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# REGGE-POLE MODEL WITH CUTS GENERATED BY ABSORPTION FOR THE REACTION $\pi^+n \rightarrow \omega p$†

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The reaction $\pi^+n \rightarrow \omega p$ is analyzed in terms of a model based on the exchange of the $\rho$ Regge pole and the associated cuts generated by absorption. The data, in particular the absence of a dip in the differential cross section and the large positive values of the density matrix element $\rho_{00}$, can be well described by this model only if the amplitudes do not vanish at nonsense, wrong-signature points.

The experimental data for the reaction[1] $\pi^+n \rightarrow \omega p$ have two outstanding features: (1) The density matrix element $\rho_{00}$ of the $\omega$ is large, $\rho_{00} \approx 0.4$. (2) There is no dip in the differential cross section. Since the only simple exchange mechanism allowed in this reaction is $\rho$ exchange, one expects[2] that $\rho_{00}=0$. Since the $\pi\omega\rho$ vertex must vanish when $\alpha_\rho=0$, one also expects[3] in the framework of the standard theory of Reggeization[4] that all helicity amplitudes and thus the differential cross section should have a dip at the nonsense, wrong-signature (NWS) point $\alpha_\rho(t)=0$, or at $t\approx -0.6$ (GeV/$c$)$^2$. The violation of the predictions shows that the $\rho$ pole alone is not able to explain the data. In this Letter, we show that the data can be understood in terms of a model including in addition to the $\rho$ pole the associated Regge cuts generated by absorption,[5,6] provided that the amplitudes do not have NWS zeros; i.e., the residue functions have Mandelstam-Wang fixed poles.[7]

The reaction $\pi^+n \rightarrow \omega p$ has also been analyzed in terms of the absorption model[8] and the pure Regge-pole model.[9] The absorption model can explain the large value of $\rho_{00}$ by the unnatural parity exchange caused by the absorption, but suffers from the usual difficulties with the energy dependence. In the pure Regge-pole model one is forced to introduce a secondary trajectory of unnatural parity corresponding to the $1^+$ particle $B(1220)$. The fit to the data then gives values for the trajectory and coupling parameters of the $B$ which are of the same order of magnitude $[\alpha_B(0)\approx\alpha_\rho(0)]$ as those of the $\rho$. This is not pleasing, since the $B$ does not seem to contribute strongly to other reactions. For a trajectory passing through $m_B^2$ one can only have $\alpha_B(0) \approx \alpha_\rho(0)$ if the slope of the $B$ trajectory is very small compared with most Regge-pole slopes; conversely, with a normal slope one would have $\alpha_B(0)\approx -\frac{2}{3}$, and at high energies (e.g., $s\approx 10$ GeV$^2$) there should be little contribution from $B$ exchange. Although one cannot exclude the $B$, we shall show that one can do well without it.

We shall calculate the cut correction by assuming in the conventional way that the absorption gives rise to diffraction elastic scattering which is spin independent and equal in the initial and the final state ($\omega p$ elastic scattering is not known).

We write the full amplitude in the form

$$M=M^\rho+\lambda M^{\rho p},$$

where $M^\rho$ is the $s$-channel $\rho$ Regge-pole exchange,

$M^{\rho p}$ is the Regge cut, and $\lambda$ (the only parameter introduced apart from those in the pole terms) is a parameter to account for the effect of absorptive diffraction inelastic scattering. $M^{\rho p}$ is given by[6]

$$M_{\lambda_c\lambda_d;\lambda_a\lambda_b}{}^{\rho p}(s,z) = -\frac{iq}{16\pi^2 W}\int d\Omega\, M_{\lambda_c\lambda_d;\lambda_a\lambda_b}{}^{\rho}(s,x) M^{\mathrm{el}}(s,y)\cos n\varphi, \tag{1}$$

where $W$ and $q$ are the c.m. energy and magnitude of the c.m. momentum, $d\Omega = dx d\varphi$, $x$ is the scattering angle for the $\rho$ exchange term, and $y$ is the scattering angle for the elastic scattering. The variables $x$, $y$, and $\varphi$ are related by $y = xz + (1-x^2)^{1/2}(1-z^2)^{1/2}\cos\varphi$. The quantum number $n = |(\lambda_c - \lambda_d) - (\lambda_a - \lambda_b)|$ gives the net amount of helicity flip. $M^{\mathrm{el}}$ is the inital or final state elastic scattering amplitude, defined below.

