Table I. Comparison of experimental and theoretical values for the L bands. All values are in units of eV.

	Experiment[a,b]	Theory
L_1	3.6	3.63
L_2	4.3	4.26
L_3	4.95	4.98
L_4	6.6	6.60
L_5	...	7.40

[a]Ref. 1. [b]Ref. 10.

hard to identify experimentally because it lies in the region of the absorption spectrum which is dominated by exciton bands and band-to-band transitions.

Work is proceeding to justify the potential used and to calculate other features of the absorption curve of KCl.

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[1]J. H. Schulman and W. D. Compton, Color Centers in Solids, International Series of Monographs on Solid State Physics (Pergamon Press, Inc., New York, 1962), Vol. II.

[2]J. C. Slater and G. F. Koster, Phys. Rev. 94, 1498 (1954).

[3]Seiichi Oyama and Toru Miyakawa, J. Phys. Soc. Japan 21, 868 (1966).

[4]We are indebted to Mr. B. H. Duane, Battelle-Northwest Laboratory, Richland, Washington, for sending us his least-squares fitting program LIKELY: Bernard H. Duane, U. S. Atomic Energy Commission, Battelle-Northwest, Report No. BNWL-390, September, 1967 (unpublished).

[5]P. D. DeCicco, Phys. Rev. 153, 931 (1967).

[6]W. Beall Fowler, Phys. Rev. 135, A1725 (1964).

[7]N. F. Mott and M. J. Littleton, Trans. Faraday Soc. 34, 485 (1938).

[8]B. S. Gourary and F. J. Adrian, in Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1960), Vol. 10.

[9]W. Beall Fowler, Phys. Rev. 151, 657 (1966).

[10]M. Hirai and M. Ueta, J. Phys. Soc. Japan 17, 566 (1962).

QUANTUM EFFECTS IN CYCLOTRON RESONANCE IN p-TYPE InSb

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Seven cyclotron resonance lines have been resolved providing quantitative data suitable for theoretical analysis of quantum effects of intraband transitions in the degenerate valence bands of indium antimonide. The heavy hole has been resolved for the first time and its mass shows anisotropy. Identification of several transitions with low quantum numbers have been made from observations of their intensities as a function of temperature.

We report the first cyclotron resonance measurements of quantum effects in the degenerate valence bands of indium antimonide. Seven transitions have been recorded by observing the absorption spectra of 0.337-mm monochromatic radiation transmitted through oriented single-crystal specimens of p-type InSb as a function of magnetic field intensities up to 175 000 G. The magnetic field was applied parallel to the direction of propagation. The lower quantum states of the valence bands were populated by holes thermally excited from an acceptor level. Absorption lines were observed for temperatures between about 20 and 80°K.

The submillimeter spectrometer using an HCN laser is the same as that described[1,2] for the observation of the quantum effects in germanium except that the cw laser now employs natural gas (methane) and nitrogen as a fuel. A brass light pipe guides the unpolarized laser radiation through an InSb disk which is 2 mm thick. The chopped radiation is detected by a Golay cell and amplified by a Princeton Applied Research Corporation Model-110 tuned amplifier. The doped InSb has a hole mobility of 7×10^3 cm^3 V^{-1} sec^{-1} and impurity concentration of 6×10^{14} cm^{-3} measured at 78°K. Such material is available from several commercial sources.[3]

Figure 1 shows the seven resolved transitions observable at this submillimeter wavelength.

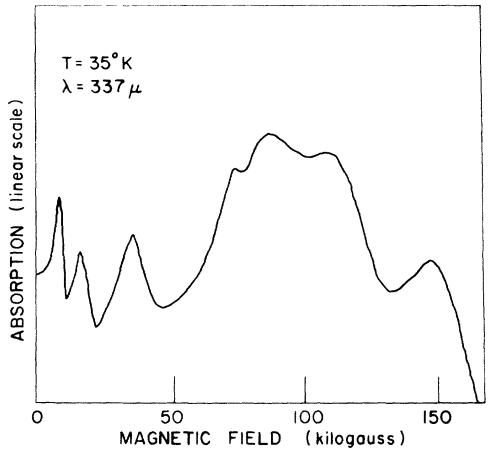

FIG. 1. Pen recording of seven cyclotron resonance transitions in the valence bands of p-InSb with the applied magnetic field parallel to the ⟨100⟩ axis.

