FINAL-STATE INTERACTION ENHANCEMENTS IN WEAK THREE-BODY DECAYS*

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Decay-rate enhancements are calculated in an exactly soluble model of a weak decay into three hadrons. Very large enhancements and de-enhancements can be produced by a purely attractive, moderately strong, pairwise, final-state interaction. The enhancements are also very sensitive to the localization of the bare weak-decay amplitude.

Direct analytical methods have provided little insight into final-state interaction (FSI) effects in weak three-body decays. We have therefore investigated numerically these effects in an exactly soluble model and present here results showing strong dependence of decay rates on the FSI. The model we studied describes the weak decay of a 0^+ particle (the G) into three identical 0^+ daughters (the H's). To simplify the three-body dynamics, we assumed nonrelativistic kinematics and separable s-wave H-H interactions.[1]

The (separable) fully off-shell H-H t matrix has the form

$$\langle \vec{q}'|t(W)|\vec{q}\rangle = v(q')\tau(W)v(q). \quad (1)$$

We chose a vertex function of the form

$$v(q) = (4\pi)^{-1/2}q^2(q^2+1)^{-2}, \quad (2)$$

which is capable of producing an s-wave H-H resonance. The range of $v(q)$ was taken as the unit of length, hence the 1 in Eq. (2). We also set $\hbar = M_H = 1$. The two-body propagator $\tau(W)$ appearing in Eq. (1) is

$$\tau(W) = -[(\pi/32\nu) + \int d^3q\, v^2(q)(W-q^2)^{-1}]^{-1}, \quad (3)$$

where ν is the (dimensionless) potential strength chosen so that $\nu = 1$ corresponds to a zero-energy H-H bound state. The H-H phase shifts for various values of ν over the energy range relevant to our three-body calculations are shown in Fig. 1. We see that for $\nu \sim 0.7$ the phase-shift is large but nonresonant over a wide range of energies, whereas for $\nu \gtrsim 0.9$ there is a sharp low-energy H-H resonance. As $\nu \to 1$, the resonance energy goes to zero and the width vanishes.

The matrix element for the decay $G \to 3H$ may be written (to first order in the weak interaction and to all orders in the strong interactions)[2]

$$M = \langle \psi_{3H}^{(-)}|\mathcal{H}_{\text{wk}}|G\rangle, \quad (4)$$

where $\psi_{3H}^{(-)}$ is the appropriate scattering wave function of the strongly interacting $3H$ system.

The matrix element M is represented as a formal expansion in powers of the H-H t matrix in Fig. 2(a). Defining the auxiliary amplitude, f, for the decay of G into an H plus a correlated H-H pair as in Fig. 2(b), we obtain the integral equation of Fig. 2(c), which sums the infinite series of diagrams of Fig. 2(a). This off-shell one-dimensional integral equation arises from the introduction of separable potentials into the three-body problem; its solution is equivalent to that of the three-body Schrödinger equation.[1] This equation for f was solved numerically using the contour-deformation method.[3] In the rest system of the G, the constraints of energy, momentum, and angular-momentum conservation make the partial decay rate a function of two variables. We choose the kinetic energies E_1 and E_2 of two of the H's. Combining the kinematic constraints with the Bose statistics of the final state, we obtain in terms of the functions f, v, and τ the fol-

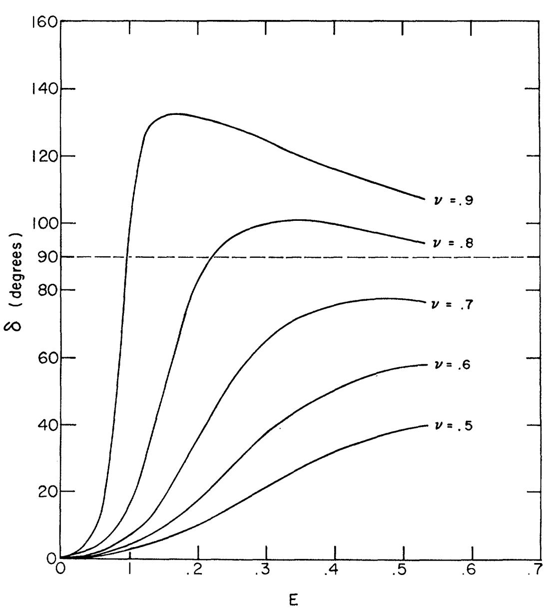

FIG. 1. The H-H phase shifts as a function of energy for various coupling strengths ν. Units are described in the text.

