Wave functions and energies (MeV) for case II. Numbers in parentheses indicate partial JT coupling. A third number refers to the seniority of the state, and an asterisk indicates the necessity of further quantum numbers. Different shell states are coupled in the order shown. Amplitudes smaller than 0.28 have been omitted. For the ground states we give the binding energies with respect to C^{12}; the other values are excitation energies.

X^A	$J^\pi T$	Wave Function	E(th)	E(exp)
O^{15}	$1/2^-,1/2$	$+0.83p^3+0.51d^2(01)p$	20.14	19.84
O^{15}	$1/2^+,1/2$	$-0.54sp^2(01)+0.54s^3-0.48d^2(01)s+0.30d^2(10)s$	4.01	5.18
O^{15}	$5/2^+,1/2$	$+0.77dp^2(01)-0.48d^3(5/2,1/2,1)$	4.07	5.24
O^{15}	$7/2^+,1/2$	$+0.96dp^2(10)$	7.29	7.28
O^{15}	$1/2^+,1/2$	$-0.59sp^2(01)+0.41sp^2(10)-0.54s^3-0.31d^2(01)s-0.31d^2(10)s$	7.32	7.55
O^{15}	$3/2^+,1/2$	$-0.72dp^2(10)-0.54ds^2(10)+0.34d^3$	7.34	6.79
O^{16}	$0^+,0$	$+0.71p^4+0.58d^2(01)p^2(01)$	35.00	35.44
O^{16}	$0^+,0$	$+0.34p^4-0.62s^4+0.39d^2(01)s^2(01)-0.40d^2(10)s^2(10)$	5.83	6.05
O^{16}	$2^+,0$	$+0.39dsp^2(01)-0.43ds^3+0.37d^2(21)p^2(01)+0.47d^3(5/2,1/2,1)s+0.34d^4(202)$	6.93	6.92
O^{16}	$4^+,0$	$-0.69d^2(41)p^2(01)-0.33d^2(50)s^2(10)-0.28d^3(9/2,1/2)s-0.43d^4(402)$	10.33	10.36
O^{16}	$6^+,0$	$-0.79d^3(13/2,1/2)s-0.50d^4(604)*$	17.42	16.20
O^{16}	$0^-,0$	$+0.75sp^3-0.62d^2(01)sp$	9.39	10.95
O^{16}	$1^-,0$	$+0.69sp^3+0.36s^3p-0.54d^2(01)sp$	7.34	7.12
O^{16}	$2^-,0$	$+0.81dp^3+0.47d^3(5/2,1/2,1)p$	8.52	8.88
O^{16}	$3^-,0$	$+0.81dp^3+0.50d^3(5/2,1/2,1)p$	6.22	6.13
O^{16}	$0^-,1$	$+0.73sp^3-0.33s^3p+0.47[d^2(01)s](1/2,3/2)p$	12.10	12.79
O^{16}	$1^-,1$	$+0.79sp^3+0.56[d^2(01)s](1/2,3/2)p$	12.82	13.10
O^{16}	$2^-,1$	$+0.79dp^3+0.49d^3(5/2,3/2,1)p$	12.46	12.97
O^{16}	$3^-,1$	$+0.81dp^3+0.53d^3(5/2,3/2,1)p$	12.82	13.26
O^{17}	$5/2^+,1/2$	$+0.69dp^4-0.31d^3(5/2,1/2,1)p^2(01)+0.50d^3(5/2,3/2,1)p^2(01)$	38.86	38.72
O^{17}	$1/2^+,1/2$	$+0.65sp^4+0.37[d^2(01)s](1/2,1/2)p^2(01)+0.50[d^2(01)s](1/2,3/2)p^2(01)$	0.32	0.87

(vi) Some very pure 2p-2h states with $T=0$ and spins 4^+, 5^+, and 6^+ appear around 15 MeV. They have the structure $[d^2(50)p^2(10)](J0)$ (see caption to Table II). The 6^+ state in particular seemed rather puzzling at first since it comes well below the 6^+ rotational state. However, these levels should be very favorably populated in the reaction $N^{14}(\alpha,d)O^{16}$. This reaction has been studied by Harvey et al.,[17,18] and they observed three strong peaks in this energy region which seem to support our description.

(vii) The $E0$ transition rate between the ground and first excited 0^+ states has been calculated using our complete state vectors. We employed radial wave functions given by a single Woods-Saxon well with parameters adjusted to reproduce approximately the observed single-particle separation energies in this mass region. Specifically, the chosen well binds $1p_{1/2}$, $1d_{5/2}$, and $2s_{1/2}$ particles by 14.64, 3.75, and 3.50 MeV, respectively.[19] For the matrix element $\langle 0_2^+|\sum_p r_p^2 \times|0_1^+\rangle$, we obtain 3.21 fm^2 (case I) and 3.80 fm^2 (case II), to be compared with the experimental value of 3.8 fm^2. The matrix element is very sensitive to the values of the radial integrals. Use of harmonic-oscillator wave functions with $\hbar\omega=13$ MeV gives 1.51 fm^2 (case II), but such functions provide a poor description of the loosely bound $2s_{1/2}$ and $1d_{5/2}$ states. For the $E2$ transitions between members of the rotational band, preliminary estimates using effective charges of $0.5e$ indicate fairly good agreement with the observed rates ($4^+\rightarrow 2^+$ and $2^+\rightarrow 0_2^+$). However, these and other relevant transitions are being calculated more accurately and will be reported in a more complete communication.

(viii) As a further check of the model we have calculated the first ten levels in F^{18} (for which

the $J^{\pi}T$ values are known). Although some shifts occur, it is interesting that we obtain the negative-parity states and the $1^{+}0$ (1.7-MeV) "intruder" state (see Ref. 16) at approximately the right positions. Since the calculation also accounts fairly well for the levels fitted by Arima et al.,[16] we conclude that the core-polarization effects they implicitly take into account are not sufficiently strong to make our choice of interaction inconsistent.

We feel that the evidence presented shows that a shell-model calculation can go a long way towards providing a detailed explanation of the structure of O^{16} and neighboring nuclei. Several problems remain, such as the possible influence of spurious states and of missing configurations, improvements of the interaction, and detailed calculations of dynamic properties. They are being investigated.

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[19]Thanks are due to Dr. E. H. Auerbach for calculating the necessary single-particle wave functions and the radial integrals.