ments and the implications for the structure of the nematic phase will be presented in a later article.

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OSCILLATIONS PRESENT IN PLASMA-ELECTRON HEATING BY AN ELECTRON BEAM*

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Oscillations of electrostatic fields present when electrons are strongly heated by electron-beam plasma interaction reveal large-amplitude oscillations with $\omega/k_{\parallel} \lesssim v_b$, the electron-beam speed; $\omega \approx \omega_{ce}/10$, where ω_{ce} is the electron gyrofrequency; coherence lengths of roughly a few wavelengths ($\ll$ plasma length); and maximum electrostatic potential in the wave approaching the applied beam voltage. A gross compatibility is found between the observed frequencies and wavelengths and the predictions of a linear theory.

In previous electron beam-plasma interaction experiments,[1] we have observed intense heating of plasma electrons (to a temperature of 100 keV) when confined in a simple magnetic mirror. Similar observations have been made elsewhere.[2] In this Letter, we report on preliminary measurements concerning the frequency, wavelength, correlation time, and correlation length of the plasma-electron oscillations. These measurements support those made elsewhere.[3] In addition, we present direct time-resolved measurements of the electric field, as well as the absolute amplitude of the electric field. These measurements suggest the mechanism by which the heating process occurs.

Two sets of experiments have been performed on the oscillating plasma. In the first set, an impedance-matched probe was placed inside the plasma, and the resulting oscillations were displayed on a traveling-wave–type oscilloscope.[4] The resulting oscillogram, displayed in Fig. 1(a), top, reveals the following information: First, the plasma is not "turbulent," but exhibits coherent oscillations at one frequency that persist for several cycles, then suddenly change phase, frequency, and amplitude in a random fashion. Second, the oscillations generally occur at frequencies near the electron plasma frequency, which in this apparatus is typically somewhat less than the electron cyclotron frequency. Third, the amplitude of the potential oscillations is of the same order as the beam voltage. The observed potentials and correlation lengths are in accord with expectations based on particle-trapping arguments[5]: for $eE_1L \sim eV_b$, background electrons can be trapped in the fluctuating wave leading to a rapid damping of the instability. Here E_1 is the fluctuating electric field strength, L is the correlation length, and V_b is the accelerating potential applied to the electron beam.

A second set of data, obtained with a sampling oscilloscope[6] and displayed in Fig. 1(b), yields information on the auto- and cross-correlation functions for oscillations studied by one and by

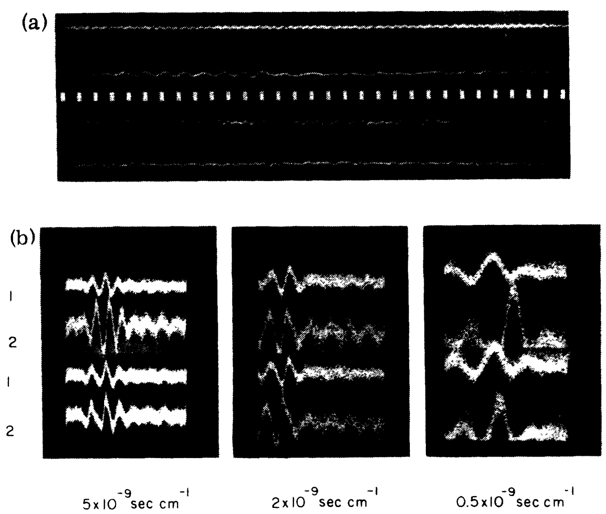

FIG. 1. (a) Fluctuating wave forms. Signals recorded at random from the steady-state plasmas. Vertical time markers are at nanosecond intervals. (b) "Correlated" plasma-probe signals. These signals were obtained by a sampling technique. Samples of V at t and $(t+\Delta t)$ over many thousands of cycles are stored. The result is a kind of correlation; i.e.,

$$\frac{1}{n}V_0\sum_{i=1}^{n}V(t_i+\Delta t),$$

where V_0 is the chosen peak voltage of an oscillating signal at which a trigger circuit was activated to record the data. Traces 1 are analogous to autocorrelation functions. Traces 2 are analogous to cross-correlation functions for two probes aligned along the electron beam. Note how the second set of traces shift relative to the first set of traces, corresponding to a shift in the spacing between the probes. The three sets of photos were obtained for three different plasma conditions. The number under each set gives the oscilloscope sweep speed used.

two probes, respectively. The data yield the following information: First, on the average, the oscillations lie below the plasma-electron and the plasma-cyclotron frequencies by a factor of between 2 and 10. Second, the oscillations are correlated for a time corresponding to one to two cycles. Third, the average wavelength corresponds to a few centimeters and does not appear to correspond to a standing wave between parts of the apparatus. Fourth, the correlation length in the plasma corresponds to a few wavelengths. Fifth, the average phase velocity of the wave along the magnetic axis, as found from the average frequency and average wavelength, is slightly less that the velocity of the incident electron beam that produces the heating.

