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NEW APPROACH TO CALCULATIONS OF NUCLEAR MOMENTS OF INERTIA

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A new approach to moment-of-inertia calculations yields a formula superior to the usual cranking-model formula and casts doubt on the apparent success achieved with pairing-force theory in cranking-model calculations.

In this Letter we present an approach to the problem of calculating the moments of inertia of deformed nuclei based on Lipkin's[1] suggestion for treating collective motions in a many-body system. This approach leads to an explicit expression for the moment of inertia which seems to be superior to the usual cranking-model[2] (CM) formula. Our calculations, both in the transuranic region and for nuclei in the s-d shell, indicate that the success achieved by Nilsson and Prior[3] in caluclating the moments of inertia of heavy deformed nuclei using the CM with pairing forces (Belyaev's formula) may be more fortuitous than real.

The argument begins by accepting as an experimental fact the existence of well-defined ground-state rotational bands in the low-energy spectra of even-even deformed nuclei,

$$E_J = E_0 + \lambda J(J+1)\hbar^2, \tag{1}$$

where $\lambda = 1/2I$ and I is the so called moment-of-inertia parameter. Following Lipkin an intrinsic Hamiltonian H' is constructed by subtracting the operator associated with the collective energy,

$$H' = H - \lambda J_{\mathrm{op}}{}^2. \tag{2}$$

Since every state in the rotational band is degenerate with respect to H', any linear combination $|\psi\rangle$ of eigenstates of H in the band is also an eigenstate of H' with eigenvalue E_0. It can then be readily shown that the conditions

$$\langle\psi|H'-E_0|\psi\rangle = \langle\psi|(H'-E_0)J_{\mathrm{op}}{}^2|\psi\rangle = 0 \tag{3}$$

are valid. Conditions similar to (3) were first employed by Goodfellow and Nogami[4] in connection with their application of Lipkin's suggestion to nuclear pairing-force theory.

The conditions (3) may be written in more illuminating form as[5]

$$E_0 = \langle\psi|H|\psi\rangle - \lambda\langle\psi|J_{\mathrm{op}}{}^2|\psi\rangle \tag{4a}$$

and

$$\lambda = \frac{\langle\psi|HJ_{\mathrm{op}}{}^2|\psi\rangle - \langle\psi|H|\psi\rangle\langle\psi|J_{\mathrm{op}}{}^2|\psi\rangle}{\langle\psi|J_{\mathrm{op}}{}^4|\psi\rangle - \langle\psi|J_{\mathrm{op}}{}^2|\psi\rangle\langle\psi|J_{\mathrm{op}}{}^2|\psi\rangle}. \tag{4b}$$

In practice the many-body intrinsic wave function $|\psi\rangle$ cannot be determined; so we are forced to calculate the averages in (4) by means of some trial wave function which may be obtained, for example, from a variational calculation. In this case expression (4b) amounts to a consistency condition imposed on the trial wave function.

It is generally believed[6] that the deformed ground-state wave function in a proper Hartree-Fock (HF) calculation is a reasonably good example of an intrinsic state of the type $|\psi\rangle$. Accordingly, such calculations should yield a reasonable value for the rotational parameter λ. Introducing a HF basis, expression (4b) can be written as[7]

$$\lambda = \frac{1}{2I} = \frac{\sum_4 \langle 0|H|4\rangle\langle 4|J_{\mathrm{op}}{}^2|0\rangle}{\sum_4 \langle 0|J_{\mathrm{op}}{}^2|4\rangle\langle 4|J_{\mathrm{op}}{}^2|0\rangle}, \tag{5}$$

where $|0\rangle$ is the HF ground state and the sum is over all two-particle, two-hole states.

Our main interest, however, is heavy nuclei, where HF calculations are virtually impossible.