Previous valence-band cyclotron resonance measurements[4-6] at microwave and millimeter wavelengths have shown only the envelope of this spectrum consisting of two absorption bands of unresolved lines which have been called the light hole and the heavy hole. This ambiguous designation of these absorption bands as individual lines has led to a controversy[7] concerning the anisotropy of the heavy hole. At the present time, however, the anisotropy can only be observed directly by magneto-optical interband measurements[8] or by far-infrared measurement of intraband quantum effects. The effective mass of the highest field transition (line No. 7) is shown in Table I for the magnetic field along the ⟨100⟩ and the ⟨111⟩ directions. The anisotropy of this transition is definitive but small. The other transitions appear isotropic within the uncertainty of this preliminary experiment. Since these latter transitions have been the largest contributors to the unresolved envelopes observed in the previous microwave measurements, we have looked for possible causes for the shift in the envelopes

Table I. Effective masses at 35°K.

Line number	m^* ⟨100⟩	m^*⟨111⟩
1	0.018±0.002	0.016±0.002
2	0.046±0.003	0.049±0.003
3	0.112±0.004	0.106±0.006
4	0.238±0.004	0.221±0.012
5	0.281±0.006	0.290±0.003
6	0.351±0.006	0.303±0.016
7	0.467±0.005	0.431±0.003

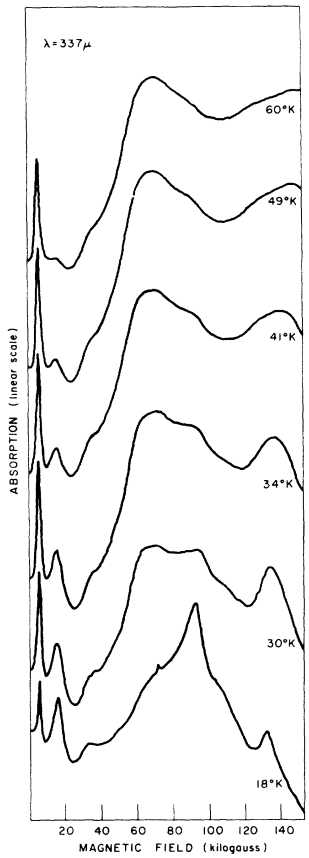

FIG. 2. Cyclotron resonance spectra showing the variation of the intensities of the seven absorption lines with temperature. The magnetic field was applied parallel to the ⟨111⟩ axis.

that have been perpetuating the controversy. Comparison of Fig. 1 with Fig. 2 shows one possible cause. Changes in selection rules and intensity of absorption with field direction produces an observable anisotropy of the envelope.

We follow the convention that the initial state of a hole transition is nearer to the band edge than the final state; then the changes in the spectra of Fig. 2 as the temperature is increased helps to identify the individual transitions. We assume that the states near the band gap become thermally populated as the temperature is increased. Those absorption lines that grow weak and disappear at higher temperatures have suffered from thermal population of their final states indicating that both initial and final states lie near the degenerate valence band edge. Moreover, only three absorption lines have survived at the highest temperature. They are broadened by thermal scattering, but their undiminished intensity implies that their initial and

final states both lie far from the band edge giving us some reason to call them light and heavy holes.

