ing the idea of DHS we would say that the t-channel resonances produce the u-channel Regge trajectories, that is, the backward diffraction peak. Now if we go to the region $u \sim 0$,

$$\Gamma(2-\alpha(t)) = \{2\pi[1-\alpha(t)]\}^{\frac{1}{2}}\{[1-\alpha(t)]/e\}^{-\alpha(t)},$$

and if we accept the idea that $\alpha(t)$ decreases with decreasing t, then $\Gamma(2-\alpha(t))$ is very large. It in fact dominates $(\nu/s_0)^{\alpha(t)}$ and makes the amplitude peak at $u=0$ [Fig. 1(c)].[4]

It no doubt would be a complete folly to attempt to get actual numbers from such a calculation[6]; however, the general features may be correct, and when this idea is put together with that of DHS, we have a general principle which may someday permit a complete bootstrap.

I would like to thank C. Schmid for suggesting this approach to me. In addition I would like to thank Kerson Huang and my colleagues at the Weizmann Institute for helpful discussions.

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[2]K. Huang and S. Pinsky, Phys. Rev. 174, 1915 (1968).

[3]See, e.g., V. Barger and M. Olsson, Phys. Rev. 151, 1123 (1966).

[4]For small t we take $\alpha = 1+t$ and $\gamma(t) = 10$, and for large t we take $\alpha = 1+\frac{1}{2}t$ and $[-2\pi\alpha(t)]^{1/2}\gamma(t) = 1$. This is in no way vital to the argument, it was just the most convenient way to make the backward and forward peaks the same size. Also $s_0 \simeq 5$.

[5]No attempt will be made here to identify the trajectories conclusively; however, it is almost obvious that the trajectory in the region from $t=-2.46$ to -6.0 is the Pomeranchuk and the one at large $-t$ is some kind of cut.

[6]However, some more ambitious attempts along this line have already been tried; see G. Veneziano, Nuovo Cimento 57A, 190 (1968); M. A. Virasoro, to be published.

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INELASTIC SHADOW EFFECTS IN NUCLEAR TOTAL CROSS SECTIONS*

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We calculate the screening correction in proton-nuclear scattering at energies high enough so that inelastic excitations of the beam proton can contribute coherently. Data from proton-proton missing-mass experiments are used. These inelastic contributions to shadowing further reduce the total cross sections from the sum of single nucleon total cross sections. The effect grows from a very small increase in screening at, say 5 GeV/c, to a significant increase at, say, 15 GeV/c.

We have calculated the "shadowing" or double-scattering effects in nucleon-nucleus scattering which are associated with inelastic intermediate states of the nucleon and find a significant decrease in the total cross section with increasing energy. The ordinary shadow effect results from the semiclassical fact that when an object scatters on a composite or extended system, some parts of that system may eclipse other parts of it. In particular, the amplitude for double scattering has opposite sign to that in which just one nucleon is struck (the dominant contribution at small momentum transfer), provided the amplitudes are mainly imaginary in phase, as elastic amplitudes *are* at high energy. The elastic shadow effect can also be thought of as an increased transmission through the nucleus, resulting from some of the flux which is scattered out of the incident beam at a point x being scattered back into it at a point y. The inelastic shadow effects correspond to the same picture except that the system which propagates from x to y is no longer identical to the incident particle. These inelastic states also increase the transmission of incident flux, if the amplitudes are also predominantly imaginary.

The existence of inelastic shadowing was first proposed by Abers et al.[1] for scattering on deuterium. Our prediction for the size of the effect is similar to theirs.

We have used the missing-mass experiment

$$p+p\to p+p^* \tag{1}$$

of Anderson et al.[2] to calculate the shadowing effects associated with the inelastic intermediate states represented by p^*. We predict a significant increase at high energies in the "screening correction" to proton-deuteron scattering, $\sigma_n+\sigma_n-\sigma_d$, over and above the value given by the usual Glauber elastic calculation.[3,4] The increase is about 1.8 mb at 30 BeV. The elastic screening correction is expected to be about 4-5 mb; the total cross section is about 75 mb. (Experimental data indeed seem to show an increase in screening at high energies, but they are open to doubt because of possible systematic errors.[5]) The predicted effects for heavy nuclei are even greater: E.g., the p-Pb total cross section may be reduced by 20% at 30 BeV. Because the inelastic shadow effect increases with nuclear mass number A, we predict a decrease at high energy in the power x which occurs in the approximate rule $\sigma\propto A^x$.[5]

In order for an inelastic state of mass m^* to contribute significantly to the shadow effect, the three-momentum transfer $\Delta\cong(m^2)/2q$ required to produce it must satisfy the coherence requirement $\Delta R\lesssim 1$, where R is the radius of the nucleus and q is the incident momentum in the laboratory frame. The coherence requirement implies that inelastic intermediate states are just beginning to be important at energies of 5-10 BeV. The entire spectrum of m^* may contribute coherently at a few hundred BeV, if that spectrum remains concentrated near threshold, as predicted by diffraction-dissociation models.[6]

