MESON-NUCLEON ELASTIC SCATTERING AND PHOTOMESON PRODUCTION VIA THE u CHANNEL*

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An explanation of the available pion-nucleon and photo-meson production data on the backward direction is proposed using baryon trajectories. Special emphasis is given to the constraints, the unequal-mass kinematics, and their implications.

Backward scattering is of great interest to the Regge pole model for several reasons. From the theoretical point of view it involves several constraints imposed by (a) the MacDowell symmetry,[1] (b) conspiracy, (c) Gribov's theorem,[2] and, in addition, the unequal-mass complications.[3] It is important to note that contrary to forward scattering, all the constraints occur in regions accessible to experiment. From the experimental point of view there are, or there will soon be available, data on several processes.[4]

We discuss here the importance of the N_α and the Δ trajectory in explaining the elastic pion-proton data. We propose an explanation for the dominance of the nucleon trajectory in π^+p elastic scattering and we point out some observable consequences of the unequal-mass kinematics. Finally, we make some general observations concerning the photoproduction processes.

For fixed u and large s, even within the backward cone, the Regge representation of the helicity amplitudes is given by[5]

$$\begin{pmatrix} F_{++}(u,z_u) \\ F_{+-}(u,z_u) \end{pmatrix} = \left\{ \sum_i \tfrac{1}{2} c_i \beta_{\alpha_i}(w) \frac{1+\eta_i \exp[-i\pi(\alpha_i - \frac{1}{2})]}{\sin\pi(\alpha_i-\frac{1}{2})} \frac{(2\alpha_i+1)}{\Gamma(\alpha_i-\frac{1}{2})} \left(\frac{s}{1\ \mathrm{BeV}^2}\right)^{\alpha-\frac{1}{2}} \mp \begin{bmatrix} \text{Same expression with } \alpha_i(w) \text{ and } \beta_{\alpha_i}(w) \\ \text{replaced by the complex conjugates} \end{bmatrix} \right\} \begin{pmatrix} \cos\frac{1}{2}\theta u \\ \sin\frac{1}{2}\theta u \end{pmatrix}, \tag{1}$$

where $\sum_i$ denotes summation over trajectories and where η_i is the signature factor, and c_i is the isospin factor. Equation (1) obtained by Reggeizing the helicity amplitudes, which are free of kinematic singularities. The unequal-mass complications are treated in the manner described in Ref. 3. The normalization is as follows:

$$\frac{d\sigma}{du} = \left(\frac{\pi s_0}{s}\right)^2 \{|F_{++}(u,z_u)|^2 + |F_{+-}(u,z_u)|^2\}, \tag{2}$$

(where $s_0 = 1$ BeV). We make the convention of always dealing with the $l = J-\frac{1}{2}$ amplitude and eliminating the $l = J+\frac{1}{2}$ amplitude by using the MacDowell symmetry. To be explicit, using

$$[\beta_{\alpha_i}(w)]_{w=-m_i} = \left\{ \frac{f_{l^+}(w)(w+m_i)(d\alpha/dw)}{q^{2(\alpha_i-\frac{1}{2})}[(w+m)^2-\mu^2]} \right\}_{w=-m_i}, \tag{3}$$

we obtain for the nucleon residues

$$\beta_\alpha(w) + \beta_\alpha(-w) = \frac{3g^2}{16\pi} \frac{M}{w} \frac{d\alpha}{dw}$$

and

$$\beta_\alpha(w) - \beta_\alpha(-w) = \frac{3g^2}{16\pi} \frac{d\alpha}{dw} \tag{4}$$

and the following ratios at the poles:

$$\left(\frac{\beta(w)}{d\alpha/dw}\right)_{\alpha=N_\alpha} : \left(\frac{\beta(w)}{d\alpha/dw}\right)_{\alpha=N_\gamma} : \left(\frac{\beta(w)}{d\alpha/dw}\right)_{\alpha=\Delta} = 11.9:1.5:1.0. \tag{5}$$