At first, it seems that a sampling oscilloscope cannot be used to study plasma oscillations which are random functions of time and amplitude. However, our experiments discussed above demonstrate that the electric field oscillations do not involve random functions. First, we have observed that the oscillations generally appear as coherent wave forms, lasting for several cycles. Also, we find experimentally that the probability of having high-amplitude noise bursts is a rapidly decreasing function of amplitude. Thus, if we set the trigger of the sampling oscilloscope to a given V, on the average the oscilloscope will be triggered by signals lying between V and $V+\Delta V$, where $\Delta V \ll V$. Thus, on the average, we are observing signals that have a peak value chosen at the value V. Let us now compute how this technique allows us to obtain a correlation function.

The definition of a correlation function $F(\Delta t)$ is

$$F(\Delta t)=\frac{1}{n}\sum_{i=1}^{n}V(t_i)V(t_i+\Delta t).$$

Here $V(t)$ is the voltage occurring at time t, and $V(t+\Delta t)$ is the voltage occurring at time $t+\Delta t$.[7] Assume that we observe the correlation function at only one chosen value of $V(t_i)=V_0$. Then V_0 is a constant and may be removed from the summation. Our somewhat more restricted "correlation function" $F^1(\Delta t)$ is now given by the more simple equation

$$F^1(\Delta t)=\frac{1}{n}V_0\sum_{i=1}^{n}V(t_i+\Delta t).$$

This is the function displayed in Fig. 1(b). From the number of cycles observed before the trace "smears" we can compute the coherence time for the oscillations. By using one probe to provide a trigger signal and a second probe to provide a signal for a second trace on the oscilloscope (marked 2), we obtain an approximation to a cross-correlation function between the signals on the two probes. The second probe was moved between the observations of the second and fourth trace. Note how the cross-correlation pattern shifted relative to the autocorrelation pattern! From the time shift between the two cross-correlation patterns Δx we compute the average velocity of the waves, $v=\Delta x/\Delta t$. This distance required to move the cross-correlation probe away from the autocorrelation probe to cause the cross-correlation signal to "smear out" yields the correlation length for the plasma oscillations. Two advantages arise from using the sampling oscilloscope as a correlation device. By changing the sensitivity of the trigger setting one can obtain the auto- and cross-correlation functions as a function of V_0. In general, we find that these values do not depend on V_0. A second advantage is that one can distinguish between loss of correlation and amplitude decay. In the first case, the amplitude distribution of dots remains constant with Δt and Δx, but the pattern "smears out." In the second case, the pattern persists, but the amplitude distribution of the dots shrinks toward the x axis. Of course, our simple oscilloscope, strictly speaking, is not a complete correlator. However, as Fig. 1 shows, much information about the characteristics of the electron oscillations may still be obtained.

From the data presented above we infer that the oscillating electric fields doing the electron heating are intense and propagate along the magnetic field with a velocity slightly less than that of the incident electron beam. Theory suggests that for the experimental parameters a very strong instability is present between the cold background plasma and the two counter-streaming electron beams. Since the oscillations can be observed when the intense heating is not present, we conjecture that the very energetic (100-keV) electrons result from multiple encounters, perhaps stochastic, with the fluctuating electric fields. Because of the short transit time of primary beam electrons, this can occur only if the electrons are trapped and confined in a strong magnetic mirror as experimentally observed.[1] Heating processes of this type were conjectured early in the experimental history[5] and have been observed recently in computer simulations of beam-plasma interations.[8]

In the absence of a theory for the spectrum of fluctuations to be expected in a turbulent plasma, we consider the scaling properties of the instabilities predicted in a linear theory using a plasma system made up of two oppositely directed monoenergetic electron beams passing through a cold background plasma in a uniform magnetic field. If the electron beams flow along the magnetic field, the appropriate dispersion relation for electrostatic waves is

$$\frac{\omega_{ce}^2}{\omega_{be}^2}\frac{k^2}{k_\perp^2}=\frac{1}{(W-H)^2-1}+\frac{1}{(W+H)^2-1}+\frac{k_\parallel^2}{k_\perp^2}\left[\frac{1}{(W-H)^2}+\frac{1}{(W+H)^2}\right]+\frac{\omega_{pe}^2}{\omega_{be}^2}\left[\frac{1}{W^2-1}+\frac{k_\parallel^2}{k_\perp^2}\frac{1}{W^2}\right].$$

