[4]A. K. Mann, private communication. An experiment that was sensitive to the decay $K_L{}^0 \to \pi^+ + \nu + \bar{\nu}$ was reported by L. B. Auerbach et al., Phys. Rev. 155, 1505 (1967); E. F. Beall [Phys. Rev. Letters 20, 947 (1968)] has also made the same observation.

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EXPERIMENTAL DETERMINATION OF Reξ FROM $K_{\mu 3}{}^0$ DECAY*

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From measurements of the polarization of muons from $K_{\mu 3}{}^0$ decay, we have found the average longitudinal component to be 0.88 ± 0.25, and the average component in the plane of decay but perpendicular to the muon momentum to be 0.369 ± 0.036. From these we determined Re$\xi = -1.75_{-0.2}^{+0.5}$, assuming constant form factors. The behavior of Re$\xi(0)$ with varying form factors is discussed.

The three components of polarization of the muon from the decay $K_L{}^0 + \pi^- + \mu^+ + \nu$ were measured at the Bevatron. The component P_T transverse to the plane of decay has been discussed elsewhere.[1] It provides a measure of the violation of time-reversal invariance in $K_{\mu 3}{}^0$ decay and was found to be consistent with zero. Therefore, we take Imξ to be zero; so Re$\xi \equiv \xi$. The nonzero components of polarization are P_L, the longitudinal component, and P_P, the component that is in the plane of decay, but perpendicular to P_L. The longitudinal component P_L is along $\vec{p}_\mu$, the momentum of the muon, and the perpendicular component P_P is parallel to $\vec{p}_\mu \times (\vec{p}_\mu \times \vec{p}_\pi)$, where $\vec{p}_\pi$ is the momentum of the pion.

The components of muon polarization are expressed relative to directions of momenta in either the $K_L{}^0$ rest system or the laboratory system. Cabibbo and Maksymowicz have given expressions for both cases.[2] The polarization is expressed in terms of $\xi(q^2)$, the ratio of the two form factors $f_-(q^2)$ and $f_+(q^2)$, where q^2 is the absolute value of the square of the four-momentum transferred to the di-lepton system, i.e., $q^2 = M_K{}^2 + m_\pi{}^2 - 2M_K E_\pi$. Assuming that the form factors are slowly varying, their q^2 dependence can be taken as

$$f_-(q^2) = f_-(0)[1 + \lambda_-(q^2/m_\pi{}^2)] \tag{1}$$

and

$$f_+(q^2) = f_+(0)[1 + \lambda_+(q^2/m_\pi{}^2)]. \tag{2}$$

Therefore,

$$\xi(q^2) = f_-(q^2)/f_+(q^2)$$
$$= \xi(0)[1 + \lambda_-(q^2/m_\pi{}^2)]/[1 + \lambda_+(q^2/m_\pi{}^2)], \tag{3}$$

where $f_-(0)$, $f_+(0)$, $\xi(0)$, λ_-, and λ_+ are constants. The goal of the experiment was to obtain information about these constants by measuring P_L and P_P.

An isometric view of the experiment is shown in Fig. 1. Conventional scintillation counter techniques were used. $K_L{}^0$ mesons decayed in flight upstream from counters U_1 and D_1. The neutral beam had a flux of ~5×10^5 K's per pulse of 5×10^{11} protons in the external beam of the bevatron. The $K_L{}^0$ momentum ranged approximately from 1 to 4 GeV/c.