The residue of the N_γ is considerably smaller than the nucleon residue (provided the two trajectories have comparable slopes); in addition, we expect its trajectory to lie below the nucleon trajectory. For these reasons we included in the calculation only the N_α and the Δ trajectories. The N_γ could be important only at the position of the dip in the π^+p cross section, but at the moment it is impossible to disentangle its contribution. We point out that the residues have no zeros. All the zeros are included in the remaining kinematic factors of Eq. (1). The trajectories

$$\alpha_\Delta = -0.075 + 0.75w + 0.70w^2, \tag{6}$$

$$\alpha_{N_\alpha} = -0.365 + 0.80w^2 \tag{7}$$

can account for the angular dependence. This suggests that the couplings appearing in the residues are slowly varying functions of w. The results of the calculation are shown in Figs. 1 and 2. The values of the residues at the poles are

$$\beta_\Delta = 0.35\ \text{BeV}^{-1}, \quad \beta_{N_\alpha} = 9.95\ \text{BeV}^{-1}. \tag{8}$$

The Δ residue is 38% of its value at the pole, while the nucleon residue is 5% larger than its value at the pole. The π^+p process is dominated by the N_α trajectory. The contribution of the Δ employed in the calculation is just what is required by isospin invariance. The interference between the two trajectories is destructive for $u < -0.2$ $(\text{BeV}/c)^2$ and constructive in the remaining region. The presence in Eq. (7) of a large term linear in w gives two helicity amplitudes of comparable magnitude at $u < -0.3$ $(\text{BeV}/c)^2$ ($|F_{++}|^2 \approx \frac{1}{2}|F_{+-}|^2$). This forces the cross section to level off at large values of u and it eliminates a dip at $u \approx -1.8$ $(\text{BeV}/c)^2$, which is present in π^-p when the trajectory is linear in u.[6] We conclude that the trajectories given in (6) and (7), together with residues which are slowly varying functions of w, can give the correct ratio of cross sections with the nucleon trajectory dominating in π^+p. This is in contrast to all previous analyses, where the extrapolated value of the nucleon residue at the pole was in reasonable agreement with the Born calculation, but the Δ residue differed by a factor of 10 (a factor of 100 in the cross section). This enormous difference comes from the different parametrization of the residues in all previous analyses.[7] To be more explicit the factor $\beta_\alpha/\Gamma(\alpha+\frac{1}{2})$ which appears in Eq. (1) was replaced by $(\alpha+\frac{1}{2})(\alpha+\frac{3}{2})\beta_\alpha$. The value of $(\alpha+\frac{3}{2})(\alpha+\frac{1}{2})$ at $u \sim 0$ is $\frac{3}{4}$ though at the pole

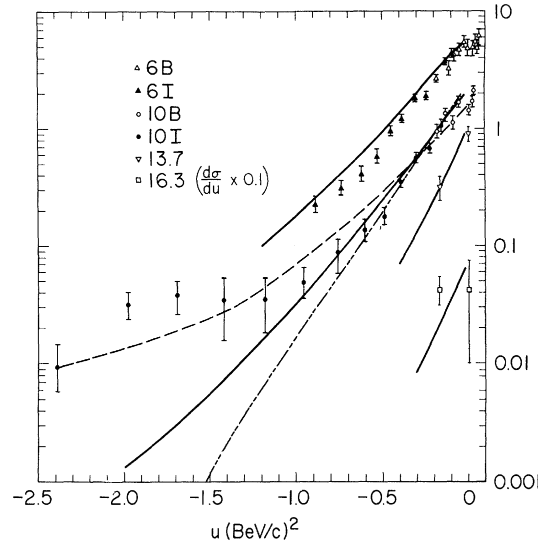

FIG. 1. The differential cross section for backward π^-p elastic scattering. The data are taken from Orear et al., Ref. 4. The open triangles, circles, and squares correspond to the backward geometry and the rest to the intermediate region. The solid curves represent the results of the calculations described in the text. The uniformly dashed curve corresponds to a similar calculation with a quadratic term $0.1u^2$ added to (6). The nonuniformly dashed curve corresponds to previous fits using a linear trajectory. Note that such fits give a dip at $u = -1.8$, which lies below our picture.