Here $W\equiv\omega/\omega_{ce}$ and $H\equiv k_\parallel v_b/\omega_{ce}$, where ω is the wave frequency, ω_{ce} is the electron gyrofrequency, $k_\parallel$ is the component of the wave vector parallel to the magnetic field, and v_b is the speed of the electron beams; ω_{be} and ω_{pe} are the plasma frequencies corresponding to the electron-beam and background-electron densities. The ions have been assumed infinitely massive.

We solve the dispersion relation for (complex) ω as a function of (real) k and display in Fig. 2 the experimental dependence of ω and k for maximum growth rate on the plasma parameters, $\omega_{be}^2/\omega_{ce}^2$ and n_{pe}/n_{be}, the relative densities of background to beam electrons. The outstanding features of the expected growing waves are (i) phase speeds ~$0.7v_b$, and (ii) frequencies less than but comparable with electron gyrofrequency; hence, roughly equal to background electron-plasma frequencies.

The significance of results from linear theory or weak turbulence theory will have to be assessed by comparison with more detailed experimental observations. The present work represents a first step in such a comparison: We

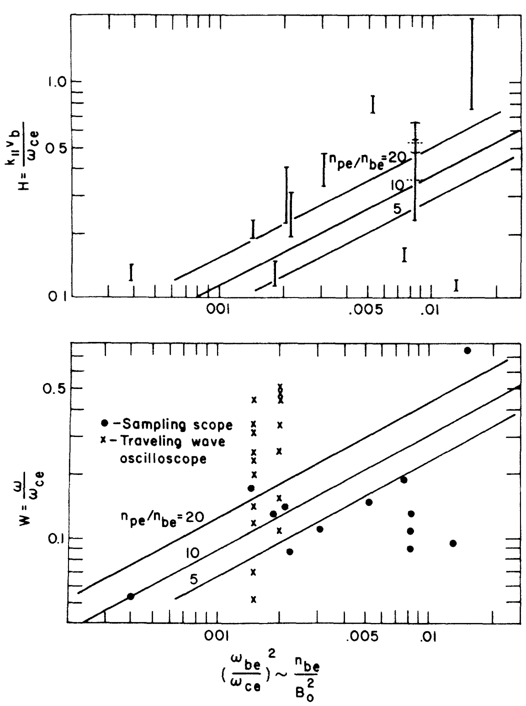

FIG. 2. The dependence of ω and k for maximum growth rate on the plasma parameters $[\omega_{pe}/\omega_{ce}]^2$ and $[n_{pe}/n_{be}]$.

have exhibited a crude technique for measuring properties of the fluctuations in a turbulently heated plasma and found a gross compatibility of observation with linear theory. As these techniques are refined, the resulting data may permit a more detailed understanding of the state of the plasma, leading hopefully to reliable scaling relations and optimization procedures.

We appreciate the help of W. D. Jones and W. Halchin in carrying out the experiments and writing the paper, and J. G. Harris in constructing the equipment in carrying out this experiment.

Note added in proof. —We have constructed a true correlator, using two sampling oscilloscopes, and have verified that the autocorrelation functions are well represented by the sampling-oscilloscope traces shown in Fig. 1(b). We wish to acknowledge the valuable contribution of William R. Wing, who did much of this work.

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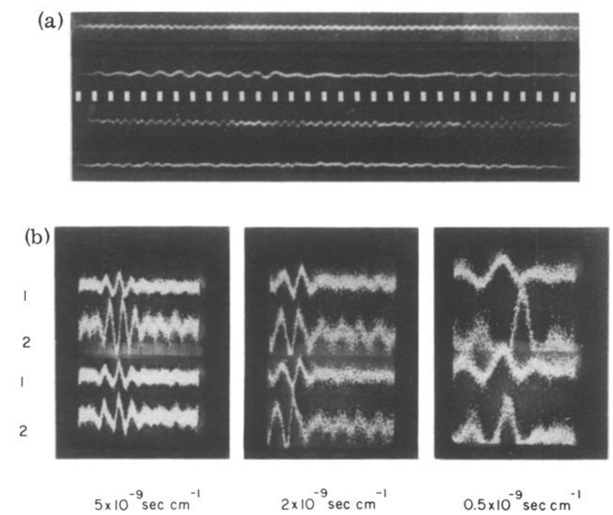

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