Encouraged by the large overlap between Nilsson wave functions and HF wave functions in the s-d shell,[8] we have carried out calculations in the transuranic nuclei using residual pairing forces in BCS approximation and Nilsson wave functions. Within the framework of the nuclear pairing-force theory the state $|0\rangle$ in expression (5) becomes the BCS ground state and the sum is over the complete set of four quasiparticle states. For this case we obtain the following expression for the moment of inertia I from formula (5):

$$I=\frac{1}{2\lambda}=\frac{\left[\sum_{1,2,\alpha} j_{12\alpha}{}^{x} j_{21\alpha}{}^{x}(u_{1\alpha}v_{2\alpha}-u_{2\alpha}v_{1\alpha})^{2}\right]^{2}+\Delta D}{2\sum_{1,2,\alpha}\Delta E_{12\alpha} j_{12\alpha}{}^{x} j_{21\alpha}{}^{x}(u_{1\alpha}v_{2\alpha}-u_{2\alpha}v_{1\alpha})^{2}}. \tag{6}$$

In this expression, the sums indicated by the indices 1 and 2 are over all single-particle states and α is the isospin index; the $j_{12\alpha}{}^{x}$ are matrix elements of the single-particle angular momentum operator (x component) and the $\Delta E_{12\alpha}$ are energies of two quasiparticles,

$$\Delta E_{12\alpha}\equiv E_{1\alpha}+E_{2\alpha}-2G_{\alpha}u_{1\alpha}v_{1\alpha}u_{2\alpha}v_{2\alpha}, \tag{7}$$

where

$$E_{1\alpha}=[(\epsilon_{1\alpha}-\lambda_{\alpha})^{2}+\Delta_{\alpha}{}^{2}]^{1/2}, \tag{8}$$

and the u and v are the usual parameters of the BCS trial wave function. Finally, the quantity ΔD, which depends upon the u and v and the $j_{12\alpha}{}^{x}$, appears to be small in all the numerical calculations and may be omitted. In the limit of no pairing, expression (6) immediately reduces to the simple expression[9]

$$I^{(0)}=\frac{(\sum_{1,2,\alpha} j_{12\alpha}{}^{x} j_{21\alpha}{}^{x})^{2}}{\sum_{1,2,\alpha}(\epsilon_{1\alpha}-\epsilon_{2\alpha}) j_{12\alpha}{}^{x} j_{21\alpha}{}^{x}}. \tag{9}$$

In Eq. (9), the index 1 refers to particle states and the index 2 to hole states. Formulas (6) and (9) are to be compared with their CM counterparts, the Belyaev formula,[10]

$$I_{c}=\sum_{1,2,\alpha}\frac{j_{12\alpha}{}^{x} j_{21\alpha}{}^{x}(u_{1\alpha}v_{2\alpha}-u_{2\alpha}v_{1\alpha})^{2}}{E_{1\alpha}+E_{2\alpha}}, \tag{10}$$

and its simplification in the limit of no pairing,

$$I_{c}{}^{(0)}=2\sum_{1,2,\alpha}\frac{j_{12\alpha}{}^{x} j_{21\alpha}{}^{x}}{\epsilon_{1\alpha}-\epsilon_{2\alpha}}.$$

It is interesting to note that the neutron contribution to the moment of inertia is independent of the proton contribution in the CM formulas (10) and (11) in contrast to formulas (6) and (9) where these contributions are mixed.

If one believes that the long-range part of the two-body forces has been well accounted for by the HF approximation and that the short-range part can be represented approximately by a pairing force, then formula (6) should be a valid expression for the rotational parameter. Further, if for some reason the short-range part of the two-body forces can be neglected, then the simpler formula (9) should give a reasonable value for I. Expressions (6) and (9) have the advantage over the CM formulas that they are derived entirely on a quantum-mechanical basis and in particular, formula (9) does not possess the "unphysical singularities" which occur in formula (11) when unoccupied and occupied states are very close together. Moreover, we can show in simple models that expression (9) gives approximately $\frac{1}{2}$ the rigid-body values for I in contrast to the CM formula (11) which gives I_{rigid}. Hence, formula (9) gives closer agreement with experiment.