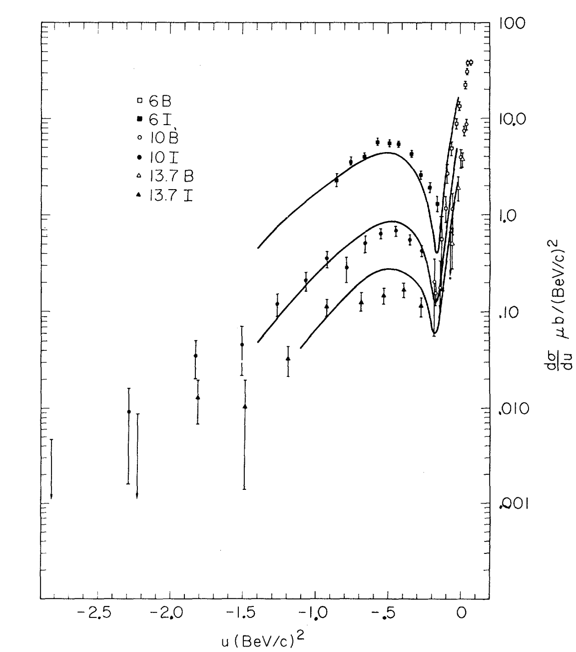

FIG. 2. Angular distribution and energy dependence for backward π^+p elastic scattering. The notation and source of the data are the same as in Fig. 1.

it is 6; it changes by a factor of 8. Correspondingly $1/\Gamma(\alpha+\frac{1}{2})$ changes only by a factor of $\sqrt{\pi}$. This allows constant residues.

The half-angles cause the following effects:

(1) The high-energy dependence of the helicity amplitudes at $u=0$ is as follows:

$$F_{++}(u,z)\sim s^{\alpha-\frac{1}{2}}, \tag{9a}$$

$$F_{+-}(u,z)\sim s^{\alpha}. \tag{9b}$$

(2) Whenever the diffraction peak is not very steep, the half-angles make some of the amplitudes level off at $u\approx 0$. For pion-nucleon scattering for example, the helicity nonflip amplitude goes to zero at $u=(M^2-\mu^2)^2/s$. This creates antishrinkage and possibly shallow minima. In photoproduction, where three of the helicity amplitudes go to zero at 180°, we predict that these effects should be much more prominent.

In regard to photoproduction, presently there are data for two photoproduction processes in the backward direction: $\gamma p\rightarrow\pi^+n$ and very preliminary data for $\gamma p\rightarrow\pi^0 p$ at one energy only. The analysis of photoprocesses is very similar to the meson-nucleon case. The Reggeized Chew-Goldberger-Low-Nambu amplitudes[8] are given by

$$F_2(u,z_u)\pm F_1(u,z_u)=\sum_i c_i\beta_{\alpha_i}(w)\frac{1+\eta_i\exp[-i\pi(\alpha_i-\frac{1}{2})]}{\sin\pi(\alpha_i-\frac{1}{2})}\frac{1}{\Gamma(\alpha_i+\frac{1}{2})}\left(\frac{s}{1\ \mathrm{BeV}^2}\right)^{\alpha_i-\frac{1}{2}}\pm\begin{bmatrix}\text{MacDowell}\\ \text{symmetric}\\ \text{contribution}\end{bmatrix}, \tag{10a}$$

$$F_4(u,z_u)\pm F_3(u,z_u)=\sum_i d_i\gamma_{\alpha_i}(w)\frac{1+\eta_i\exp[-i\pi(\alpha_i-\frac{1}{2})]}{\sin\pi(\alpha_i-\frac{1}{2})}\frac{1}{\Gamma(\alpha_i-\frac{1}{2})}\left(\frac{s}{1\ \mathrm{BeV}^2}\right)^{\alpha_i-\frac{3}{2}}\pm\begin{bmatrix}\text{MacDowell}\\ \text{symmetric}\\ \text{contribution}\end{bmatrix}, \tag{10b}$$