The identification of the dominant transitions shown in Fig. 2 where the magnetic field was applied along a $\langle 111 \rangle$ direction can be made in terms of the quantum magnetic ladder as calculated from the expressions of Luttinger and Kohn.[9] The first line represents two light-hole transitions of quantum numbers $0 \rightarrow 1, 1 \rightarrow 2$ for the two b^+ and a^+ series,[10] respectively, which cannot be resolved except at very low temperature. At high temperature in Fig. 2 this corresponds primarily to the 1 to 2 transition. Line No. 2 is a $0 \rightarrow 1$ transition of the a^+ series. If we take the anisotropy to be small, $\gamma_2 \approx \gamma_3 = \bar{\gamma}$ in terms of Luttinger's notation,[11] we obtain $\Delta\mathcal{E}_l = s(\gamma_1 + 2\bar{\gamma})$ and $\Delta\mathcal{E}_l = s(\gamma_1 - \bar{\gamma})$ for the energy spacings of light-hole lines Nos. 1 and 2 of Table I, where $s = eH/m_0c$ and m_0 is the free electron mass. For the highest field transition (line No. 7) which we identify with the heavy-hole transition, the energy spacing is given by $\Delta\mathcal{E}_h = s(\gamma_1 - 2\bar{\gamma})$. Our experimental data provide the values $\gamma_1 + 2\bar{\gamma} = 63 \pm 8$ and $\gamma_1 - 2\bar{\gamma} = 2.2 \pm 0.2$ from which we obtain $\gamma_1 = 32.6 \pm 4.1$ and $\bar{\gamma} = 15.1 \pm 2.0$. These parameters are in good agreement within experimental error with those of Pidgeon and Brown,[8] who determined their values from magnetoabsorption. Using these numbers, we can identify the large peak (line No. 5) at 18°K, $H \parallel \langle 111 \rangle$ as the transition $2 \rightarrow 3$ for a^- or b^- heavy-hole series which can be approximated by

$$\Delta\mathcal{E}_h \approx \gamma_1 - 2\bar{\gamma} + \frac{(\gamma_1 - \kappa)^2 - \frac{3}{4}\gamma^2}{\bar{\gamma}(4n^2 - 1)}.$$

When the appropriate numbers are inserted for $n = 2$, this gives $\Delta\mathcal{E} \approx 3.1$ within experimental error in agreement with the data of Table I.

A more detailed quantitative analysis of the seven transitions is now in progress. More accurate determinations of the effective masses will be used in the comprehensive theoretical treatment. The accuracy can be increased, particularly for the low-field lines, by repeating the experiments at higher frequencies using a water-vapor laser. The approximate analysis used above which gave only semiquantitative agreement with experiment is being refined to make use of the technique of Pidgeon and Brown, that is, an 8×8 matrix involving the influence of the conduction band and, to a lesser extent, the split-off valence band. Additional correction for $k_H \neq 0$ will be taken into account in the forthcoming analysis.

The use of the 4×4 matrix of Luttinger and Kohn which was appropriate for the valence bands of germanium is not as satisfactory for indium antimonide. Nevertheless, the crude analysis given here definitely establishes the existence of the quantum effects in this material and has provided a correlation between the quantum treatment of Luttinger and Kohn and these experimental observations. At this time, however, we shall not try to identify the remaining weaker transitions until the more exact theory is employed.

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[1]K. J. Button, H. A. Gebbie, and B. Lax, IEEE J. Quantum Electron. QE-2, 202 (1966).

[2]C. C. Bradley, K. J. Button, B. Lax, and L. G. Rubin, in the International Quantum Electronics Conference, 1968 (unpublished); and to be published.

[3]The particular specimens used here were purchased from Cominco American Products, Inc., Spokane, Wash.

[4]D. M. S. Bagguley, M. L. A. Robinson, and R. A. Stradling, Phys. Letters 6, 143 (1963).

[5]M. L. A. Robinson, Phys. Rev. Letters 17, 963 (1966).

[6]H. T. Tohver and G. Ascarelli, Bull. Am. Phys. Soc. 13, 94 (1968).

[7]H. T. Tohver and G. Ascarelli, in Proceedings of the Ninth International Conference on Physics of Semiconductors, Moscow, 1968 (to be published).

[8]C. R. Pidgeon and R. N. Brown, Phys. Rev. 146, 575 (1966).

[9]J. M. Luttinger and W. Kohn, Phys. Rev. 97, 869 (1955).

[10]L. M. Roth, B. Lax, and S. Zwerdling, Phys. Rev. 114, 90 (1959).

[11]J. M. Luttinger, Phys. Rev. 102, 1030 (1956).