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s-WAVE $K\pi$ INTERACTION IN THE K_{l3} DECAY MODE

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Recent measurements of the $K_{\mu3}/K_{e3}$ relative branching ratio make possible an evaluation of the matrix element of the divergence of the vector current which is independent of f_+. The matrix element of the divergence does not depend on the mass of a possible intermediate vector boson W. The experimental data are best fitted by a resonance in the isospin-$\frac{1}{2}$ s-wave $K\pi$ system in the interval 750-1200 MeV.

There have been several studies[1-4] of the dispersion relation which the matrix element of the divergence of the strangeness-changing vector current is believed to satisfy in the K_{l3} decay mode. Application of the elastic unitarity condition to determine the phase of the matrix element on the branch cut from the $K\pi$ threshold leads to a relation between the divergence of the vector current and the s-wave $K\pi$ interaction. We wish to show that recent experimental measurement[5] on the $K_{l3}{}^+$ decay modes make possible an evaluation of the divergence of the current which is sufficiently accurate to give direct information on the isospin-$\frac{1}{2}$ s-wave $K\pi$ interaction. The calculation of the divergence depends on two considerations:

(a) The mass of a possible intermediate vector boson W does not enter the relation between the divergence of the vector current and the s-wave $K\pi$ interaction. The matrix element of the weak current in K_{l3} decay is

$$f_+(P_K+P_\pi)_\mu+f_-q_\mu=(1-s/m_W{}^2)^{-1}[g_+(P_K+P_\pi)_\nu+g_-q_\nu](\delta_\mu{}^\nu-q^\nu q_\mu/m_W{}^2), \tag{1}$$

where $q_\mu = (P_K - P_\pi)_\mu$ and $s = q_\mu q^\mu$ is the square of the invariant mass of the leptons. The form factors $f_\pm, g_\pm$ are functions of s. The $f_\pm$ are the form factors which are experimentally measured, and the $g_\pm$ define the structure of the π, K, and W boson vertex:

$$f_+ = g_+(1-s/m_W^2)^{-1}, \quad (2)$$

$$f_- = g_- - g_+[(m_K^2 - m_\pi^2)/m_W^2] \times (1-s/m_W^2)^{-1}. \quad (3)$$

The form factor g_+ satisfies a dispersion relation which can be solved in terms of the p-wave $K\pi$ phase shift. Therefore, the mass of the W boson enters the relation between the experimentally measured form factor f_+ and the p-wave $K\pi$ interaction. However, the divergence of the current $D(s)$ can be written in terms of either pair of form factors and is independent of the mass of the W boson,

$$(m_K^2 - m_\pi^2)f_+ + sf_- = (m_K^2 - m_\pi^2)g_+ + sg_-. \quad (4)$$

The dispersion relation satisfied by $[D(s)-D(0)]/s$ can be solved[6,7] in terms of the phase δ of $D(s)$ on the branch cut from the $K\pi$ threshold s_0 to give

$$D(s) = D(0)e^{u(s)}; \quad u(s) = \frac{s}{\pi}\int_{s_0}^{\infty} ds' \frac{\delta(s')}{s'(s'-s)}. \quad (5)$$

A relation suitable for experimental evaluation can be obtained by an expansion in powers of s,

$$\left(\frac{\partial D}{\partial s}\right)_{s=0} = \left(\frac{m_K^2}{m_\pi^2} - 1\right)\lambda_+ + \xi$$

$$= \frac{1}{\pi}(m_K^2 - m_\pi^2)\int_{s_0}^{\infty} ds' \frac{\delta(s')}{s'^2}, \quad (6)$$

where $\lambda_+ = m_\pi^2(\partial f_+/\partial s)_{s=0}$, $\xi = f_-(0)/f_+(0)$, and $f_+(0) = 1$. The elastic unitarity condition states that δ is equal to the s-wave $K\pi$ phase shift for s' below the inelastic thresholds in the $K\pi$ s-wave system. To the extent that the semileptonic $|\Delta I| = \frac{1}{2}$ rule is correct, the phase shift must be for the isospin-$\frac{1}{2}$ channel. It has been assumed implicitly that the lepton current is local, and that there are no large time-reversal-nonconserving phases in $f_\pm$. The only low-lying inelastic two-body threshold in the s-wave $K\pi$ system is that for $K\eta$ at 1043 MeV, and it is reasonable to suppose that the elastic unitarity condition is good for masses below this.