where c_i, d_i are isospin factors and $\beta_{\alpha_i}(w), \gamma_{\alpha_i}(w)$ are the residues which are related to the electric and magnetic multipoles. The helicity amplitudes $A_{\mu\lambda}(u,z_u)$ (μ being the final helicity in the c.m. system and λ the initial-state helicity) are related to the Chew-Goldberger-Low-Nambu amplitudes as follows:

$$A_{1/2,\,3/2}=-(1/\sqrt{2})[(F_3+F_4)\sin\theta\cos\tfrac{1}{2}\theta],\quad A_{1/2,\,1/2}=(1/\sqrt{2})[2(F_3-F_4)\sin\theta\sin\tfrac{1}{2}\theta],$$

$$A_{-1/2,\,3/2}=(1/\sqrt{2})[(F_3-F_4)\sin\theta\sin\tfrac{1}{2}\theta],\quad A_{-1/2,\,1/2}=(1/\sqrt{2})[2(F_1+F_2)\sin\tfrac{1}{2}\theta+(F_3+F_4)\sin\theta\cos\tfrac{1}{2}\theta]. \tag{11}$$

Several properties are of special importance to the analysis of the data: (1) For real trajectories the helicity amplitudes have zeros at "nonsense values of wrong signature." (2) At 180° of the s-channel only the $A_{-\frac{1}{2},\frac{1}{2}}(u,z_u)$ amplitude survives, in accordance with conservation of angular momentum. (3) As $w\rightarrow 0$, $F_1+F_2\rightarrow w^{-2}$, $F_1-F_2\rightarrow w^{-1}$, $F_3+F_4\rightarrow w^{-2}$, $F_3-F_4\rightarrow w^{-3}$. The singularities, when translated in terms of singularities of the residues, result in several possibilities of evasive or conspiratory solutions. In our analysis we assumed only the conspiracy solutions, where in the above limit $\beta_\alpha(w)+\beta_\alpha(-w)\rightarrow w^{-2}, \beta_\alpha(w)-\beta_\alpha(-w)\rightarrow w^{-1}$, and there are similar relations for the γ_α's.

Since the data at the moment are still very crude, we can only discuss some very distinct features and comment on their implications:

(a) Although the $\gamma p\rightarrow\pi^+n$ cross section can, in general, come from an interference of the Δ and the N_α trajectory, the absence of a dip in the angular dependence of the data of Anderson et al. eliminates the possibility of dominance by the nucleon trajectory. In this analysis the most important contribution comes from the Δ trajectory, in contrast to previous expectations. If we further assume, as in the π-N case, that the couplings which appear in the residues are constants (independent of u), we have two arbitrary parameters. One is determined from the overall normalization, in agreement with the vector-dominance model discussed below in (b). The other parameter is determined by choosing the contribution of $A_{1/2,3/2}$, $A_{1/2,1/2}$, $A_{-1/2,3/2}$ to the cross section at $u\approx -0.4(\mathrm{BeV}/c)^2$ and $s=19.26$ to be three times the contribution of $A_{-1/2,1/2}$. Such a large ratio makes the cross section have a dip at $u\approx 0$ and decrease with energy like

$$d\sigma/du\sim s^{2\alpha_\Delta-3}. \tag{12}$$

Figure 3 shows the data and our calculations. The curves give the observed angular distribution and energy dependence. We observe a small dip at $u\approx 0$, but no shrinkage.

(b) In the vector-meson-dominance model the

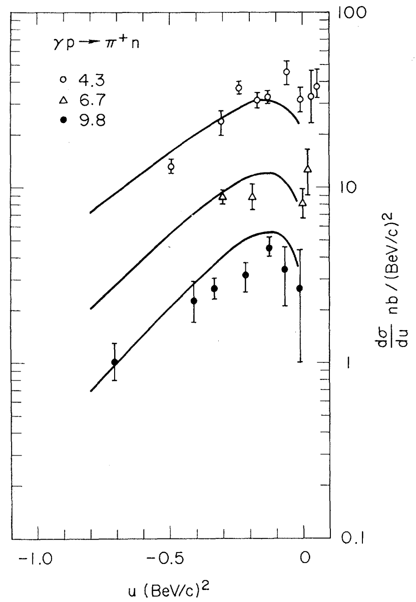

FIG. 3. Photoproduction of π^+ mesons at 4.3, 6.7, and 9.8 BeV. The curves correspond to fits using only the Δ trajectory. The data were taken at the Stanford Linear Accelerator Center by Anderson et al.

