duce that the time evolution in our system has some different aspects. In this regard one is encouraged to investigate in detail the solution of the nonlinear Schrödinger equation (14).

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CAPACITANCE OBSERVATIONS OF LANDAU LEVELS IN SURFACE QUANTIZATION*

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Capacitance observations of Landau levels in a two-dimensional electron gas induced in the inversion layer on a (100) surface of p-type silicon are reported. Evidence for surface quantization and the associated lifting of the spin and valley degeneracy are presented. An observed increase in the carrier threshold with increasing magnetic field is shown to be further evidence of surface quantization.

Recently, Fowler *et al.*[1,2] have shown that a two-dimensional electron gas can be induced at the surface of p-type silicon using a metal-oxide semiconductor-field-effect transistor (MOSFET).[3] This two-dimensional gas is produced by quantization of the density of states shielding charge at the silicon surface, a previously anticipated result.[4-8] Fowler *et al.* employed the Shubnikov-de Haas effect to observe this surface quantization. Their experiments were sufficiently sensitive to observe Landau levels associated with the lifting of the electron spin and valley degeneracy predicted by Fang and Howard.[8] In this Letter we report an observation of an oscillatory capacitance due to Landau levels in a two-dimensional electron gas generated in the inversion layer on a (100) surface of p-type silicon.

In the absence of thermal or collision broadening, an electric and a magnetic field perpendicular to the surface will induce a density of states given by a series of delta functions,[9] i.e., Landau levels. Assuming neither collision broadening nor Landau-level splitting due to electron spin or valley degeneracy, the surface space charge density Q_{sc} in an inversion layer of p-type silicon is[2,9]

$$Q_{sc} = \frac{e^2 H}{2\pi\hbar c} g_s g_v \times \sum_{n=0}^{\infty} \int_0^{\infty} \frac{\delta[E_{z1} + (n+\frac{1}{2})\hbar\omega_c - E]}{1 + \exp[(E - E_F)/kT]} dE, \qquad (1)$$

where $\delta(x)$ is the Dirac delta function of argument x, e the electronic charge, H the magnetic field perpendicular to the surface; c the velocity of light, $\hbar$ Planck's constant, g_s and g_v the spin and valley degeneracy, respectively, kT the thermal energy of the electron, E_F the Fermi energy, E_{z1} the energy of the first quantum state for $H = 0$ (only the first state will be considered here), and $\omega_c = eH/m_t^* c$ the cyclotron frequency associated with m_t^*, the transverse effective mass of the electron. The summation is over all

Landau levels of the first state E_{z1}. The small-signal space charge capacitance is then

$$C_{sc}=\frac{\partial Q_{sc}}{\partial \Phi_s}=\frac{e^2H}{2\pi\hbar c}g_s g_v\frac{\partial}{\partial \Phi_s}\times\sum_{n=0}^{\infty}\int_0^{\infty}\frac{\delta[E_{z1}+(n+\frac{1}{2})\hbar\omega_c-E]}{1+\exp[(E-E_F)/kT]}dE. \quad (2)$$

For $\hbar\omega_c \gg kT$, it is evident that C_{sc} will oscillate with increasing Fermi energy E_F, with both the period and amplitude being proportional to H. The carrier threshold, i.e., turn-on for MOSFET's, will also be a function of H due to the zero-point energy $\frac{1}{2}\hbar\omega_c = e\hbar H/2m_t^*c$.

To observe both of these effects capacitance experiments were performed using 100-Ω cm p-type substrate MOSFET's with a coaxial source-drain structure. Except for having an oxide thickness of approximately 2900 Å, these specimens were identical to those employed by Fowler et al.[1,2] The gate (field plate) was one terminal of the capacitor and the n^+ source and drain electrodes were shorted to the substrate to form the other terminal. The capacitance C_g of this structure was measured at a frequency of 100 kHz as a function of gate voltage and magnetic field perpendicular to the surface.