We note the following properties[6] of the total amplitude $M = M^\rho + \lambda M^{\rho p}$: (1) Only one new parameter, $\lambda$, is introduced; all other parameters are in $M^\rho$ and $M^{\mathrm{el}}$. (2) The cut and pole contributions are entirely separate, different kinds of singularities in the $J$ plane; there is no double counting.[6] (3) The phases of $M^\rho$ and $M^{\mathrm{el}}$ are roughly opposite so that one has destructive interference in $|M^\rho + \lambda M^{\rho p}|$. (4) The size of the cut depends significantly on the helicities through the quantum number $n$ [cf. Eq. (1)], so that $|M^\rho| \approx \lambda |M^{\rho p}|$ at different values of $t$ for different helicities, and in the sum over helicities

$$\sum_{\mu\lambda'\lambda} |M^\rho + \lambda M^{\rho p}|^2,$$

different terms have dips or breaks at different values of $t$. This leads to a fairly smooth $d\sigma/dt$. (5) If the amplitudes have NWS zeros, all pole terms in the sum

$$\sum_{\mu\lambda'\lambda} |M^\rho + \lambda M^{\rho p}|^2$$

are strongly suppressed at the same value of $t$ and the cut correction is not sufficient to fill in the dip in $d\sigma/dt$.

The parameter $\lambda$ arises from the contribution of coherent inelastic states in the intermediate state between the quantum-number exchange and the diffraction elastic scattering; it is fully discussed in Ref. 6. It is shown there that the physical interpretation of $\lambda$ requires its value to be about 2, with an allowed reasonable range of perhaps $\pm 30\%$. Analyses[6] (under the same assumptions as the present work) of $\pi^- p \to \pi^0 n$, $\gamma p \to \pi^+ n$, and backward $\pi^\pm p$ elastic scattering all seem to give a value within $30\%$ of $\lambda = 2$.

We parametrize the amplitudes as follows. The spin-independent elastic-scattering matrix element $M^P$ is written in the form

$$M^{\mathrm{el}}(s,t) = -i2qW\sigma_T e^{At/2}, \tag{2}$$

where $A[\approx 7.5\ (\mathrm{GeV}/c)^{-2}]$ measures the width of the forward diffraction peak and $\sigma_T$ is the $\pi N$ total cross section, $\sigma_T \approx 24$ mb.

To obtain the $\rho$-pole contributions to the $s$-channel amplitudes (we write $M_{\lambda'\lambda}{}^{\mu} = M_{\mu\lambda';\lambda}{}^{\rho}$), we go to the $t$ channel, impose the requirement that only the quantum numbers of the $\rho$ occur there, take only the Regge-pole contributions, and cross back to the $s$ channel.[2] The resulting Regge-pole amplitudes can be written as

$$\begin{aligned} &M_{+-}{}^{0} = M_{++}{}^{0} = 0,\\ &M_{-+}{}^{+} = -M_{+-}{}^{+} = -i\beta_1\sqrt{2}\, stT/M^4,\\ &M_{--}{}^{+} = M_{++}{}^{+} = i(\beta_1-\beta_2)2\sqrt{2}\, s(-t)^{1/2}T/M^3, \end{aligned} \tag{3}$$

where

$$T = \left(\frac{s}{s_0}\right)^{\alpha_\rho(t)-1}\Gamma(-\alpha_\rho(t))[1-e^{-i\pi\alpha_\rho(t)}]$$

and the parameters are the $\rho$ trajectory $\alpha_\rho(t)$, the energy scale $s_0$, and the two residue functions $\beta_1$ and $\beta_2$. The remaining six amplitudes are obtained by parity conservation: $M_{-\lambda',-\lambda}{}^{-\mu} = (-)^{\lambda-\lambda'+\mu}M_{\lambda'\lambda}{}^{\mu}$. The amplitudes (3) have the following properties: (i) They are the leading terms for large $s$. (ii) In the $\omega$ rest frame $\rho_{00} = 0$, $\mathrm{Re}\rho_{10} = 0$ [Eq. (4)]. (iii) Only longitudinal $\omega$'s occur in the $t$-channel center of mass. (iv) They correspond to an evasive solution[10] of the constraint equations. (v) They are nonzero at $\alpha_\rho = 0$. Introducing the complete amplitudes $\tilde{M}_{\lambda'\lambda}{}^{\mu}$, including absorption, and performing the Lorentz transformation to the $\omega$ rest frame, we find