(b) The second consideration concerns the experimental evaluation of $(\partial D/\partial s)_{s=0}$. By using only measurements of the $K_{\mu 3}/K_{e3}$ relative branching ratio R, this evaluation can be made almost completely independent of the value of λ_+ and therefore of the p-wave $K\pi$ interaction. In order to calculate ξ from an experimental measurement of R it is necessary to assume μ-e universality and a value for λ_+. The dependence of ξ on $\lambda_- = [f_-(0)]^{-1}m_\pi^2(\partial f_-/\partial s)_{s=0}$ is considerably less important[8] and for present purposes we set $\lambda_- = 0$. From the relation[9] between R, ξ, and $\lambda_\pm$,

$$\partial\xi/\partial\lambda_+ \approx -11,$$

the error in $(\partial D/\partial s)_{s=0}$ caused by $\delta\lambda_+$ is

$$(12.4 + \partial\xi/\partial\lambda_+)\delta\lambda_+ \approx 0.01$$

for $\xi \approx 0$, and the present[10] best value $\lambda_+ = 0.023 \pm 0.008$. This is an order of magnitude less than the error caused by the experimental error in R.[11] The existing determinations of ξ[5,12-17] through measurement of the $K_{\mu 3}/K_{e3}$ relative branching ratio have been averaged to give a best value of $\xi = 0.00 \pm 0.10$ for $\lambda_- = 0$ and $\lambda_+ = 0.023$. We find $a\chi^2$ of 4.7 for six degrees of freedom. The experimental estimate of $(\partial D/\partial s)_{s=0}$ is, therefore, 0.285 ± 0.10. The integral in Eq. (6) has been evaluated for

$$\tan\delta(s) = \frac{K}{K+m_\pi}\frac{m_r\Gamma}{m_r^2 - s}$$

representing a single resonance of mass m_r and width Γ. The first factor, in which K is the c.m. momentum, gives δ the correct behavior at threshold. The calculated $(\partial D/\partial s)_{s=0}$ was fitted to the experimental estimate and the values of χ^2, for one degree of freedom, are shown in Fig. 1 as a function of m_r and Γ. The effect of a nonresonant background for $s > m_r^2$ $(s < m_r^2)$ would be to move the minimum in χ^2 to lower (higher) values of m_r. For the hypothesis that there is no s-wave $K\pi$ interaction, we find $\chi^2 = 8.1$ for one degree of freedom. The value $\xi = -1.0 \pm 0.3$ which has been determined[18] from measurements of the μ polarization in $K_{\mu 3}^+$ decay leads to the experimental estimate $(\partial D/\partial s)_{s=0} = -0.71 \pm 0.32$. This measurement of ξ is independent of μ-e universality but appears to be inconsistent with the principles adopted in this work unless the s-wave phase shift is large and negative over a wide interval of s'.

We conclude that the present experimental

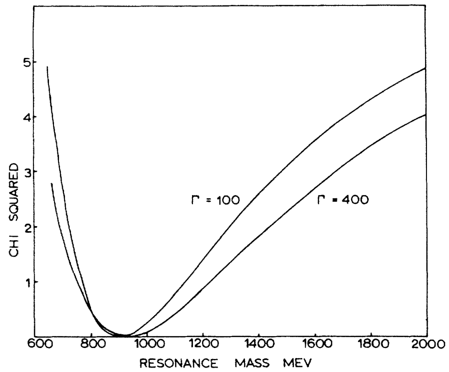

FIG. 1. The experimental evaluation of $(\partial D/\partial s)_{s=0}$ is compared with values calculated for an s-wave $K\pi$ resonance of mass m_r and width Γ. The values of χ^2 are for one degree of freedom.

measurements of the $K_{\mu 3}/K_{e3}$ relative branching ratio favor a nonzero $K\pi$ interaction. The data favor a resonant state in the 750- to 1200-MeV mass interval. It is not possible to estimate the width, which could be so large that the state would not be seen in $K\pi$ invariant-mass distributions in strong production processes. In the quark model, this state could be a member of a $^3P_0$ nonet.[19]

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$$\xi\lambda_- = \tfrac{1}{2} m_\pi^2 (m_K^2 - m_\pi^2)\left(\frac{\partial^2 e^u}{\partial s^2} - \frac{1}{f_+(0)}\frac{\partial^2 f_+}{\partial s^2}\right)_{s=0}.$$

If $D(s)$ and f_+ have poles at $s = m_r^2$ and m_π^2/λ_+, respectively,

$$\xi\lambda_- \approx (m_K^2 - m_\pi^2)\left(\frac{m_\pi^2}{m_r^4} - \frac{\lambda_+^2}{m_\pi^2}\right) \lesssim 0.01.$$

The condition $\lambda_- = 0$ leads to an error $\delta\xi \approx +4\xi\lambda_-$.

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