lowing expression for the partial decay rate:

$$\mathcal{R} \equiv (\partial^2 R/\partial E_1 \partial E_2) = N\theta(E_1E_2-(\tfrac{1}{2}E-E_1-E_2)^2)|F(E_1, E_2; E)+v(E-\tfrac{3}{2}E_1)\tau(E-\tfrac{3}{2}E_1)f(E_1)$$
$$+v(E-\tfrac{3}{2}E_2)\tau(E-\tfrac{3}{2}E_2)f(E_2)+v(\tfrac{3}{2}(E_1+E_2)-\tfrac{1}{2}E)\tau(\tfrac{3}{2}(E_1+E_2)-\tfrac{1}{2}E)f(E-E_1-E_2)|^2, \quad (5)$$

where E is the total kinetic energy released in the decay. The constant N appearing in Eq. (5) contains purely numerical factors, including the weak-decay coupling constant. Since we are only interested in enhancements, we do not specify N. The function F appearing in (5) is the bare weak-decay amplitude

$$F(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \langle \vec{p}_1, \vec{p}_2, \vec{p}_3 | \mathcal{H}_{wk} | G \rangle, \quad (6)$$

subject to energy and momentum conservation. F is completely symmetric in the H coordinates. We chose the simple form

$$F(\vec{p}_1, \vec{p}_2, \vec{p}_3) = \beta^2[\beta^2 + \tfrac{1}{2}(p_1^2+p_2^2+p_3^2)]^{-1}, \quad (7)$$

where β^{-1} is a length determining the spatial extension of the bare weak amplitude.

The enhancement of the three-body decay rates due to strong final-state interactions is defined as

$$\mathcal{E}(\nu, E, \beta^2) = \int dE_1 dE_2 \, \mathcal{R}(\nu, E, \beta^2) / \int dE_1 dE_2 \, \mathcal{R}(0, E, \beta^2), \quad (8)$$

that is, as the total decay rate divided by the total rate in the absence of strong interactions. We have studied this enhancement for two values of β^2 (0.1 and 10), three values of the kinetic energy (0.0466, 0.107, and 0.400), and many values of ν between 0 and 1. When $\beta^2 = 0.1$ the weak vertex is spread out in configuration space compared with the range of the strong interaction. Conversely, $\beta^2 = 10$ corresponds to a nearly point vertex. The energies 0.0466 and 0.107 were chosen so that when the two-body resonances could overlap, they would be quite narrow and quite broad, respectively. The third value, $E = 0.4$, was chosen to favor the strong but nonresonant interactions occurring at $\nu \lesssim 0.75$.

In Fig. 3(a) we have plotted the enhancement as a function of ν for $E = 0.0466$. We first note the striking difference between the enhancement for $\beta^2 = 10$ and that for 0.1. From Eq. (4) we see that when $\beta^2 = 10$, the decay rate is sensitive to the three-body wave function at small distances, while for $\beta^2 = 0.1$ the decay rate samples a large volume of the wave function. Clearly the strong interactions can (and do) affect the former more than the latter. For $\beta^2 = 10$ the enhancement varies over five orders of magnitude, from a de-enhancement of less than 0.03 to a maximum of 2500. We emphasize that this surprisingly large range of $\mathcal{E}$, including as it does substantial de-enhancements, results from a purely attractive H-H interaction of only moderate strength. The enormous de-enhancement near $\nu = 0.4$ arises from nearly total destructive interference beween the first and second terms of Fig. 2(b). The first value of ν for which final-state H-H resonances can occur in the kinematically allowed region (Dalitz plot) at this energy is $\nu = 0.94$. As ν increases from 0.94 to 1 and the H-H resonance

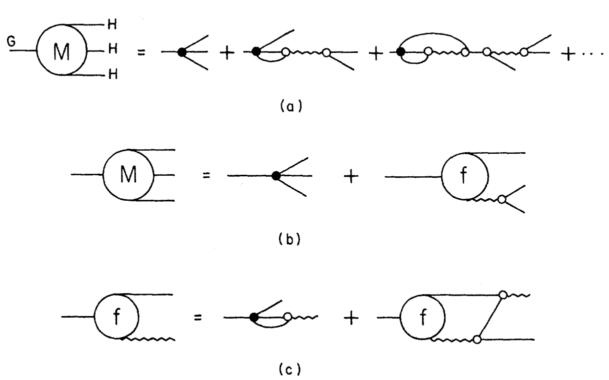

FIG. 2. (a) Diagrammatic representation of the multiple scattering expansion for $G \to 3H$ weak decay. (b) Diagrammatic definition of f, the amplitude for decay of the G into H plus correlated H-H pair. (c) Diagrammatic representation of the off-shell integral equation for f.

