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$K_1^0K_1^0$ THRESHOLD ENHANCEMENT IN $\pi^-p \to K_1^0K_1^0n$ AT 4 AND 5 GeV/c

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The reaction $\pi^-p \to K_1^0K_1^0n$ has been investigated at 4 and 5 GeV/c in a spark-chamber experiment. The threshold enhancement in the s-wave $K_1^0K_1^0$ system can be fitted by either a complex scattering length or by a resonance form.

We have studied the reaction $\pi^-p \to K_1^0K_1^0n$ at 4 and 5 GeV/c, with emphasis on the threshold enhancement observed in the $K_1^0K_1^0$ mass spectrum.[1] Our data indicate that this enhancement in the s-wave $K_1^0K_1^0$ system can be interpreted in terms either of a complex scattering length or of a resonance. However, several factors, discussed below, complicate the resonance interpretation. A complete description of the experiment will be published elsewhere.

Previous investigations of the $K_1^0K_1^0$ threshold enhancement at momenta between 1.5 and 4.2 GeV/c indicated that the effect was well described by a constant scattering length in the s-wave, isoscalar $K^0\bar{K}^0$ system.[1] In two more recent experiments[2,3] at 6 GeV/c and 5, 7, and 12 GeV/c, the enhancement is interpreted as an isoscalar, s-wave resonance, the S^*, although different resonance widths are quoted.

Our data were obtained using a conventional magnet-spark-chamber spectrometer at the zero-gradient synchrotron at Argonne National Laboratory. Details of the spectrometer have been published elsewhere.[4] A π^- beam with a momentum bite of $\pm\frac{3}{4}\%$ was incident on a 2-in.-diam cylindrical liquid-hydrogen target with its axis normal to the beam. The target was surrounded by a 3-in.-o.d. scintillation cup used in anticoincidence to restrict triggering to neutral final states. The triggering logic required that at least one charged secondary from a neutral decay pass through the spectrometer.

Approximately 240 000 and 200 000 pictures were taken at 4 and 5 GeV/c, respectively. Most of the film was scanned twice for double-vee candidates having at least one track passing through the magnet. Efficiencies for finding measurable events are estimated to be 80 to 90% for most of the film.[5] Events with clearly paired tracks (two 90° stereo views were used) were selected, and a sketch was made to aid in measurement. This two-stage scan yielded 4006 and 4961 candidates for measurement at 4 and 5 GeV/c, respectively.

Candidates were measured with image-plane digitizers and reconstructed in real space to an accuracy of about 1 mm. As a test, the K_1^0 mass was calculated from the geometry output and was found to be 0.492 ± 0.030 GeV/c^2. The kinematic analysis was performed using GRIND, and all possible double-vee hypotheses were tried. Events with only one measured momentum were subject to four constraints in the simultaneous three-vertex fit; approximately one-quarter of the final sample had only one-constraint fits at the production vertex.

Ambiguities between $K_1^0K_1^0n$ and $\Lambda K_1^0\pi^0$ and/or $\Sigma^0K_1^0$ hypotheses sometimes occurred when vee fits were ambiguous. These events were separated on the basis of the ratio of K_1^0 to Λ survival probability for the measured path length. For an ambiguous decay to be accepted, the K/Λ probability ratio was required to be ≥ 2 at 4 GeV/c and ≥ 3 at 5 GeV/c. Since most Λ's were vetoed by the target anticoincidence counter, we assumed that most of the ambiguous slow-Λ fits were actually K_1^0 decays accidentally fit as Λ's. The $K_1^0K_1^0$ mass distribution for events selected in this way (about 25% in the threshold region, and 40% for the entire sample) is consistent with the distribution for unambiguous events; this is not true for the ambiguous events rejected.