In order to estimate the inelastic shadow effects, we modify the standard eikonal model. The wave function of the incident particle inside a nucleus (taken for simplicity to be spherical and homogeneous) is given by

$$\psi^{(+)}(z,\rho)=e^{iqz}e^{-\frac{1}{2}\lambda D}, \tag{2}$$

where z and ρ are cylindrical coordinates, $D\equiv z+(R^2-\rho^2)^{1/2}$ is the depth penetrated, and

$$\tfrac{1}{2}i\lambda=[1-i\,\mathrm{Re}f(0)/\,\mathrm{Im}f(0)]3\sigma_N Ai/8\pi R^3 \tag{3}$$

is the complex index of refraction. The wave function of an excited system of mass m^* is

$$\psi^{(+)}(z,\rho,m^*)=(\tfrac{1}{2}\bar{\lambda})\int_{-(R^2-\rho^2)^{1/2}}^{z}dz'\,e^{iqz'}\times e^{iq'(z-z')}e^{-\frac{1}{2}\lambda D}, \tag{4}$$

where

$$\bar{\lambda}=-3iAf(0,m^*)/qR^3,$$

and the cross section for the Reaction (1) is given by $d^2\sigma/dtdm^*=|f(\theta,m^*)|^2$. We assume that the index of refraction for p^* is the same as for p, and neglect the possible effects of spin flip and isospin dependence.

The forward elastic nucleon-nucleus amplitude in this model is

$$f_A(0)=f_N(0)(3A/4\pi R^3)\int e^{-\frac{1}{2}\lambda D}dr+\int dm^*\left(\frac{d^2\sigma}{dtdm^*}\right)_0\left(\frac{2q}{\Delta}\right)\left(\frac{1-i\eta}{1+i\eta}\right)\left(\frac{3A}{4\pi R^3}\right)^2\int(1-e^{-i\Delta D})e^{-\frac{1}{2}\lambda D}dr,$$
$$\eta=\mathrm{Re}f(0,m^*)/\,\mathrm{Im}f(0,m^*) \tag{5}$$

using the approximate kinematics $q-q'=(m^{*2}-m^2)/2q$. The first term in Eq. (5) is the usual shadow effect; the second is the inelastic shadow effect, treated in lowest order. Performing the integrals and employing the optical theorem, we obtain

$$\sigma_A=\sigma_0-\sigma_{\mathrm{inel}},\quad \sigma_0=4\pi\,\mathrm{Re}[\lambda\varphi(\lambda)],\quad \sigma_{\mathrm{inel}}=\int dm^*(d^2\sigma/dtdm^*)F(q,m^*), \tag{6}$$

where σ_{inel} is the decrease in total cross section due to inelastic intermediate states, σ_0 is the total

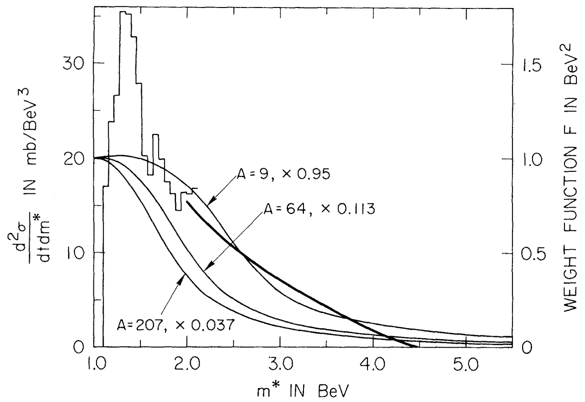

FIG. 1. Differential cross section at 0° and weight functions $F(m^*)$ for nuclei of mass number A and proton lab momentum 15 BeV/c. The heavy curve shows the extrapolated cross section assumed in numerical work.

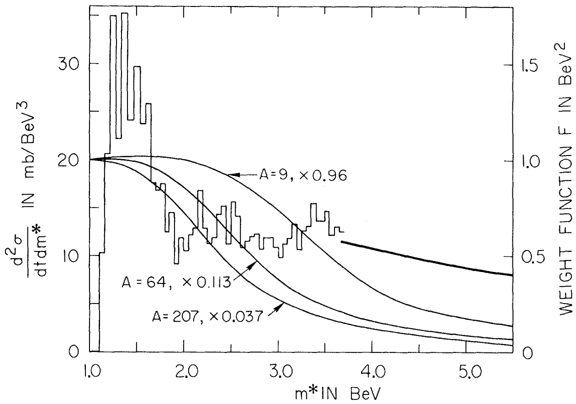

FIG. 2. As in Fig. 1, for proton momentum 30 BeV/c.

