suggested by Leask et al.[6] might account for several of the anomalously large crystal-field parameters in CMN obtained from analysis of susceptibility data. Other effects, such as the possible admixture of $5d$ states into the ground state by large crystal fields,[15] could also influence the g factor and the saturation moment. They could also affect α, and if such an effect were field dependent, could influence our calculation of $M'(H, T)$. A theory which proposes to account for the observed discrepancy must explain why M', although perturbed from a Brillouin-function dependence, apparently remains a function of H/T.

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ELECTRIC DIPOLE TRANSITION FROM THE $2f_{7/2}$ ISOBARIC ANALOG RESONANCE TO THE $2d_{5/2}$ GROUND STATE IN ^{141}Pr †

H. Ejiri,* P. Richard, S. Ferguson, R. Heffner, and D. Perry
Department of Physics, University of Washington, Seattle, Washington

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Electric dipole γ rays from the $2f_{7/2}$ isobaric analog state $(2T_0)^{-1/2}T_-|i\rangle$ to the $2d_{5/2}$ ground state $|f\rangle$ in ^{141}Pr were measured with a Ge(Li) crystal. The matrix element of the $E1$ γ transition, $|\langle f|m_\gamma T_-(2T_0)^{-1/2}|i\rangle|$, and that of the analogous first forbidden β transition, $|\langle f|m_\beta|i\rangle|$, were obtained.

A measurement of electric dipole γ rays from isobaric analog states (IAS) in heavy nuclei is interesting since it provides information on the IAS and the low-lying states[1-4] as well as the matrix element $\langle\vec{r}\rangle$ for the $E1$ γ decay $\langle m_\gamma\rangle$, and for the analogous first forbidden β decay[1-3] $\langle m_\beta\rangle$ (Fig. 1). These matrix elements are related by

$$\langle f|m_\beta|i\rangle=\langle f|[m_\gamma, T_-]|i\rangle$$

$$\approx (2T_0)^{1/2}\langle f|m_\gamma|\mathrm{IAS}\rangle, \qquad (1)$$

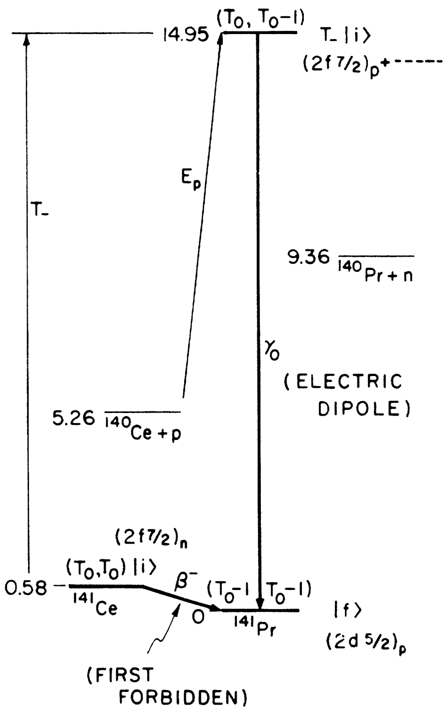

FIG. 1. Schematical diagram of isobaric analog states, and the electric dipole γ and the analogous first forbidden β transitions. The initial state $|i\rangle$, the analog state $T_-|i\rangle$, and the final state $|f\rangle$ have configurations $(2f\frac{7}{2})_n|0\rangle$, $(2f\frac{7}{2})_p|0\rangle + (2f\frac{7}{2})_n\sum_\delta(a_\delta{}^\dagger b_\delta)|0\rangle$, and $(2d\frac{5}{2})_p|0\rangle$, where $|0\rangle$ is the ^{140}Ce core.

where $|\mathrm{IAS}\rangle = (2T_0)^{-1/2}T_-|i\rangle$ and where $\langle f|T_-m_\gamma \times |i\rangle \approx 0$.[1] Since the IAS in heavy nuclei are located in the high excitation-energy region, they decay mainly by particle emission, so that the electromagnetic radiation branches are very small. $E1$ γ transitions from IAS in medium nuclei with $N = 50$ have been measured with a large NaI crystal.[5] However, for heavy nuclei with closely spaced low-lying levels, well-isolated high-energy γ rays from the IAS to such low-lying states may be observed by use of good resolution Ge(Li) crystals despite an extremely small detection efficiency.