Two cuts were applied to the entire data sample to exclude possibly misidentified events: The χ^2 probability for the $K_1^0K_1^0n$ fit was required to be greater than 1%, and the missing mass (i.e., the neutron mass) was required to be between 0.65 and 1.23 GeV/c^2. The full width at half-height of the experimental missing-mass-squared distribution is approximately 0.35 (GeV/$c^2)^2$. After these cuts, the final data contained 933 and 1036 events at 4 and 5 GeV/c, respectively.[5]

Figures 1(a) and 1(b) show the $K_1^0K_1^0$ invariant-mass distributions for the final data. The data have been scaled by the smoothed-potential path correction functions shown. Both sets of data exhibit an enhancement in the mass region near threshold. A large, rather broad peak near 1.3 GeV/c^2 is also evident. We attribute this peak to the $K_1^0K_1^0$ decay mode of the f^0 and A_2 mesons, whose contributions are not separately resolved. There is no evidence for $f'(1500)$ production. Since the K_1^0n mass distributions were found to be consistent with phase space, we conclude that $Y^*(1520)$ production is small.

We have tested the sensitivity of our results to

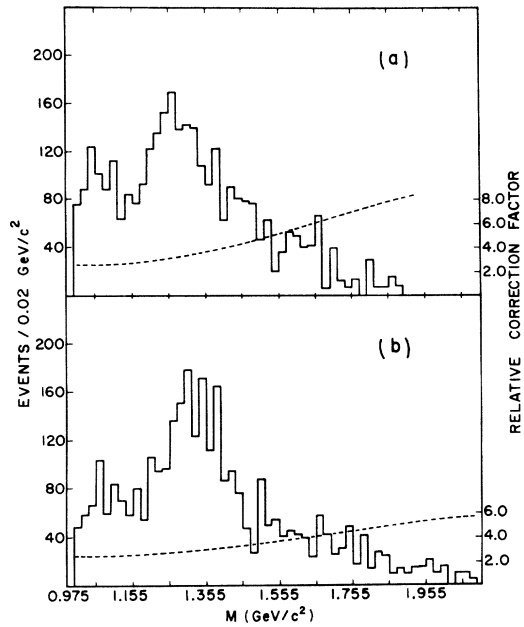

FIG. 1. Weighted $K_1^0K_1^0$ invariant-mass distributions (a) at 4 GeV/c, and (b) at 5 GeV/c. The dashed curves show the potential path weighting functions used to correct the raw data for decay losses. Monte Carlo calculations yield correction curves of similar shape.

contamination from the reaction $\pi^-p \rightarrow K_1^0K_1^0n\pi^0$, with low π^0 energy in the laboratory. Because of the peripheral character of the K_1^0-pair production, these events can sometimes fit the $K_1^0K_1^0$ hypothesis, particularly at 5 GeV/c. After varying the upper limit of the missing-mass cut, we conclude that our results are insensitive to this contamination. We estimate the ratios of $K_1^0K_1^0n$ + neutrals to $K_1^0K_1^0n$ production as about 0.1 and 0.3 at 5 and 5 GeV/c, respectively, consistent with bubble-chamber values.[2,6]

Figures 2 and 3 show several characteristics of the reaction $\pi^-p \rightarrow K_1^0K_1^0n$ at 4 and 5 GeV/c, respectively, for $K_1^0K_1^0$ invariant masses less than 1.135 GeV/c^2. The distribution of the square of the four-momentum transfer from target proton to neutron, $-t$, is shown in Figs. 2(b) and 3(b). The reaction is quite peripheral, and the data are well fitted by the form $e^{\alpha t}$ with $\alpha = 6.3 \pm 0.8$ and 6.9 ± 0.8 (GeV/$c)^{-2}$ at 4 and 5 GeV/c, respectively. We have also examined the Gottfried-Jackson angular distributions[7] and find them to be consistent with isotropy, and thus consistent with single-pion exchange and an s-wave interaction at the meson vertex.

Two interpretations of the enhancement in the low-mass region have been investigated. First,