Negative pions from accepted events passed

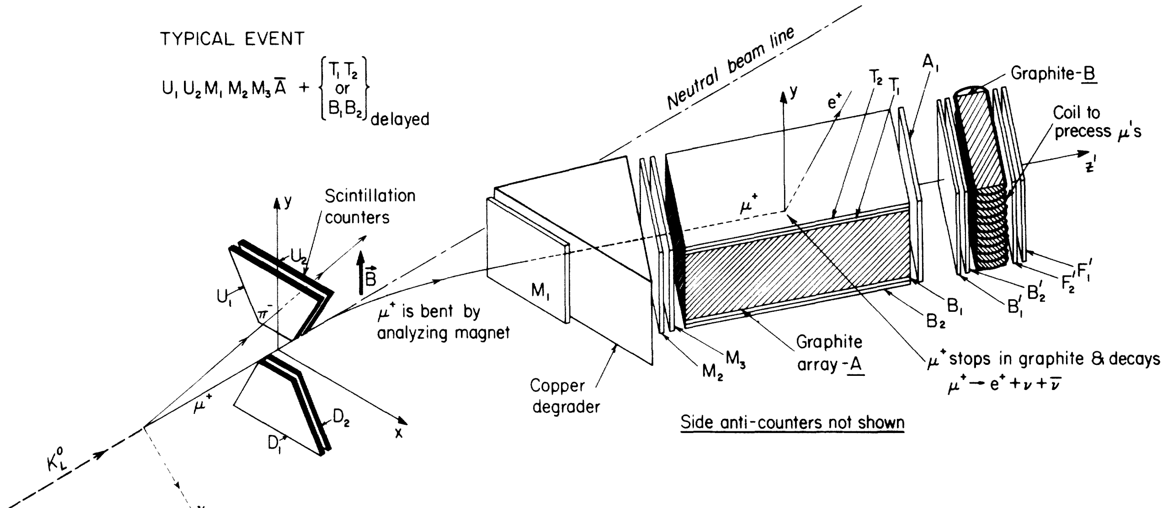

FIG. 1. Isometric view of apparatus.

through counters U_1 and U_2 or D_1 and D_2. Positive muons were bent out of the neutral beam by the analyzing magnet, degraded in copper, and stopped either in the first (larger) array of graphite (A), or in the second (smaller) block of graphite (B) The mean thickness of copper was about five nuclear mean free paths, so that hadrons and electrons were effectively removed. The first graphite array was used to measure P_P, and the second was used to measure P_L. In most of the accepted events the muon went generally forward with the pion going roughly at 90° in the $K_L{}^0$ rest frame. For the P_P measurement the decay plane was constrained to be a vertical plane on the average, due to the requirement that the pion pass through the U or D counters. For the P_L measurement the orientation of the decay plane was not important; however, in order to keep experimental conditions as nearly comparable as possible for the measurements of P_L and P_P, the detection of a pion in U_1U_2 or D_1D_2 was required in both cases.

In the P_P measurement the vertical component of the muon polarization was determined by measuring the vertical asymmetry of the positrons from the muon decay by means of counters above and below the graphite (counters T_1T_2 and B_1B_2 in Fig. 1). The requirement for a P_P event was a fast coincidence in $M_1M_2M_3$ with no anticounter pulse (indicating a muon stopped in the first array of graphite), together with a coincidence in either U_1U_2 or D_1D_2 (signifying a pion). This had to be followed by a delayed count in either T_1T_2 or B_1B_2 in the time interval $0.1 < t < 2.2$ μsec (indicating the detection of a positron from the muon decay). The requirement for a P_L event was similar, with the decay positron detected in either $F_1'F_2'$ or $B_1'B_2'$. The asymmetries ϵ_P and ϵ_L are defined as $\epsilon_P = (T-B)/(T+B)$ and $\epsilon_L = (F'-B')/(F'+B')$, where T, B, F', and B' represent the coincidences discussed above.

A Monte Carlo program was written to optimize the geometry of the experiment and to provide a means of relating the muon polarization as measured in the laboratory to the polarization in the $K_L{}^0$ rest frame and to Reξ.

The asymmetry that was measured with the T_1T_2 and B_1B_2 counters included a contribution due to the nonuniformity of the vertical distribution of the stopped muons. The pion and muon directions were correlated such that pions detected in the upper pion counters were associated with stopped muons having an average vertical position slightly below the median plane, and those detected in the lower pion counters were associated with stopped muons having an average vertical position slightly above the median plane. The magnitude of this false asymmetry was easily determined in two different ways, and corrections were made. The first was by measuring the vertical distributions of the stopped muons for the U and D counters and using this information as input to a Monte Carlo program that simulated the decay of the muons in the graphite array. The second was by reversing the magnetic field in the analyzing magnet to bend negative muons into the graphite. It has been shown[3] that negative muons are depolarized in graphite and retain only ~16% of their initial polarization. Hence by comparing the asymmetry measured with negative muons with that measured with positive muons it was possible to determine the background asymmetry. The results of these two completely different methods agreed very well.