In order to re-examine, from the point of view presented here, the role of pairing forces in the calculation of the rotational parameter, we have carried out detailed numerical work to compare formula (6) with (9) and with the corresponding CM formulas. Figure 1 shows the experimental results for the transuranic nuclei and also for our calculations using both formula (6) and formula (9). We have also repeated the calculations of Nilsson and Prior using Belyaev's formula with the choice of parameters suggested by Szymanski.[11] Our calculations with Belyaev's formula (which confirm the Nilsson-Prior results) are shown in Fig. 1 for comparison. It is quite remarkable that the values estimated from the simple expression (9) give extremely good agreement with experiment, except for the transition

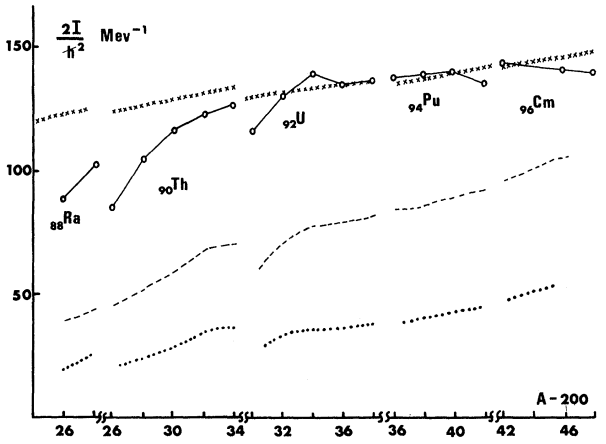

FIG. 1. Experimental values of the moment of inertia $2I/\hbar^2$ (circles) compared with predictions calculated from Belyaev's formula (10) (dashes), from our formulas (6) (dots), and (9) (crosses).

regions between spherical and deformed nuclei where the rotational model fails in any case. However, the results obtained from formula (6) are rather unsatisfactory. This seems to suggest that the importance of pairing forces in the calculation of the rotational energy is overemphasized.

It is even more remarkable that expression (9) appears also to give good agreement with experiment for the moments of inertia of s-d shell nuclei. Figure 2 shows a comparison of the experimental values for Ne^{20}, Mg^{24}, and Si^{28} with our calculations using expression (9) and Nilsson wave functions. Also shown in Fig. 2 are the CM calculations which we carried out using Nilsson wave functions. The agreement with experiment which we obtained using expression (9) would be less satisfactory, however, if we had used HF theory because of the gap which appears in the spectrum; this gap, which does not exist in the unshifted Nilsson spectrum, presumably accounts for the differences in Fig. 2 between the values obtained in our CM calculations and those of Kelson and Levinson.[8]

The present investigation seems to cast doubt on the significant role attributed to pairing forces in moment-of-inertia calculations. If one accepts that the present formulation is superior to the CM, then some of the quantitative judgments based upon the CM (e.g., the results of Nilsson and Prior), in which the pairing force plays a very essential part, should be treated with more reservation.

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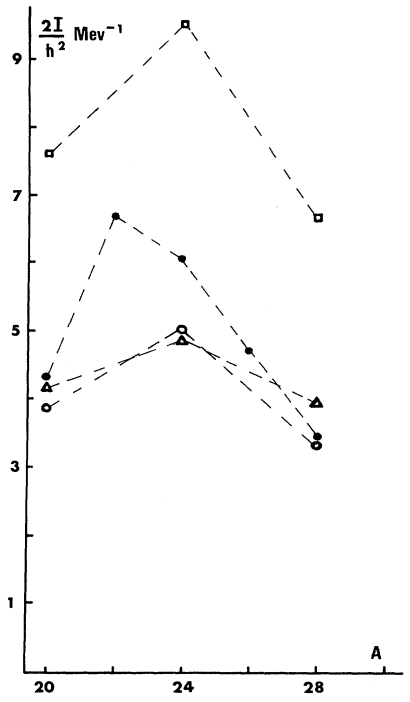

FIG. 2. Experimental values of the moment of inertia $2I/\hbar^2$ (circles) compared with predictions calculated from CM formula (11) using the HF wave functions of Kelson and Levinson (solid dots), from formula (11) with Nilsson wave functions (squares), and from our formula (9) using Nilsson wave functions (triangles).