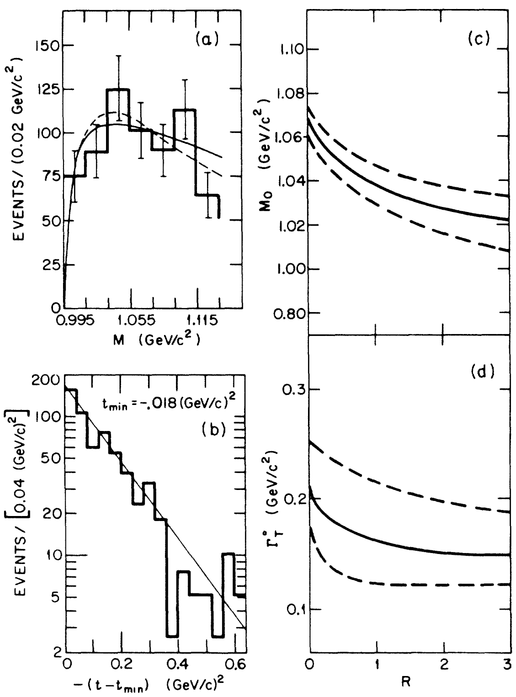

FIG. 2. 4-GeV/c data (251 events) for $K_1^0K_1^0$ invariant mass less than 1.135 GeV/c^2. (a) Weighted $K_1^0K_1^0$ invariant-mass distribution. The solid curve represents the scattering-length–model fit; the dashed curve represents the resonance-model fit. (b) Distribution of the square of the four-momentum transfer from target proton to neutron. (c) Fitted resonance mass M_0 as a function of the $\pi\pi$ to $K\bar{K}$ branching ratio R. (d) Fitted resonance width $\Gamma_T^0 = \Gamma_T(M_0)$ as a function of R. The dashed curves represent the limiting values (diagonal error terms) as determined from $\chi_{min}^2 + 1$; in addition, the errors are highly correlated.

we have considered the production mechanism to be single-pion exchange. The virtual process $\pi^-\pi^+ \to K^0\bar{K}^0$, occurring at the meson vertex, is assumed to be s wave, and to be adequately represented by a complex-scattering length of the form $A = a + ib$ in the isoscalar[2] $K^0\bar{K}^0$ system. Following in essence Alexander et al.,[8] we have fitted our data with the mass distribution

$$\frac{dN}{dM} \propto \frac{M^2 q}{P_{\rm inc}^{\;2} p} \times \left[\int \frac{t}{(m_\pi^{\;2} - t)^2} F^2(t)\,dt\right] \frac{b}{(1+bq)^2 + (aq)^2}.$$

Here, M is the $K_1^0K_1^0$ invariant mass, $P_{\rm inc}$ is the momentum of the incoming beam pion, q is the K_1^0 momentum in the $K_1^0K_1^0$ barycentric system, and p is the barycentric momentum of a pion in a $\pi\pi$ system of total energy M. $F(t)$ is a form factor introduced to describe the behavior of the t distribution. The form used, $F(t) = (\Lambda^2 - m_\pi^{\;2})/(\Lambda^2 - t)$, gives a good representation of our data with $\Lambda^2 = 0.165$ (GeV/c^2)^2. This analysis yields only $|a|$, while b must necessarily be positive.

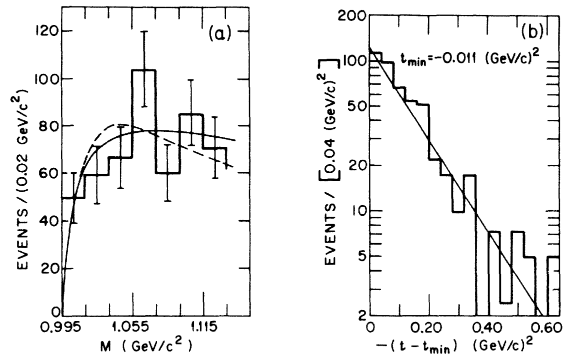

FIG. 3. 5-GeV/c data (199 events) for $K_1^0K_1^0$ invariant mass less than 1.135 GeV/c^2. (a) and (b) are the same as in Fig. 2.

Maximum-likelihood fits yield similar results at both momenta:

$$|a| = 1.18_{-0.44}^{+0.31}\ \text{F},\quad b = 0.58 \pm 0.62\ \text{F at 4 GeV}/c,$$

$$|a| = 0.98 \pm 0.28\ \text{F},\quad b = 0.18_{-0.72}^{+0.58}\ \text{F at 5 GeV}/c.$$

The corresponding curves are shown in Figs. 2(a) and 3(a).