Figures 1 and 2 show the capacitance C_g as a function of gate voltage for $H = 90$ kOe and $H = 150$ kOe, respectively, at a temperature of 1.3°K. At the notches one observes that C_g tends to zero. From Eqs. (1) and (2) it is apparent that the density of states and the small-signal space charge capacitance become very small when E_F is midway between two Landau levels. Since the oxide capacitance C_{ox} is in series with C_{sc}, $1/C_g = 1/C_{ox} + 1/C_{sc}$, C_g must also go to zero when C_{sc} tends to zero. The flatness of the capacitance curves at their maxima is due to the fact that $C_{sc} \gg C_{ox}$ whenever E_F is not almost exactly midway between two Landau levels. There is also some distortion of the notch shapes due to detector nonlinearities [$\frac{1}{2}(C_{g\,\max}-C_{g\,\min})$ occurs about $0.3(C_{g\,\max}-C_{g\,\min})$ above $C_{g\,\min}$].

Since each full Landau level contains $eHg_sg_v/2\pi\hbar c$ electrons/cm^2,[9] the total carrier density and carrier density change between notches can be calculated from $N = k_{ox}(V_g - V_T)/4\pi ed$, where k_{ox} is the silicon-dioxide dielectric constant, V_g and V_T are the gate voltage and threshold voltage for inversion, respectively, and d is the oxide thickness.[1,2] Assuming $m_t^* = 0.19m_e$ (with m_e, the mass at a free electron) and $g_s = g_v = 2$[1,2,8] for the (100) surface of silicon, the periods of the Landau levels are approximately 11.8 and 17.7 V for $H = 90$ kOe and $H = 150$ kOe, respectively, values that correlate favorably with our experimental data. In both Figs. 1 and 2, the fourth, sixth, and seventh notches occur after each of the first three Landau levels has been filled with electrons. The other notches are due to spin and valley splittings in the first Landau level and spin splitting in the second Landau level. The last notch in Fig. 1 occurs after four Landau levels have been filled with electrons. The combination of Landau-level spacing and decreasing notch amplitudes and linewidth as the applied voltage increases indicates that the mobility of the electrons in the inversion region is largest a few volts above the carrier threshold and decreases rapidly thereafter, a result consistent with the field-effect mobilities and effec-

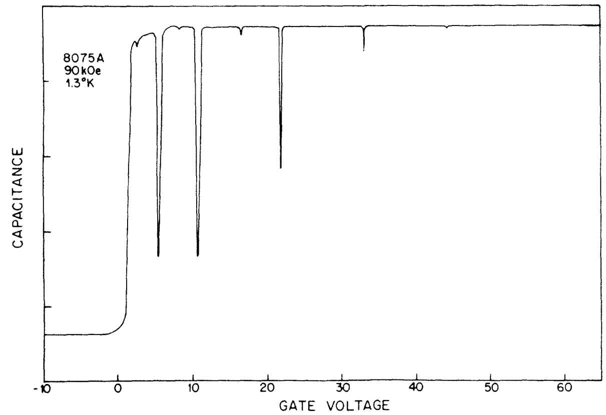

FIG. 1. The capacitance as a function of gate voltage or surface field at 90 kOe.

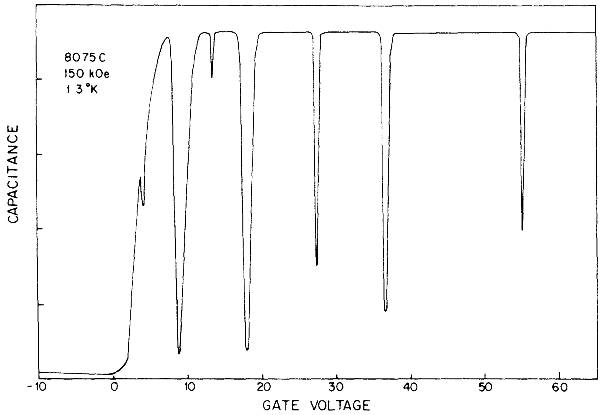

FIG. 2. The capacitance as a function of gate voltage or surface field at 150 kOe.

tive mobilities measured by Fang and Fowler[10] for similar conditions.