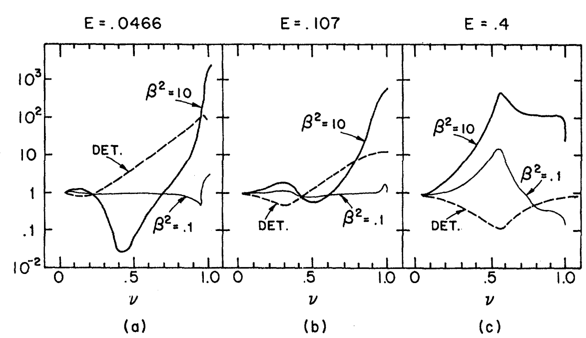

FIG. 3. The enhancements in $G \to 3H$ decay as a function of the H-H coupling strength ν for weak-decay vertex ranges $\beta^2 = 10$ and $\beta^2 = 0.1$. Also shown is the absolute value for the three-body Fredholm determinant. These are show for kinetic energies of (a) 0.0466, (b) 0.107, and (c) 0.400.

bands move through the Dalitz plot crossing in various places, we see no evidence for anomalous behavior of the enhancement. The very large enhancements for $\nu \gtrsim 0.9$ come almost entirely from the last rescattering. For this case ($E = 0.0466$, $\beta^2 = 10$, $\nu > 0.9$) the solution f of the integral equation Fig. 2(c) has a magnitude close to that of the inhomogeneous term, but differs from it in phase by ~90°. Thus the impulse approximation (one final-state rescattering) would give the correct magnitude for $\mathcal{E}$. We must not conclude from this that it is safe to treat these problems in the Born or impulse approximations. In the case $\beta^2 = 0.1, E = 0.0466$, there is actually a region of de-enhancement when the H-H resonance bands are in the Dalitz plot. Recall in comparing these cases that the strong interactions are identical. To produce a de-enhancement in the presence of overlapping resonances, the sum of all rescattering corrections must alter f so that the first and second terms in Fig. 2(b) can interfere destructively. In fact we can show that the Neumann series for Fig. 2(c) diverges strongly for $\nu > 0.5$; so these results cannot be reproduced in any finite order of perturbation theory.

We show the enhancements for $E = 0.107$ and $E = 0.4$ in Figs. 3(b) and 3(c), respectively. Again we note a marked difference between $\beta^2 = 10$ and $\beta^2 = 0.1$. For $E = 0.107$ the resonances enter the Dalitz plot around $\nu = 0.88$; for $E = 0.4$, they enter around $\nu = 0.78$ although at this value of ν the H-H resonance is very broad. As ν approaches unity and the resonance energies approach zero, the diminishing three-body phase-space causes the enhancement to decrease, accounting for the drop in $\mathcal{E}$ near $\nu = 1$ in Figs. 3(b) and 3(c), since the f's vary imperceptibly there. This diminution of $\mathcal{E}$ is also present at $E = 0.0466$ but occurs too close to $\nu = 1$ to appear in Fig. 3(a).

Also plotted in Fig. 3 are the absolute values of the Fredholm determinants of the integral equation of Fig. 2(c). Note that the parameter β^2 is associated with the weak-decay amplitude and only enters the integral equation in the inhomogenous term; hence the determinant does not depend on β^2. Each of the determinants has a minimum which corresponds to a three-body resonance. This resonance becomes more pronounced and moves to higher energy with ν. This behavior may seem contrary to intuition, but it occurs also in several other models. There seems to be no reason to believe that all resonances are the analytic continuation of bound states to weaker coupling. In fact there is a three-body bound state in our model which is unrelated to the resonance and whose binding energy can be shown to increase with ν. The effect of the $3H$ resonance on the enhancement is twofold: It produces a local maximum in $\mathcal{E}$ (as a function of ν) and changes the sign of the real part of f, accounting for the interference minimum at slightly higher ν. Whether this resonance maximum, or the interference minimum, or the final-rescattering enhancement, is the most striking feature of $\mathcal{E}$ as a function of ν depends on the detailed behavior of all the amplitudes involved.

We conclude that strong enhancements or de-enhancements can only be produced by FSI when the weak-decay amplitude is strongly localized in space. We further note that it is unnecessary for resonances to appear in the Dalitz plot, or even for the two-body force to produce resonances in order to have large enhancements or de-enhancements. We therefore conclude that it is impossible to make general statements about the effects of strong FSI on any particular weak-decay process in the absence of a detailed dynamical calculation. However, it is reassuring that apparent puzzles such as the anomalously fast $\eta \rightarrow 3\pi$ decay or the apparently inhibited $X^0 \rightarrow \eta\pi\pi$ decay[4] could be accounted for within the framework described here without recourse either to new two-body resonances or to new selection rules.

There are many other interesting features of our simple model and these will be discussed at greater length in a forthcoming article. We also hope to study strong production of hadronic three-body final states. The sensitivity of our results on weak decay to the structure of the bare weak-decay vertex leads us to expect great sensitivity to the mechanism of production.

We wish to acknowledge many interesting discussions with Professor H. Primakoff about the relevance of our calculations to high-energy physics.

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