We measured the $E1$ γ rays from the $2f_{7/2}$ IAS to the $2d_{5/2}$ ground state in ^{141}Pr ($N = 82$). This γ transition corresponds to the first forbidden β decay $^{141}\mathrm{Ce} \to {}^{141}\mathrm{Pr}$ (see Fig. 1). The $2f_{7/2}$ resonance analog to the ground state of ^{141}Ce was excited by the proton-capture reaction on ^{140}Ce at $E_p = 9.75$ MeV.[6] Proton beams of 0.6-1.0 μA were provided by the University of Washington High Voltage Engineering Corporation Model FN tandem accelarator. The target used was self-supporting natural Ce (88.48% of ^{140}Ce) with a thickness of 0.91 mg/cm^2. This thickness was obtained from the Rutherford scattering yield of 5-MeV protons at 35°. The γ-ray detector was a 20.7-cm^3 Ge(Li) crystal with energy resolution $\approx$30 keV for 15-MeV γ rays. In order to attenuate low-energy γ rays and neutrons, an absorber of 103-mm-thick paraffin containing 15% Li$_2$CO$_3$ followed by a 9.6-mm-thick PbSn alloy was inserted between the target and the detector. An absolute detector efficiency was obtained by observing the 15.106-MeV γ rays from the reaction ^{12}C$(p,p'\gamma)$,[7] which is very close to the γ-ray energy of present interest.

The $2f_{7/2}$ isobaric analog resonance was measured in an excitation function of the reaction ^{140}Ce$(p,p')^{140}$Ce*(4 MeV) by observing the inelastic-scattering protons with a Si detector. γ-ray spectra were subsequently observed at several proton energies on and off resonance at 90° and 125° to the beam. Apart from γ rays due to $(p, p'\gamma)$ reactions on oxygen contaminants, we found clearly a single isolated resonant line of 14.95 ± 0.05 MeV at $E_p = 9.768$ MeV (Fig. 2). This line is identified as the $E1$ γ-ray transition from the proton capture state ($2f_{7/2}$ IAS) to the ground state ($2d_{5/2}$) in ^{141}Pr. The anisotropy of this line, after subtraction of the off-resonance contributions, was found to be $Y(90°)/Y(125°) = 1.29 \pm 20\,\%$. This value also supports the assignment of the γ rays to the transition $f_{7/2}$-$d_{5/2}$ in view of the calculated anisotropies $Y(90°)/Y(125°) = 1.18$ and 0.763 for transitions from IAS $\frac{7}{2}^-$ $(2f_{7/2})$ to the ground $\frac{5}{2}^+$ $(2d_{5/2})$ state and to the 145-keV first excited $\frac{7}{2}^+$ $(1g_{7/2})^{-1}$ $(2d_{5/2})^2$ state, respectively.

The γ transition width Γ_γ was obtained from the on-resonance γ-ray yield corrected for the off-resonance contribution. Assuming little effects of interference with nonresonant contributions to the IAS resonance cross section,[8] we used the single-resonance formula for calculating the preliminary value Γ_{γ_0}:

$$\frac{d\sigma(p,\gamma_0)}{d\Omega} = \frac{\lambda^2}{4}\,\frac{2J+1}{(2s+1)(2I+1)} \times \frac{\Gamma_p\Gamma_{\gamma_0}[1+A_2P_2(\cos\theta)]}{(E_p-E_0)^2+(\Gamma/2)^2}, \qquad (2)$$

where the resonance parameters are $\Gamma_p = 12$ keV, $\Gamma = 61$ keV,[6] $J = \frac{7}{2}$, $s = \frac{1}{2}$, and $I = 0$. The transition probability obtained is

$$\Gamma_{\gamma_0}(\mathrm{exptl}) = 24 \pm 10\ \mathrm{eV} \quad [\tau_m = (2.7 \pm 1.0)\times 10^{-17}\ \mathrm{sec}]. \qquad (3)$$