Oscillations were also observed at magnetic fields as low as 50 kOe and in MOSFET's with 10-Ω-cm *p*-type substrates.

Figure 3 shows the change in capacitance as a function of gate voltage near the carrier threshold at a temperature of 1.3°K for various constant magnetic fields. This change is due to the increase of $\frac{1}{2}\hbar\omega_c$ in the threshold energy. Assuming that $kT \ll \frac{1}{2}\hbar\omega_c$ and $\omega_c\tau \gg 1$ (with τ the carrier relaxation time), the triangular potential well approximation of Coleman et al.[11] for $H=0$ gives

$$E_{z1} = \left(\frac{e^2\hbar^2K_r^{\,2}}{2m_l^*d^2g_v^{\,2}}\right)^{1/3} \gamma_0 V_g^{\,2/3}, \tag{3}$$

where K_rV_g/dg_v is the electric field at the silicon surface, K_r the ratio of the dielectric constant of the $\mathrm{SiO_2}$ insulator to the silicon substrate, m_l^* the longitudinal effective mass of the electron, and $\gamma_0 = 2.338$. When the magnetic field is applied, the first allowed level becomes $E_{z1} + \frac{1}{2}\hbar\omega_c$. This energy increase is related to the shift in threshold voltage by Eq. (3), so that

$$\tfrac{1}{2}\hbar\omega_c \approx \left(\frac{e^2\hbar^2K_r^{\,2}}{2m_l^*d^2g_v^{\,2}}\right)^{1/3} \times\gamma_0[V_T^{\,2/3}(H)-V_T^{\,2/3}(0)], \tag{4}$$

where $V_T(H)$ and $V_T(0)$ are the threshold gate voltages in a magnetic field H and in zero magnetic field, respectively. Letting $\delta V_T = \delta V_T(H) = V_T(H) - V_T(0)$ represent the change in threshold voltage, Eq. (4) can be written as

$$\delta V_T(H) = \left[\left(\frac{em_l^*d^2g_v^{\,2}}{4m_t^{*3}K_r^{\,2}c^3}\right)^{1/3}\frac{H}{\gamma_0} + V_T^{\,2/3}(0)\right]^{3/2} - V_T(0). \tag{5}$$

Using the intercepts of the tangent lines of maximum $\partial C_g/\partial V$ at a constant magnetic field as an approximate measure of threshold voltage, one obtains $\delta V_T = 980$ meV for $H = 100$ kOe. From Eq. (5) we obtain $\delta V_T = 703$ meV, assuming that $K_r = 0.33$ and $m_l^* = 0.98$ on the (100) surface of silicon.[1,2,8] Since δV_T was found to decrease rapidly with increasing temperature (presumably due to thermal broadening), the above value serves to indicate the order of magnitude of this effect.

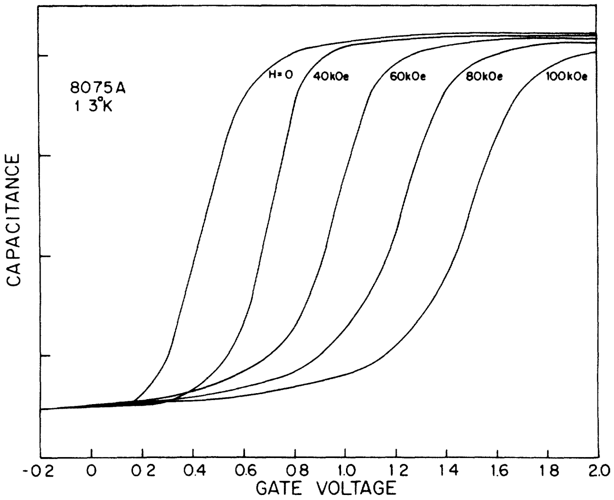

FIG. 3. The capacitance as a function of gate voltage or surface field near threshold for various magnetic field strengths.

