t-CHANNEL REGGE AMPLITUDE FROM s-CHANNEL RESONANCES

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A counter example is constructed to an alleged theorem that direct-channel resonances lying on rising Regge trajectories cannot asymptotically simulate a crossed-channel Regge amplitude in the direct-channel physical region. The crucial point is that the residue of a Regge pole falls strongly with energy.

In a recent Letter, Mandular and Slansky[1] have alleged a theorem that "finitely spaced Regge trajectories of resonances do not yield Regge asymptotic behavior." They thus conclude the impossibility of the suggestion of Dolen, Horn, and Schmid[2] and Schmid[3] that the sum of direct-channel resonances can yield an amplitude which resembles, after some energy averaging, a crossed-channel Regge exchange amplitude. But the attempted proof of the theorem contains a fallacy, namely, the statement that even when many s-channel trajectories of resonances are included, "the asymptotic s dependence of the right-hand side [i.e., s-channel resonances] is still dominated by the highest trajectory." This is not so if the residue of the trajectory falls sufficiently rapidly with s. In fact, it is well known that for trajectories rising faster than $\sqrt{s}$ such a fall (a) is necessary in order to avoid violation of polynomial boundedness, a property of Regge amplitudes,[4] and (b) is observed experimentally.

This behavior is explicitly exhibited by a counter example to the theorem of Ref. 1, which is simply constructed as follows: We take a Regge amplitude $A_s(s,t)=\beta(t)s^{\alpha(t)}$, project it into s-channel partial waves,[3] and observe what distribution of s-channel resonances is required to yield these partial-wave amplitudes.[6] We can do the projection analytically if we make the following approximations, which are reasonable when s is large: (i) $A_s(s,t)\sim e^{ct}s^{b+at}$ for small $|t|$, (ii) $P_l(\cos\theta)\approx J_0(l\theta)$, and (iii) the lower limit -1 on the $\cos\theta$ integration is ignored, the integration being carried to $-\infty$. Then

$$
\begin{aligned}
A_l(s) &\equiv \tfrac{1}{2}\int_{-1}^{1} dz\, e^{ct}s^{b+at}P_l(z), \quad t=\tfrac{1}{2}s(z-1)\\
&\approx \tfrac{1}{2}\int_0^\infty d\theta \exp(-\tfrac{1}{4}\tilde{a}s\theta^2)J_0(l\theta)\\
&=\exp[-l^2/\tilde{a}s]s^b/\tilde{a}s, \qquad (1)
\end{aligned}
$$

where $\tilde{a}=c+a\ln s$. We notice that A_l is large for $l^2<\tilde{a}s$, but (exponentially) small for $l^2\gg\tilde{a}s$. Obviously, $2(\tilde{a})^{1/2}$ plays the role of the size of the interaction region.

We now make the hypothesis that the partial-wave absorptive amplitudes $A_l(s)$ are given by the sum of s-channel resonances, energy averaged; that is, $A_l(s)$ is the density of (partial) widths of l-wave resonances. We further hypothesize that these resonances lie on rising trajectories of which the leading one is the same as the leading trajectory in the t channel which determined A_s originally. These conditions do not determine the s-channel trajectories; we arbitrarily assume them to be linear, parallel, and equispaced: $\alpha_N(s)=as+b-N$, $N\geq 0$. The trajectory residues $\beta_N(s)$ are then proportional to the (partial) widths of the resonances on the trajectories, and hence the hypothesis implies that $\beta_N(s)$ be proportional to $A_{\alpha_N}(s)$, i.e.,

$$\beta_N(s)\sim\exp[-(as+b-N)^2/\tilde{a}s]s^{b-1} \qquad (2)$$

for $as+b\geq N\geq 0$. This is a reasonable result, except for one apparent difficulty: $A_l(s)$ is non-vanishing for $l>as+b$, whereas $\beta_N(s)$, Eq. (2), must vanish for $\alpha_N>as+b$, because $N\geq 0$ [i.e., there is a highest trajectory]. But when s is large, $A_l(s)$ is exponentially small in this region, and setting it to zero there makes only an exponentially small error in the $A_s(s,t)$ which is reconstructed from $A_l(s)$.

Thus we have exhibited a particular set of s-channel Regge trajectories whose resonances constitute an amplitude which closely approximates a t-channel Regge form. In this model one has, for large s,

$$\beta_N(s)\sim\exp[-(a^2/\tilde{a})s].$$

This shows explicitly that the leading s-channel trajectory does not dominate at large s because its residue falls exponentially with s.

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[1]J. E. Mandula and R. C. Slansky, Phys. Rev. Letters 20, 1402 (1968).
[2]R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).
[3]Christoph Schmid, Phys. Rev. Letters 20, 689 (1968).
[4]N. N. Khuri, Phys. Rev. Letters 18, 1094 (1967).
[5]H. Goldberg, Phys. Rev. Letters 19, 1391 (1967).
[6]We simply equate the "energy-averaged resonance" and Regge amplitudes, rather than equating the derivatives of their integrals (i.e., the finite-energy sum rules as done in Ref. 1.

MEASUREMENTS OF π^0 AND η^0 PHOTOPRODUCTION AT INCIDENT GAMMA-RAY ENERGIES OF 6.0-17.8 GeV*

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We report on cross-sectional measurements on the photoproduction of π^0 and η^0 mesons on hydrogen for a range in t of -0.2 to -0.9 $(\mathrm{GeV}/c)^2$ and for a range of energies 6.0-16.0 GeV. The sharp dip observed at lower energies at $t=-0.5$ for π^0 production becomes less pronounced at higher energies. This implies in the Regge framework the B^0 meson trajectory dominates at higher energies. Our one angular distribution for η^0 production at 6.0 GeV shows no dip at $t=-0.5$ as do the π^0 data.

Measurements have been made on forward π^0 photoproduction $\gamma+p\rightarrow\pi^0+p$ in the range 2-5.8 GeV.[1] The results show a pronounced dip in the cross section at a value of the square of the four-momentum transfer, $t=-0.5$ $(\mathrm{GeV}/c)^2$. There has been considerable speculation on the energy dependence of this process at higher energies.[2] We have recently completed measurements at incident photon energies up to 17.8 GeV.

A collimated beam of bremsstrahlung photons from the Stanford Linear Accelerator Center linear accelerator irradiated the liquid-hydrogen target, a 12-in. long by 2-in. diam thin Mylar cylinder. The yield of protons recoiling from the target was measured as a function of angle for a variety of proton momenta and primary energies.

The measurements were made with a 100-in. radius, 90°-bend, second-order–corrected spectrometer[3] which focused momenta and production angles in a single focal plane normal to the impinging particles. The dispersion of this spectrometer was 1.66 in. per percent in momentum, and 0.32 in. per mrad in production angle. Protons were identified and separated from pions on the basis of ionization loss in three trigger counters and by vetoing pions with a Lucite Cherenkov counter. At all momenta, pion contamination was less than a few percent of the proton signal. The trigger counters were put in coincidence with eight hodoscope counters 10 in. by $\frac{3}{4}$ in. by $\frac{1}{4}$ in. thick located at the focal plane. The whole counter assembly was rotatable so that the axes of the hodoscope counters could be aligned with kinematic "missing-mass lines" in the focal plane. The resolution of the apparatus was limited by the multiple Coulomb scattering of particles in the liquid-hydrogen target.

The beam intensity was continuously monitored with both a Cherenkov monitor and secondary-emission quantameter. The secondary-emission