$$\rho_{00} = \sin^2\chi|\tilde{M}_{+-}{}^{+}+\tilde{M}_{-+}{}^{+}|^2 / \sum_{\mu\lambda'\lambda} |\tilde{M}_{\lambda'\lambda}{}^{\mu}|^2,$$

$$\mathrm{Re}\rho_{10} = \cot\chi\rho_{00}/\sqrt{2},$$

$$\rho_{1,-1} = (4|\tilde{M}_{++}{}^{+}|^2 + |\tilde{M}_{+-}{}^{+}-\tilde{M}_{-+}{}^{+}|^2 - \cos^2\chi|\tilde{M}_{+-}{}^{+}+\tilde{M}_{-+}{}^{+}|^2)/2\sum_{\mu\lambda'\lambda}|\tilde{M}_{\lambda'\lambda}{}^{\mu}|^2, \quad (4)$$

where

$$\sum_{\mu\lambda'\lambda} |\tilde{M}_{\lambda'\lambda}{}^{\mu}|^2 = 2(|\tilde{M}_{+-}{}^{+}|^2 + |\tilde{M}_{-+}{}^{+})^2 + 2|\tilde{M}_{++}{}^{+}|^2)$$

$$= 32\pi\Delta(s, \tilde{M}^2, \mu^2)d\sigma/dt,$$

$$\Delta(x, y, z) = x^2+y^2+z^2-2xy-2xz-2yz,$$

and ($\theta^{(s)}$ is the $s$-channel scattering angle)

$$\sin\chi = \frac{m}{\sqrt{s}}\frac{[\Delta(s, M^2, \mu^2)]^{1/2}}{[\Delta(t, m^2, \mu^2)]^{1/2}}\sin\theta^{(s)}.$$

When performing the fit we take all the parameters $[\alpha_3(t), s_0]$ except $\beta_1$ and $\beta_2$ from the fit to the reaction $\pi^- p \to \pi^0 n$ done in Ref. 6. One combination of $\beta_1$ and $\beta_2$ is obtained by fitting the magnitude of $d\sigma/dt$, so that we have one arbitrary parameter, $\beta_1/\beta_2$, plus the coherent inelastic factor $\lambda$, which is allowed to vary around a value $\lambda \approx 2$ by perhaps 30 % to fit the shape of $d\sigma/dt$ (with no dip) and the density matrix elements.

Since the integrals in Eq. (1) contain a factor $\cos n\rho$ ($n = |\mu-\lambda'+\lambda|$) which oscillates faster for larger $n$, we see that the cut correction is large ($\approx$100 %) for $M_{+-}{}^{+}(n=0)$, intermediate for $M_{++}{}^{+}(n=1)$, and small ($\approx$0) for $M_{-+}{}^{+}(n=2)$. This is the mechanism for obtaining a large and positive $\rho_{00}$ in this model [Eq. (3)]; for the pole contributions $M_{+-}{}^{+}+M_{-+}{}^{+}=0$, but after the cut corrections $\tilde{M}_{+-}{}^{+}+\tilde{M}_{-+}{}^{+}\neq 0$. Note that this effect will persist to high energies. On the other hand, in the case with no NWS zeros, the destructive interference between the pole and cut contributions does not give a dip since $d\sigma/dt$ gets contributions from different amplitudes, the real and imaginary parts of which have dips in different places, and the resulting curve is smooth. If the amplitudes have NWS zeros, the cut correction is never sufficient to compensate for the zeros of the pole terms, and $d\sigma/dt$ has always a deep dip in this case.

The results of the numerical calculations are shown in Figs. 1 and 2 and compared with data from the Michigan group.[1] It did not appear to us

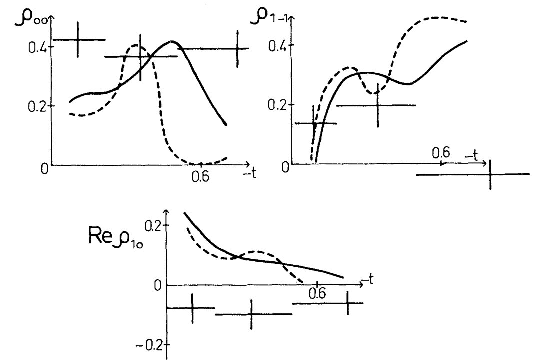


FIG. 1. Comparison of the theoretical curves with experimental data (Ref. 1) at $p_{\mathrm{lab}} = 3.65$ GeV/$c$ for the density matrix elements. The continuous (dashed) curves show the predictions when the amplitudes do not vanish (do vanish) at nonsense, wrong-signature points. The values of the parameters used were $\alpha_\rho = 0.4+1.1t$, $s_D = 1$ (GeV)$^2$, $\lambda = 2.25$, and $\beta_1 = \beta_2$.