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OAKES THEORY OF WEAK INTERACTIONS AND PARITY-NONCONSERVING NUCLEAR FORCES

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The parity-nonconserving nucleon-nucleon potential is calculated from the Oakes theory of weak interactions on the assumption that π and ρ exchange terms dominate. It is found that this theory predicts a circular polarization of $\pm(1.8\pm0.9)\times10^{-4}$ for 482-keV γ ray from ^{181}Ta. The observed polarization is $-(0.06\pm0.01)\times10^{-4}$.

Recent experiments[1-4] indicate the existence of a parity-nonconserving nucleon-nucleon potential. Believing that this is a manifestation of the $\Delta S=0$ nonleptonic weak interactions, we obtain a further test of theories of the weak interaction. Unfortunately the present experiments all involve heavy nuclei, and there is a significant possibility that the uncertainties in the nuclear physics destroy the reliability of the predictions. Until experimental information becomes available on simpler nuclear systems,[5] we feel justified in proceeding on the belief that the calculations are reliable at least as to orders of magnitude. Support for this view may be drawn from the fact that the calculations of Wahlborn[6] and Manqueda and Blin-Stoyle,[7] using different approximations for nuclear physics, arrive at nearly identical results, -0.6×10^{-4} and -0.7×10^{-4}, for the polarization of the 482-keV γ ray in ^{181}Ta.

Assuming that the dominant contributions to the parity-nonconserving (pv) NN potential are weak boson exchange and pion exchange,[8] we previously calculated the parity nonconservation predicted by the Cabibbo Hamiltonian[9] and the d'Espagnat Hamiltonian.[10] The Cabibbo Hamiltonian was consistent with the data while the d'Espagnat Hamiltonian was in disagreement by almost an order of magnitude. In the present paper we report the results of a similar calculation using the Hamiltonian recently proposed by Oakes,[11] which introduces neutral currents as a mechanism for CP nonconservation.

The Oakes Hamiltonian H_O may be written as a sum of charged and neutral current terms

$$H_O=H_C+H_N. \tag{1}$$

The term involving the charged currents is the standard Cabibbo Hamiltonian.[12] The neutral current term is also of current×current type:

$$H_N=(G/\sqrt{2})J_\lambda^{(0)}J^{(0)\lambda}, \tag{2}$$

where G is the Fermi constant and $J^{(0)}$ is a neutral current which we call the Oakes current. It is of $V+A$ form and is universal in the sense that it is the sum of a hadronic current $J_{\text{had}}^{(0)}$,

$$J_{\text{had}}^{(0)}=\cos\varphi(V+A)^{3-\sqrt{38}}+2\sin\varphi(V+A)^{7}, \tag{3}$$

and a leptonic current. φ is found to be approximately 10^{-3} from the observed CP nonconservation in the neutral kaon system, which is proportional to $\sin\varphi$. In calculating the parity-nonconserving effects we neglect this small CP nonconservation.

The π-exchange graph is calculated from the weak parity-nonconservation $NN\pi$ vertex,[13] which is estimated by an extension of the Suzuki-Sugarawa current-algebra analysis of the pv nonleptonic hyperon decays.[14]

We relate $\langle N_f\pi^\alpha|H_O^{\text{pv}}|N_i\rangle$ to $\langle N_f\pi^\alpha|H_C|N_i\rangle$ by noting the following:

(i) The part of the H_N^{pv} which contributes to the hyperon decays is of order $\sin\gamma$. Such terms are neglected in our analysis.[15] Thus the reduced matrix elements $\langle B\|(H_O^{\text{pv}})\underline{8}\|B\rangle$ calculated from the hyperon decay data are equal to the reduced matrix elements of H_C^{pv} calculated from the same data. In particular $\langle B\|(H_O^{\text{pv}})\underline{27}\|B\rangle\simeq0$.

(ii) Only the $\Delta I=1$ part of H^{pv} contributes to $\langle N_f\pi^\alpha|H^{\text{pv}}|N_i\rangle$ if CP is conserved.[13,16] With no contribution from $(H^{\text{pv}})\underline{27}$ the only contribution is from $(H^{\text{pv}})_3\underline{8}$ – the third component of the oc-