Other false asymmetries that were not correlated with the pion counters were eliminated by combining the data from the U and D counters, by careful alignment of the apparatus, by the periodic interchange of major portions of the electronics, and by other precautions. A very important check for possible instrumental asymmetries was made by measuring the asymmetry without requiring the detection of a pion. This asymmetry was found to be 0.0011 ± 0.0018, consistent with zero, as it should be.

Any false asymmetry in the P_L determination was measured by applying a magnetic field to graphite (at B) to precess the muons rapidly about a vertical axis, thus effectively depolarizing them. The data were then corrected for this background asymmetry.

Small corrections were necessary for several effects that tended to reduce the measured polarization from the true value. These were the following:

(a) Accidental counts in the electron counters (~4 %).

(b) Events associated with an accidental count in the pion counter (~2 %).

(c) Precession of the muon spin by stray magnetic fields in graphite analyzer B (~4 % for P_L). This effect was negligible for P_P, because analyzer A was carefully magnetically shielded.

(d) ~7 % of the muons came from pion decay, where the pions resulted from K_L^0 decay. The muons were slightly longitudinally polarized, making the correction ~8 % for P_L.

(e) Less than 1 % of the pions from K_L^0 decay penetrated the copper and stopped in the graphite analyzers. This had a negligible effect on the final result.

(f) There was a negligible effect from the neutrons in the beam interacting with the He gas in the decay region. The net result of these corrections was a dilution factor of 0.87 for P_P and 0.83 for P_L.

The analyzing power of the graphite was calculated to be 0.39 ± 0.01 for P_P and 0.34 ± 0.01 for P_L, using the standard expression for the distribution of the decay electrons from polarized muons.[3]

The final laboratory average values determined are

$$\langle P_L^{\text{lab}} \rangle = 0.88 \pm 0.25$$

and

$$\langle P_P^{\text{lab}} \rangle = 0.369 \pm 0.036$$

averaged over the sample of events selected by our apparatus. These values, of course, are related only to our specific experimental setup. By means of the Monte Carlo program these values can be related to $\xi(q^2)$ and to $\langle P_L^{\text{c.m.}} \rangle$ and $\langle P_P^{\text{c.m.}} \rangle$, the values of P_L and P_P in the K_L^0 rest frame, averaged over the entire Dalitz plot.

The curve in Fig. 2, determined by the Monte Carlo program, shows the relationship between $\langle P_L^{\text{lab}} \rangle$ and $\langle P_P^{\text{lab}} \rangle$ (for this experiment) as $\text{Re}\xi$ is varied. The shaded portion of the curve illustrates uncertainties due to factors that are not well known, such as the shape of the momentum spectrum of the K_L^0 beam, etc. The data points with errors represent our measured values. $\text{Re}\xi(q^2)$ can be determined from either P_L or P_P. However, because of the possibility of unknown dilution factors or depolarization effects, it is much safer to determine $\text{Re}\xi$ from the ratio of P_L and P_P. The samples of events studied for the two cases are essentially the same, and the experimental arrangements used were very similar. Therefore, unknown dilution or depolarization effects for the two measurements should be about the same. Such effects would shift the experimental point in Fig. 2 toward the origin, but the ratio of P_L to P_P should change very little. The technique actually used to extract $\text{Re}\xi$ from the data is illustrated in Fig. 2. The result for

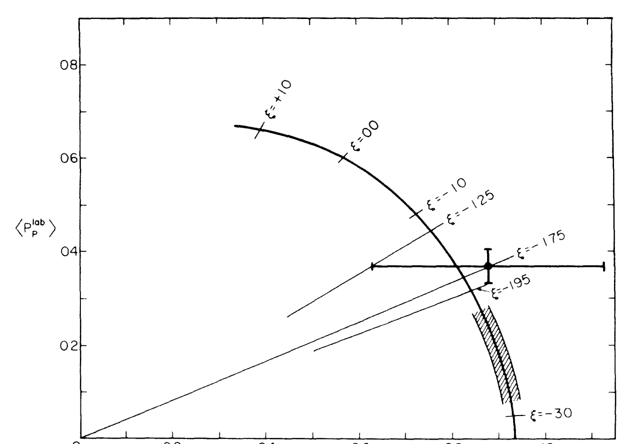

FIG. 2. Curve of Monte Carlo calculation showing relation between $\langle P_P^{\text{lab}} \rangle$ and $\langle P_L^{\text{lab}} \rangle$ for our geometry for various values of $\text{Re}\xi$, assuming that $\text{Im}\xi = 0$. The experimental point with error flags represents our measurements.