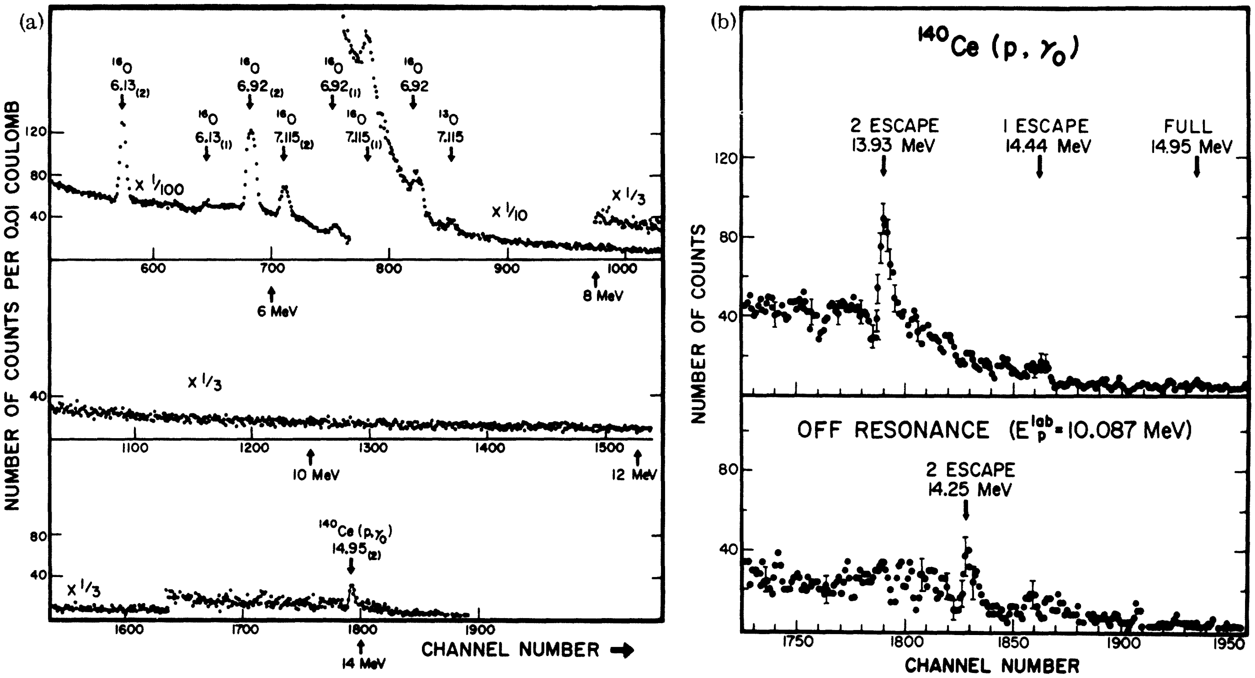

FIG. 2. (a) Energy spectrum of γ rays at $E_p = 9.768$ MeV ($2f_{7/2}$ resonance) in ^{141}Pr at $\theta = 90°$. The 2-escape and 1-escape peaks are labeled $E_{(2)}$ and $E_{(1)}$, respectively. (b) Expanded energy spectra of the ground-state transition (γ_0) at $E_p^{\text{lab}} = 9.768$ MeV ($2f_{7/2}$ resonance) and at $E_p = 10.087$ MeV (off resonance). This plot was obtained by summing up counts of three nearest channels. The overall energy resolution is about 40 keV, which is due to the Ge detector and the target thickness.

The single-particle matrix element $M_{\text{sp}} = {}_0\langle f| \times m_\beta |i\rangle_0$ for single-particle wave functions $|i\rangle_0$ and $|f\rangle_0$ with pure T spins is related to the γ matrix element between the IAS $(2T_0)^{-1/2}T_-|i\rangle_0$ and the final state $|f\rangle_0$ by

$$
\begin{aligned}
{}_0\langle f|m_\gamma|\text{IAS}\rangle_0 &= (2T_0)^{-1/2}\,{}_0\langle f|[m_\gamma, T_-]|i\rangle_0 \\
&= (2T_0)^{-1/2}M_{\text{sp}}, \qquad (4)
\end{aligned}
$$

where $T_+|f\rangle_0 = 0$,

$$
em_\gamma = e\sum_{\lambda\nu}\Big\{\Big(1-\frac{Z}{A}\Big)a_\nu^\dagger\langle\nu|\vec{\mathrm{r}}|\lambda\rangle a_\lambda - \Big(\frac{Z}{A}\Big)b_\nu^\dagger\langle\nu|\vec{\mathrm{r}}|\lambda\rangle b_\lambda\Big\},
$$

$$
T_- = \sum{}_\delta a_\delta^\dagger b_\delta,
$$

and

$$
[m_\gamma, T_-] = m_\beta = \sum_{\lambda\nu} a_\nu^\dagger\langle\nu|\vec{\mathrm{r}}|\lambda\rangle b_\lambda .
$$

The $a_\nu^\dagger$ ($b_\nu^\dagger$) and a_λ (b_λ) are creation and annihilation operators for proton (neutron). It is interesting to note that the main $E1$ transition from $T_-|i\rangle$ is due to a coherent sum of a proton-transition amplitude $(2f_{\frac{7}{2}})_p \to (2d_{\frac{5}{2}})_p$ with effective charge $e_{ff} = (1-Z/A)e$[9] and a neutron one $(2f_{\frac{7}{2}})_n \to (2d_{\frac{5}{2}})_n$ with $e_{ff} = -(Z/A)e$.[9] They sum up with opposite signs, resulting in a single-particle transition matrix M_{sp} with $e_{ff} = e$, as shown in Eq. (4).