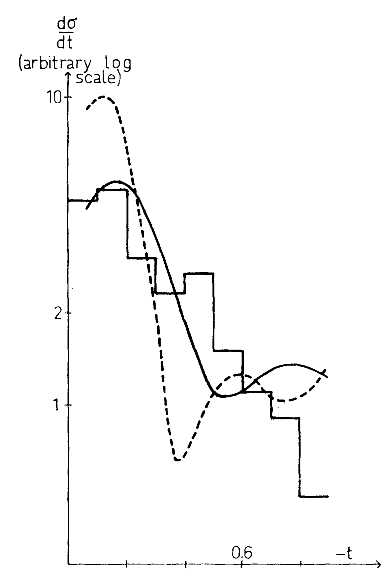


FIG. 2. Same as Fig. 1 for differential cross section.

that the experimental results were sufficiently well established to justify detailed $\chi^2$ fitting of the data. The best fit was obtained for $\beta_1=\beta_2$; the remaining parameters, according to Ref. 6, were taken to be $\alpha_\rho(t)=0.4+1.1t$, $s_0=1$, and $\lambda=2.25$. Note that $\mathrm{Re}\rho_{10}$ is positive here, as it is also in the $\rho+B$ Regge model[9] and in the absorption model for $|t|<0.5$.[8] One experimental group[11] has reported a positive value, $\mathrm{Re}\rho_{10}=+0.25$, with no errors given, for the reaction $\pi^+n\rightarrow\omega p$ at 3.25 GeV/$c$. It is not clear theoretically how one could get $\mathrm{Re}\rho_{10}<0$. The same set of parameters appears to give the best fit for both cases, when NWS zeros are and are not present.

It should be emphasized that the $\rho$ trajectory obtained in the $\pi^-p\rightarrow\pi^0n$ analysis and used here is significantly different from the usual one; the latter is an effective trajectory considering both pole and cut. Thus the dip in the case with NWS zeros is at $\alpha_\rho=0$. Using $\alpha_\rho=0.55+t$ gives less good agreement with data; although the dip is moved out to $-t\approx0.6$, the slope is steep and near the peak of the data the theoretical curve is a full order of magnitude too large.

The form of the theory for $-t\gtrsim0.6$ may not be quantitatively correct because we have neglected a second cut which will interfere destructively with the first in the large $t$ region. This cut arises from the first break in the elastic scattering; so to include it the elastic scattering would be treated as an appropriate sum of two destructively interfering, approximately exponential terms. But the qualitative result that there must be a break in the slope of $d\sigma/dt$ will persist.

We should perhaps mention that the $\omega$-exchange contribution to the reaction $\pi^-p\rightarrow\bar{\rho}p$ cannot be simply related to the present reaction in a model such as ours because the $\rho N\bar{N}$ vertex has a significant tensor coupling, which (it turns out) gives a large unnatural-parity contribution to the cut, while the $\omega N\bar{N}$ vertex is largely charge coupling, which gives mainly a natural-parity cut term. Thus in the $\rho$ production fewer amplitudes contribute significantly and it may be possible that the pole-cut interference gives rise to a dip rather than a break as here. Similarly, the $\rho$ and $\omega$ density matrices will not be the same.

In summary, we have shown that one can understand the data for the reaction $\pi^+n\rightarrow\omega p$ rather satisfactorily by a model with a $\rho$ Regge pole and its associated cut generated by absorption, if the amplitudes have no NWS zeros. If the amplitudes vanish at NWS points, there is probably a further contribution to this reaction in addition to the $\rho$ pole and the associated cuts.

Additional experimental data, designed to clarify the following points, are desirable: (1) It is important to measure $\rho_{00}$ in smaller bins, to detect its shape, and to see if it decreases to smaller values for $-t\approx0.1$. (2) The sign of $\mathrm{Re}\rho_{10}$ must be established; if $\mathrm{Re}\rho_{10}$ is negative, some previously unsuspected mechanism is present. (3) Cross-section data at larger angles are necessary to see if the expected break is present. It is not absolutely clear that the theory requires a break, but there does not appear to be any simple way to avoid it.

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