Table I. The variation of $\mathrm{Re}\xi(0)$ and $\mathrm{Re}\xi(2.5m_\pi^2)$ with λ_- and λ_+.

λ_-	λ_+	$\mathrm{Re}\xi(0)$	$\mathrm{Re}\xi(2.5m_\pi^2)$
0	0	−1.75	−1.75
0	+0.1	−2.35	−1.88
−0.08	0	−2.35	−1.88
0	−0.1	−1.20	−1.60
+0.13	0	−1.20	−1.60

$\mathrm{Re}\xi$ is

$$\mathrm{Re}\xi = -1.75_{-0.2}^{+0.5}.$$

It is clear from Fig. 2 that the experimental point is consistent with no depolarization or dilution. If we use the results for P_P alone we find $\mathrm{Re}\xi = -1.7 \pm 0.2$. However, we do not believe that extracting $\mathrm{Re}\xi$ from a single component of polarization is completely reliable. These results assume that $\mathrm{Re}\xi$ is independent of q^2. Table I shows the values of $\mathrm{Re}\xi(0)$ and $\mathrm{Re}\xi(\bar{q}^2)$ for various values of λ_- and λ_+, assuming the relation stated in Eq. (3), where $\langle q^2\rangle$ is the mean value of q^2 for events selected by the apparatus and had a value of $\langle q^2\rangle = 2.5m_\pi^2$. Note that $\mathrm{Re}\xi(2.5m_\pi^2)$ is nearly independent of λ_- and λ_+.

Again, from the Monte Carlo calculation, we find $\langle P_L^{\mathrm{c.m.}}\rangle = 0.88_{-0.07}^{+0.03}$ and $\langle P_P^{\mathrm{c.m.}}\rangle = 0.18_{-0.05}^{+0.11}$.

The results of this experiment are consistent with other determinations of $\mathrm{Re}\xi$ based on muon polarization.[4] However, it can be seen from Fig. 2 that our results for both $\langle P_P^{\mathrm{lab}}\rangle$ and $\langle P_L^{\mathrm{lab}}\rangle$ are inconsistent with $\mathrm{Re}\xi \simeq +0.7$, the value found in $K_{\mu 3}/K_{e3}$ branching ratio and energy-spectra experiments.[4]

The time-reversal-nonconserving phase for $K_{\mu 3}^0$ decay can be calculated by using the value of $\mathrm{Re}\xi(0)$ quoted in this report and our previously reported value of $\mathrm{Im}\xi$.[1] Assuming constant form factors, we find

$$\arg(\xi) - \pi = \tan^{-1}\left(\frac{-0.014 \pm 0.066}{-1.75}\right) \simeq (0.5 \pm 2.2)^0,$$

for $q^2 \simeq 2.5m_\pi^2$. Since the determination of both $\mathrm{Re}\xi$ and $\mathrm{Im}\xi$ involved essentially the same portion of the Dalitz plot, this result is nearly independent of possible variations of ξ with q^2.

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[3]G. Feinberg and L. Lederman, Ann. Rev. Nucl. Sci. 13, 431 (1963).

[4]The present world average for $\mathrm{Re}\xi$ from muon polarization measurements is -1.15 ± 0.35 for $K_{\mu 3}^0$ decay, and -1.25 ± 0.32 for $K_{\mu 3}^+$ decay. The average from measurements of spectra and $K_{\mu 3}/K_{e3}$ branching ratios is $+0.7 \pm 0.3$ for K^0 decay and $+0.3 \pm 0.4$ for K^+ decay. See W. J. Willis, in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967, edited by H. Filthuth (North-Holland Publishing Company, Amsterdam, The Netherlands, 1968).