In summary, we have used a capacitance technique to obtain further evidence for a two-dimensional electron gas at the silicon (100) surface, for its associated surface quantization, and for the reduction in valley degeneracy predicted by Fang and Howard.[8] The increase in threshold voltage with increasing magnetic field also substantiates the two-dimensional band model previously proposed.[1,2,8,12] This capacitance technique provides information about the Landau levels near threshold better than it does about those of higher order. In contrast, conductance techniques seem better for studying Landau levels of large order.

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SINGLET EXCITON-EXCITON INTERACTION IN ANTHRACENE*

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The interaction of two singlet excitons to produce electron carriers has been observed in anthracene for the wavelength range 315-405 nm. With an applied field of 10^4 V cm^{-1}, the rate constant for carrier production is $0.9\times10^{-12\pm0.4}$ $\mathrm{cm}^3\ \mathrm{sec}^{-1}$. The rate constant for singlet-exciton second-order disappearance is $1.5\times10^{-8\pm0.3}$ $\mathrm{cm}^3\ \mathrm{sec}^{-1}$

Recent experiments[1,2] have demonstrated that free carriers may be generated in crystalline anthracene via photoionization of singlet excitons. The investigators agree on a cross section σ_c for free-carrier production of ca. 10^{-19} cm^2 at 694 nm. At this wavelength the singlet excitons are generated by direct two-photon absorption and the observed intensity dependence of the photocurrent is cubic. As a result of his experiments, Kepler has suggested[1] that the important photoconductivity experiments using weakly absorbed light of $\lambda > 415$ nm reported by Silver, Olness, Swicord, and Jarnagin[3] may be more reasonably interpreted in terms of exciton photoionization than by the exciton-exciton interaction ionization mechanism assumed by those investigators. Indeed, assuming an exciton photoionization mechanism, Kepler measured[1] a free-carrier production cross section of 5×10^{-19} cm^2 at 425 nm. This value makes very reasonable his suggestion that exciton photoionization is the likely carrier generation mechanism at 425 nm, as he has proved it to be at 694 cm. Strome[4] has found excellent agreement at 421 nm with Kepler's cross-section value at 425 nm. Kepler and Strome each discuss other observations which lead them to favor exciton photoionization as the carrier generation mechanism in the weakly absorbed light region. Thus, it appears that no unequivocal observation of singlet exciton-exciton carrier generation has been reported up to the present time.

We have measured the number of electron carriers generated in single crystals of anthracene at an applied field of 10^4 V/cm using pulses of approximately monochromatic, unpolarized light at wavelengths between 315 and 410 nm. Figure 1 illustrates the observed nearly quadratic dependence of the number of carriers generated on the incident light intensity. Further, the wavelength dependence and magnitude of the number of carriers generated argue strongly that exciton-exciton interaction rather than exciton photoionization is the dominant carrier generation mechanism in the range 330-400 nm. The observation of near quadratic intensity dependence is in contrast to the linear dependence found in this region of strong light absorption by earlier investigators[5,6] who were specifically examining the possible importance of the exciton-exciton mechanism in the near uv region. The present results eliminate the "paradox"[7] that the exciton-exciton mechanism seemed to be important with weakly absorbed light but unobservable with strongly absorbed light.

The single crystals used in our experiments were cleaved and mounted under a nitrogen atmosphere, and all measurements were done in a photoconductivity cell evacuated to about 10^{-5} Torr. Typically, depending on wavelength, 10^6 to 10^7 electron carriers were generated when a flash of ca. 10^{12} photons, in a pulse with width at half-peak-height $= 0.30\times10^{-6}$ sec, was incident