The ratios of the observed $E1$ matrix element $|M|_{\text{exptl}} = |\langle f|m_\gamma|\text{IAS}\rangle|_{\text{exptl}}$ to the matrix element $|{}_0\langle f|m_\gamma|\text{IAS}\rangle_0| = |M|_{\text{sp}}(2T_0)^{-1/2}$ estimated using j-j coupling single-particle wave functions[10] with pure T spin is obtained as

$$
\frac{2T_0|M|_{\text{exptl}}^2}{|M|_{\text{sp}}^2} = \frac{2T_0\Gamma_\gamma(\text{exp})}{\Gamma_\gamma(\text{sp})} = 0.075 \pm 0.03. \qquad (5)
$$

The experimental $|M|_{\text{exptl}}^2$ is hindered by a factor ≈13 with respect to the single-particle estimate $|M|_{\text{sp}}^2/2T_0$. A part of the hindrance factor, $F_c \approx 4$, may be attributed to the effect of collective states[3] and a factor $F_p = [U_f^2(2f_{\frac{7}{2}})U_i^2(2d_{\frac{5}{2}})]^{-1} \approx 1.3$, to the effect of pairing correlations.[11] Quite recently a similar order of hindrance has been found in $E1$ transitions from IAS in the medium-weight nucleus ^{89}Y.[5]

The β-decay matrix $\langle f|m_\beta|i\rangle$ is simply obtained from the γ matrix $\langle f|m_\gamma|\text{IAS}\rangle$ by using Eq. (1). The ratio $|m_\beta|^2/|m_\beta|_{\text{sp}}^2$ is given by the same expression, Eq. (5), since $|m_\beta|_{\text{sp}}$

$= |M|_{\mathrm{sp}}$ and $|m_\beta| \approx (2T_0)^{1/2} |m_\gamma|_{\mathrm{exptl}}$ for the IAS. Furthermore, the first-forbidden-transition operator for the transition $^{141}\mathrm{Ce} \rightarrow {}^{141}\mathrm{Pr}$ can be expressed on the basis of the ξ approximation[12] ($\xi = 12.5 > W_0 = 2.1$) as

$$-C_v i\xi M_\beta = -C_v i\xi m_\beta(\Lambda - 1.2\Lambda_1 - 1), \tag{6}$$

where $m_\beta = \langle \vec{r} \rangle$, $\Lambda = -i\langle \alpha \rangle / \xi \langle \vec{r} \rangle$, $\Lambda_1 = i\langle \vec{\sigma} \times \vec{r} \rangle / \langle \vec{r} \rangle$, and $\xi = \alpha Z/2R$. By using $(2T_0)^{1/2} |\langle f | m_\gamma | \mathrm{IAS} \rangle|_{\mathrm{exptl}}$ for the $|m_\beta|$ matrix element and the experimental $|M_\beta|$ obtained from the β-decay probability,[13] we get from Eq. (6)

$$(\Lambda - 1.2\Lambda_1 - 1)^2 = 0.13 \pm 0.05. \tag{7}$$

The experimental β-decay probability $|\langle f | M_\beta | i \rangle|^2$ is hindered with respect to the single-particle estimate $|m_\beta|_{\mathrm{sp}}^2 = |M|_{\mathrm{sp}}^2$ by a factor of 100, which we see is due to the hindrance factors 13 for $|m_\beta|^2 = |\langle \vec{r} \rangle|^2$ (the same as for the $E1$ γ transition) and 8 due to cancellation as given in Eq. (7).

Furthermore, if we can use Fujita's value $\Lambda = 2.4$ obtained on the basis of the theory of conservation of vector currents (CVC)[14] and Ahrens-Feenberg approximation,[1,15] we get from Eq. (7),[16] $\Lambda_1 = i\langle \vec{\sigma} \times \vec{r} \rangle / \langle \vec{r} \rangle = 0.9 \pm 0.2$, in accord with the value $\Lambda_1 = 1$ estimated by using a j-j coupling shell-model relation $\langle i(\vec{\sigma} \times \vec{r}) \rangle / \langle \vec{r} \rangle = \langle [\vec{r}, (\vec{\sigma} \cdot \vec{L})] \rangle / \langle \vec{r} \rangle = j_i(j_i + 1) - l_i(l_i + 1) - j_f(j_f + 1) + l_f(l_f + 1)$. This indicates nearly the same hindrance factor[3] for both of the $\langle \vec{r} \rangle$ and the $\langle \vec{\sigma} \times \vec{r} \rangle$. In other words, the CVC and Ahrens-Feenberg theories[1] are consistent with the present experiment as long as we use the shell-model value for Λ_1,[13,16] although some arguments on the utility of the CVC theory have been made.[1,17,18]

The present work gives a relation between γ and β transitions in heavy nuclei and an experimental method of obtaining the matrix elements m_β of first forbidden β decay from the corresponding $E1$ γ matrix element.

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