cross section with the usual elastic shadowing,

$$\varphi(\lambda) \equiv [(1+\lambda R)e^{-\lambda R} - 1 + \tfrac{1}{2}\lambda^2 R^2]/\lambda^3, \quad (7)$$

and

$$F(q,m^*) = 18A^2R^{-6}\Delta^{-1}\operatorname{Im}[\varphi(\lambda) - \varphi(\lambda+2i\Delta)] \times (1+i\eta)/(1-i\eta). \quad (8)$$

Experimental values for $d^2\sigma/dtdm^*$ at 15 and 30 BeV are shown in Figs. 1 and 2, respectively. The cross section does not change greatly between these two energies, and is thus consistent with a large amount of diffraction dissociation or "Deck effect."[6] Integrating the cross section over m^* we obtain, at 30 BeV,

$$\int_{\pi N \text{ threshold}}^{3.7 \text{ BeV}} \left(\frac{d^2\sigma}{dtdm^*}\right)_0 dm^* = 38 \text{ mb/BeV}^2, \quad (9)$$

which is sizable compared with the forward elastic cross section $(d\sigma/dt)_0 = 80$ mb/BeV^2.

Figures 1 and 2 also show the weight function $F(q,m^*)$ of Eq. (7) for various values of the mass number A. The weight functions indeed cut off around $\Delta R \approx 1$ because of the coherence requirement, which can be understood in the eikonal picture as follows: $\Psi^{(+)}(z,\rho,m^*)$ in Eq. (4) can be large only if the contributions from various depths $z' + (R^2-\rho^2)^{1/2}$ can add in phase with each other, despite the fact that their momenta (wave numbers) differ by Δ.

Performing the integral in Eq. (6), we obtain the results shown in Table I. In calculating these results, we have used $R = r_0 A^{1/3}$ with $r_0 = 1.3$ F, $\sigma_N = 40$ mb, and $\eta = \operatorname{Re}f(0)/\operatorname{Im}f(0) = -0.2$. The results are not overly sensitive to these choices. We have extrapolated the spectrum of $(d^2\sigma/dtdm^*)_0$ to masses beyond those for which it was measured, in the manner shown in the figures. Our results are rather insensitive to this extrapolation, because of the coherence cutoff: E.g., for $A = 9$ at 15 BeV the extrapolated region contributes only 25% of the inelastic effect; for $A = 207$ at 30 BeV, only 6%. Other reasonable extrapolations would therefore give similar results.

We have used the entire cross section $d^2\sigma/dtdm^*(pp \to p^*p)$ regardless of the stability of the p^*. At low energy for large nuclei, this approximation exaggerates σ_{inel} because the state p^* will spread in time and not be absorbed on a nucleon with the same amplitude with which it was produced. For a medium-mass nucleus the radius $R = 5$ F. The approximation requires that the p^* spread $R/\gamma \ll 1$ F in traversing a distance R. So $\gamma \ll 5$ implies $m^* \ll p_{\text{lab}}/s$ or $m^* \ll 3$ BeV for

Table I. Total cross sections (in units of mb) at high energy.

	Energy (BeV)	σ_0	σ_{inel}	σ_{net}	σ_{Jones}[a]
$A=9$	15	243	28	215	
	30	243	38	205	250
$A=64$	15	1275	179	1096	
	30	1275	246	1029	1090
$A=207$	15	3185	466	2719	
	30	3185	655	2530	2630

[a]L. W. Jones, M. J. Longo, J. O'Fallon, and M. N. Kreisler, in the Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, August, 1968 (to be published).

$p_{\rm lab} = 15$ BeV/c, and $m^* \ll 6$ BeV for $p_{\rm lab} = 30$ BeV/c. Referring to the figures and discussion of the extrapolation above, we see that the approximation is accurate except for medium nuclei below 15 BeV/c and heavy nuclei for $p_{\rm lab} \approx 15$ BeV/c and below. It will be difficult to accurately calculate $\sigma_{\rm inel}$ at low energies. The method used here would give too large an effect; so we can state that at, say, 5 BeV, $\sigma_{\rm inel}$ is much smaller than at 15-30 BeV. It is premature to compare our results with experiment[8] because of possible systematic experimental errors in the separate experiments at different energies and with different beams, and because of the crudity of our model of the nucleus. Our only real prediction is for the size of the inelastic effects. The most promising way to test those predictions is to measure the energy dependence of proton-nucleus cross sections, extract the small variation due to the energy dependence of σ_{pp}, and compare the remaining variation with our prediction.