The second possibility is that the threshold enhancement results from a scalar $K^0\bar{K}^0$ resonance, the S^*.[2,3] To investigate this interpretation, we have used the resonance form suggested by Jackson,[9] who pointed out that the energy dependence of the resonance width and the existence of other decay modes can seriously affect the shape of broad resonances occurring near threshold. These considerations are particularly important here, where the resonance appears just above threshold and may also decay via a $\pi\pi$ mode (a consideration neglected in previous analyses[2,3]).

Consequently, we have fitted the data with the expression

$$\frac{dN}{dM} = \left[N_1 + N_2 \frac{M}{q} \frac{\Gamma_{K_1^0K_1^0}(M)}{(M^2 - M_0^{\;2})^2 + M_0^{\;2}\Gamma_T^{\;2}(M)}\right]\frac{dF_3}{dM},$$

where N_1 and N_2 are constants determining the amount of resonance and background (assumed proportional to three-body invariant phase space, dF_3/dM), M_0 is the resonance mass, $\Gamma_T(M) = \Gamma_{KK}(M) + \Gamma_{\pi\pi}(M)$ is the total width, $\Gamma_{KK}(M) = \Gamma_{KK}^{\;0}(q/q_0)$ is the partial width for s-wave decay via the $K\bar{K}$ mode, $\Gamma_{K_1^0K_1^0}(M) = \frac{1}{4}\Gamma_{KK}(M)$ is the partial width for decay into $K_1^0K_1^0$, $\Gamma_{\pi\pi}^0(M) = \Gamma_{\pi\pi}^0(p/p_0)$ is the s-wave partial width for a possible $\pi\pi$ mode, q and p are defined above, and q_0 and p_0 are the corresponding quantities for a mass M_0. The background in the threshold region was estimated to be about 20%.

The fits are quite sensitive to the $\pi\pi$ to $K\bar{K}$ branching ratio $R = \Gamma_{\pi\pi}^0/\Gamma_{KK}^0$; unfortunately, evidence for a $\pi\pi$ enhancement in this mass region is inconclusive,[10] although an upper limit of $R \leq 2.5$ has been reported.[2] The dependence of the fitted resonance parameters on R at 4 GeV/c is shown in Figs. 2(c) and 2(d). (The shapes of the resonance curves are essentially identical over the entire range of R.) At 5 GeV/c, the data do not yield well-defined resonance parameters. The resonance curve shown in Fig. 3(a) is calculated using parameters obtained from the 4-GeV/c data with $R = 0$. The χ^2 for this curve is 9.7 with six degrees of freedom as compared with 6.4 with four degrees of freedom for the 4-GeV/c fits. Corresponding χ^2 for the scattering-length fits are 7.7 and 6.8 for 4 and 5 GeV/c, respectively, each with four degrees of freedom.

The data from our experiment can be fitted by either a complex scattering length or a resonance form. The results of our scattering-length fits are compatible with the Berkeley values[1] obtained at lower momenta. Our invariant-mass distributions appear similar to those measured at CERN.[3] However, the CERN group could not obtain satisfactory scattering-length fits, and interpreted their threshold enhancement as a scalar meson having $M_0 = 1.085^{+0.020}_{-0.014}$ GeV/c^2 and $\Gamma_0 = 0.182^{+0.057}_{-0.047}$ GeV/c^2 (at 5 GeV/c); the effect of a $\pi\pi$ decay mode on the parametrization was not considered. On the other hand, our data appear substantially different from those obtained at Brookhaven[2] at 6 GeV/c, in which a comparatively narrow peak, having $M_0 = 1.068 \pm 0.010$ GeV/c^2 and $\Gamma_0 = 0.080 \pm 0.015$ GeV/c^2, is observed.[11]

We conclude by remarking that the existence of the S^* resonance cannot be established solely by more precise investigations of the $K\bar{K}$ system, in view of the threshold and multichannel complications discussed above. Investigation of the $\pi\pi$ system, particularly $\pi^0\pi^0$, for which ρ decay is forbidden, may prove more rewarding. Finally, the possibility that this enhancement is due to both a large s-wave scattering length and a scalar resonance, as mentioned by Goldhaber,[1] although unpleasant, should not be ignored.

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