part of the trace of $t_{\mu\nu}$ be negligible, and it is not unreasonable to infer that this is true for the tensor as a whole.

[8]J. D. Bjorken and J. D. Walecka, Ann. Phys. (N.Y.) 38, 35 (1966).

[9]We use the multipole analysis of F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. B4, 54 (1968).

[10]F. J. Gilman and H. J. Schnitzer, Phys. Rev. 150, 1362 (1966).

CRUCIAL TEST OF A THEORY OF CURRENTS*

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We discuss interpretation and possible crucial tests of a theory in which the energy-momentum tensor is written in terms of currents.

In the preceeding Letter one of us (DG) has described a way of extracting testable consequences from a theory which expresses the energy-momentum tensor in terms of currents.[1] Here we shall discuss in some detail the basic ideas of this program and propose a crucial test of its relevance to the real world.

The fundamental feature of this theory is that the energy-momentum tensor of hadrons is written directly in terms of currents having algebra-of-fields commutation relations. Although the reasons for doing this are clear enough, the interpretation of the resulting theory perhaps is not. One possiblility is that we are dealing with a canonical field theory in disguise. Examples of equivalent field theories have actually been given,[2] but they have the common defect of describing only bosons. To date, no one has been able to construct an equivalent field theory involving fermions, and one might even guess that it is impossible. Therefore, this interpretation is probably of no use if we wish to describe the real world.

A speculative way of getting around this problem is to regard $\theta_{\mu\nu}$ as a way of selecting acceptable representations of the local current algebra. Given a set of current operators having the correct algebra, the energy-momentum tensor constructed according to Sugawara's prescription is guaranteed to have the commutation relations with itself demanded by Lorentz invariance. Much more is required, however, if the theory is to make physical sense: The physical states must be eigenstates of $P_\mu = \int d\vec{x}\,\theta_{0\mu}$ and must transform correctly under the action of the supposed Lorentz generators $M_{\mu\nu} = \int d\vec{x}(x_\mu\theta_{\nu 0} - x_\nu\theta_{\mu 0})$. Most representations of the current algebra will not have this property,[3] but those that do (if any exist) would appear to give a reasonable physical theory. Since, from this algebraic point of view, there is no apparent difference between bosons and fermions, it seems not impossible that solutions involving fermions can be found. They may well not be equivalent to a field theory, but that is acceptable so long as the theory makes physical sense.

In spite of its unconventional nature, this theory has much to recommend it. It relates the strong, electromagnetic, and weak interactions of hadrons in an appealing way, does not single out any particles as being fundamental, and the Hamiltonian cannot be sepeated into free and interacting parts. In fact, the theory, if consistent, is a rather well defined way of carrying out the bootstrap program.

These remarks of course say nothing about how to find a solution. Before attempting that massive task, though, we should look for a simple crucial test whose failure would tell us that the real world is not described by a theory of this kind. Such a test actually follows from the requirement $\langle p|\theta_{\mu\nu}|p\rangle = p_\mu p_\nu/m$, where $|p\rangle$ is a nucleon state of momentum p, and m is the nucleon mass. This embodies part of the information contained in the identification of $\int d\vec{x}\,\theta_{0\mu}$ with the momentum operator.

We make use of the equation (see preceding Letter)

$$\delta(x_0)[J_{0i}^{\;a}(x), J_j^{\;b}(0)] = -(i/2c)f_{ars}f_{brt}\{[V_j^{\;t}(0), V_i^{\;s}(0)]_+ + [A_j^{\;t}(0), A_i^{\;s}(0)]_+\}\delta(x), \tag{1}$$

where both J's are either vector or axial-vector currents and $J_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu$. Taking nucleon matrix elements of this equation and using isospin invariance, we can then show that

$$\delta(x_0)\tfrac{1}{2}\langle p|[J_{0i}^{K^+}(x),J_j^{K^-}(0)]|p\rangle+\delta(x_0)\tfrac{3}{2}\langle p|[J_{0i}^{\gamma}(x),J_j^{\gamma}(0)]|p\rangle+(p\rightarrow n)=-\tfrac{3}{2}(i/c)\delta(x)\langle p|\textstyle\sum_a\{[V_i^a,V_j^a]$$
$$+[A_i^a,A_j^a]\}|p\rangle=-(3i/m)\delta(x)[p_ip_j+\lambda\delta_{ij}], \tag{2}$$

where J^γ is the electromagnetic current, J^{K^+} is the total strangeness-changing weak current, and the last equality follows from the explicit form of Sugawara's $\theta_{\mu\nu}$ (see preceding Letter).

Following Bjorken,[4] we observe that if

$$T_{\mu\nu}^{[a,b]}=\tfrac{1}{2}\int d^4x\,e^{iq\cdot x}\langle p|T(J_\mu^{a}(x)J_\nu^{b}(0))|p\rangle+(a\leftrightarrow b), \tag{3}$$

(here and henceforth, the removal of vacuum expectation values from time-ordered products is assumed) then

$$T_{ij}^{[ab]}\xrightarrow[|q_0|\rightarrow\infty]{}-\frac{1}{q_0^2}\int d^4x\,e^{iq\cdot x}\delta(x_0)\langle p|[\dot{J}_i^{a}(x),J_j^{b}(0)]|p\rangle$$
$$=-\frac{1}{q_0^2}\int d^4x\,e^{iq\cdot x}\delta(x_0)\langle p|[J_{0i}^{a}(x),J_j^{b}(0)]|p\rangle.$$

The explicit form of the algebra-of-fields current commutators has been used to eliminate the q_0^{-1} term and replace $\dot{J}_i$ by J_{0i}.