isovector part of the process $\gamma p \to \pi^+ n$ is related to $\rho^0 p \to \pi^+ n$, which in turn is related by isospin invariance and time reversal to $\pi^- p \to \rho^0 n$ through the relation[9]

$$\frac{d\sigma}{du}(\gamma p \to \pi^+ n) = \frac{e^2}{f\rho}\frac{d\sigma}{du}(\pi^- p \to \rho^0 n)\rho_{11}{}^{\text{hel}}(u) + \begin{matrix}\text{isoscalar}\\ \text{contribution}\end{matrix}, \quad (13)$$

where $\rho_{11}{}^{\text{hel}}(u)$ is the density matrix element. From Fig. 3 and Eq. (13) we can predict $[d\sigma(\pi^- p \to \rho^0 n)/du]\rho_{11}{}^{\text{hel}}(u)$ for several incident energies and scattering angles. Preliminary results for all existing data of $\pi^- p \to \rho^0 n$ at 4 BeV are in agreement with the vector-dominance model.[10] The above discussion does not necessarily imply that $\pi^- p \to \rho^0 n$ is dominated by the Δ trajectory. On the contrary, the density matrix element[10] $\rho_{11}{}^{\text{hel}} \approx 0.25$ allows considerable contribution from the nucleon trajectory. In fact, if the ρp interaction is similar to the πp interaction, we expect the nucleon trajectory to be very important in the charge-exchange modes. The obvious implications will be more structure in the angular distribution and a faster fall of the differential cross section with increasing energy.

(c) The absence of a sharp dip at $u \approx -0.2$ (BeV/c)^2 in $\gamma p \to \pi^0 p$ eliminates the possibility that it is purely nucleon exchange. Could it then be mostly Δ exchange? A pure Δ exchange involves only the isovector part of the photon and it implies that

$$\left(\frac{d\sigma}{du}\right)_{\gamma p \to p\pi^0} = 2\left(\frac{d\sigma}{du}\right)_{\gamma p \to \pi^+ n}. \quad (14)$$

Recent experimental results show that[11]

$$\left(\frac{d\sigma}{du}\right)_{\gamma p \to p\pi^0} \approx \left(\frac{d\sigma}{du}\right)_{\gamma p \to \pi^+ n} \quad (15)$$

for -0.3 (BeV/c)$^2 \leq u \leq 0.0$ (BeV/c)^2. Therefore both the Δ and the nucleon contribute in $\gamma p \to \pi^0 p$. Equation (15) implies that the contribution of the $I = \frac{1}{2}$ trajectories at the position of the dip cannot be equal to zero, but that there is a remainder which interferes destructively with the Δ. Such a remainder can come from either an imaginary part in the nucleon trajectory, or from some other $I = \frac{1}{2}$ trajectory. We predict for this process (1) the contribution of the $I = \frac{1}{2}$ trajectories to be comparable with the contribution of the Δ and (2) small minima at $u \approx 0$ and a characteristic energy dependence similar to that given by Eq. (12).

We conclude that a simple Regge-pole model can account for the pion-proton and $\gamma p \to \pi^+ n$ data. There are two features in the model which should be present in all analyses: (a) Estimates of the residues can be obtained from the Born diagrams, and (b) the half-angles should be present, eliminating shrinkage at small u but creating some other noticeable effects. Both aspects could and should be checked in other processes. On the experimental side, the photoproduction data should be extended to larger ranges of u and s. These processes allow studies of interferences between several trajectories. Production of isoscalar mesons allows the study of $I = \frac{1}{2}$ trajectories alone. With new data accumulating, this should become a very attractive region in checking the Regge-pole model.

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