Following Eq. (2), we then get

$$\lim_{q^2\rightarrow-\infty}(-q^2)[\tfrac{3}{2}T_{ij}^{\gamma,p}+\tfrac{1}{2}T_{ij}^{K^-,p}+(p\rightarrow n)]=-(3i/m)[p_ip_j+\lambda\delta_{ij}],$$

where an obvious notation has been used.

The quantities $T_{\mu\nu}$ define invariant amplitudes as follows:

$$T_{\mu\nu}(p,q)=Ap_\mu p_\nu+Bq_\mu q_\nu+C(p_\mu q_\nu p_\nu q_\mu)+Dg_{\mu\nu}.$$

We really have, therefore, a condition on the asymptotic behavior of A:

$$\lim_{|q_0|\rightarrow\infty}(-q_0^2)[\tfrac{3}{2}A^{\gamma,p}+\tfrac{1}{2}A^{K^-,p}+(p\rightarrow n)]=-3i/m.$$

Regge-pole arguments suggest that A satisfies an unsubtracted dispersion relation in $\nu=p\cdot q$ for fixed q^2, so that if a is the discontinuity of A, we have

$$\lim_{q^2\rightarrow-\infty}(-q^2)\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}\{3a_p^{\gamma}(q^2,\nu)+a_p^{K^+}(q^2,\nu)+a_p^{K^-}(q^2,\nu)+(p\rightarrow n)\}=\frac{6\pi}{m}.$$

The a's are in turn related to weak and electroproduction cross sections as follows:

$$\lim_{E_l\rightarrow\infty}\frac{d^2\sigma_N(lN)}{dq^2d\nu}=\frac{2\alpha^2}{q^4}m^2a_N^{\gamma}(q^2,\nu),$$

$$\lim_{E_l\rightarrow\infty}\frac{d^2\sigma_N}{dq^2d\nu}(\nu N,\Delta S\neq0)=\left(\frac{Gm\sin\theta}{4\pi}\right)^2a_N^{K^+}(q^2,\nu),$$

$$\lim_{E_l\rightarrow\infty}\frac{d^2\sigma_N}{dq^2d\nu}(\bar{\nu}N,\Delta S\neq0)=\left(\frac{Gm\sin\theta}{4\pi}\right)^2a_N^{K^-}(q^2,\nu),$$

where the cross sections are for the familiar type of reaction in which a lepton is incident on a nucleon target and only the final lepton momentum is measured. We therefore have a sum rule for measurable quantities which follows directly, with no assumptions, from the basic equations of the theory.

All of the elements of this sum rule are measurable, although it will be some time before the weak-production data are available. In the meantime, one should note that the electroproduction data alone must satisfy

$$\lim_{q^2 \to -\infty} (-q^2) \times \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} [a_p^{\ \gamma}(q^2,\nu) + a_n^{\ \gamma}(q^2,\nu)] < \frac{2\pi}{m}.$$

If this is violated, one has proof positive that this theory does not apply to the real world.

Another way of getting a more accessible sum rule is to use SU(3) symmetry to eliminate undesirable cross sections. Since the amplitude of Eq. (3) is determined by four SU(3) invariants, one requires only four independent pieces of experimental information to determine it. The obvious thing to do is to eliminate $a_n^{K^\pm}$ in favor of $a_p^{K^\pm}$, $a_p^{\pi^\pm}$, and $a_{p,n}^{\ \gamma}$. Better yet, if the $\underline{27}$ contribution to $T_{\mu\nu}^{\ \mathrm{ab}}$ is neglected, as is suggested by the properties of the nonleptonic weak Hamiltonian, one can eliminate $a_{p,n}^{K^\pm}$ entirely. The resulting sum rule is

$$\frac{3\pi}{m} = \lim_{q^2 \to -\infty} (-q^2) \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \{6a_p^{\ \gamma}(q^2,\nu) + 6a_n^{\ \gamma}(q^2,\nu) - 2a_p^{\ \pi^+}(q^2,\nu) - 2a_p^{\ \pi^-}(q^2,\nu)\}.$$

This is easier to evaluate than the original sum rule, but less decisive because of the assumptions going into it.

We have so far exploited only a minute fraction of the information implied by a correct physical interpretation of P_μ and $M_{\mu\nu}$. One might, for example, try to obtain a condition from the identification of M_{ij} with the angular-momentum operator. The trouble is that if we use the methods described here we learn something about an asymptotic limit of virtual nonforward Compton scattering. One can write a dispersion relation for this in terms of measurable quantities but they are no longer just cross sections, but much more complicated objects which are extremely hard to measure. The same problem arises in every other case we can think of, and poses the challenge of finding other ways of extracting testable conditions from this theory.

Even if the sum rule works, that does not prove the theory right since there are so many other constraints which must be satisfied. One would, however, feel a very strong encouragement to begin work on the larger problem of solving this very interesting theory.

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[1]H. Sugawara, to be published.

[2]K. Bardakci and M. B. Halpern, to be published.

[3]A good example is provided by the gauge field theories discussed by T. D. Lee, S. Weinberg, and B. Zumino [Phys. Rev. Letters 18, 1029 (1967)]. One can show directly that if H^S is the Hamiltonian constructed according to Sugawara's prescription and H^C is the canonical Hamiltonian, then $[H^C, H^S] \neq 0$. Therefore, the physical states are not eigenstates of Sugawara's Hamiltonian.

[4]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).