To calculate the inelastic shadow effect in the case of light nuclei (small A), a homogeneous model for the nucleus is inadequate. A multiple-scattering approach such as the Glauber approximation is needed. As an example, we have calculated the inelastic correction to the proton-deuteron total cross section.[1]

The contribution to the forward p-d amplitude from an inelastic state of mass m^* is

$$f_d(0,m^*) = \frac{1}{4\pi^2|\vec{q}|}\int \frac{d\vec{k}S(\vec{k})f(k,m^*)f(-k,m^*)}{(\vec{q}^2+m^2)^{1/2}-[(\vec{q}-\vec{k})^2+m^{*2}]^{1/2}+i\epsilon}, \tag{10}$$

where $f(k,m^*)$ is the amplitude for the Reaction (1) at momentum transfer $\vec{k}$ and $S(\vec{k}) = \int e^{i\vec{k}\cdot\vec{r}} \times|\psi(\vec{r})|^2 d\vec{r}$ is the nonrelativistic deuteron form factor. The energy denominator in Eq. (10) results from the propagation of the inelastic state and contains the coherence requirement discussed above. For simplicity, we make the approximations $f(k,m^*) = f(0,m^*)e^{-\frac{1}{2}\delta\vec{k}^2}$ and $S(k) = e^{-\frac{1}{4}\alpha\vec{k}^2}$. We take $\delta = 5$ BeV^{-2} in rough accord with the data of Glauber,[8] and $\alpha = 134$ BeV^{-2} as in Ref. 3. Our results are insensitive to δ; so it is not even necessary to let it vary with m^*. Using the optical theorem we obtain

$$\sigma_{\rm inel} = \int dm \left(\frac{d^2\sigma}{dt\,dm^*}\right)_0 F(q,m^*), \tag{11}$$

where

$$F(q,m^*) = \frac{8}{4\delta+\alpha}\frac{1}{1+\eta^2}\left[(1-\eta^2)e^{-z^2}+\frac{4\eta}{\pi^{1/2}}D(z)\right],$$

$$z = (\delta+\tfrac{1}{4}\alpha)^{1/2}(m^2-m^{*2})/2q,$$

$$D(z) = e^{-z^2}\int_0^z e^{t^2}dt. \tag{12}$$

Performing the integral, we obtain $\sigma_{\rm inel} = 1.3$ mb at 15 BeV and 1.8 mb at 30 BeV. It should be possible to measure these effects by the energy dependences of σ_{pd}, σ_{pp}, and σ_{np}. Measurements at a single energy cannot establish the inelastic shadow effect, because of theoretical uncertainties in calculating the contribution of ordinary shadowing.

In πd total cross sections, an energy-dependent contribution to screening has apparently been observed.[5] Using data of Walker et al.,[9] we estimate that effects of inelastic states containing three pions may be large enough to explain that result.

Another approach to detecting inelastic shadowing would be to measure $d\sigma/dt$ on deuterium at momentum transfers large enough that single scattering is negligible. The momentum-transfer dependence of the inelastic effect will be similar to the elastic, but its energy dependence will be different.

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REGGE-POLE MODEL WITH CUTS GENERATED BY ABSORPTION FOR THE REACTION $\pi^+n \rightarrow \omega p$†

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The reaction $\pi^+n \rightarrow \omega p$ is analyzed in terms of a model based on the exchange of the ρ Regge pole and the associated cuts generated by absorption. The data, in particular the absence of a dip in the differential cross section and the large positive values of the density matrix element ρ_{00}, can be well described by this model only if the amplitudes do not vanish at nonsense, wrong-signature points.

The experimental data for the reaction[1] $\pi^+n \rightarrow \omega p$ have two outstanding features: (1) The density matrix element ρ_{00} of the ω is large, $\rho_{00} \approx 0.4$. (2) There is no dip in the differential cross section. Since the only simple exchange mechanism allowed in this reaction is ρ exchange, one expects[2] that $\rho_{00} = 0$. Since the $\pi\omega\rho$ vertex must vanish when $\alpha_\rho = 0$, one also expects[3] in the framework of the standard theory of Reggeization[4] that all helicity amplitudes and thus the differential cross section should have a dip at the nonsense, wrong-signature (NWS) point $\alpha_\rho(t) = 0$, or at $t \approx -0.6$ (GeV/c)^2. The violation of the predictions shows that the ρ pole alone is not able to explain the data. In this Letter, we show that the data can be understood in terms of a model including in addition to the ρ pole the associated Regge cuts generated by absorption,[5,6] provided that the amplitudes do not have NWS zeros; i.e., the residue functions have Mandelstam-Wang fixed poles.[7]

The reaction $\pi^+n \rightarrow \omega p$ has also been analyzed in terms of the absorption model[8] and the pure Regge-pole model.[9] The absorption model can explain the large value of ρ_{00} by the unnatural parity exchange caused by the absorption, but suffers from the usual difficulties with the energy dependence. In the pure Regge-pole model one is forced to introduce a secondary trajectory of unnatural parity corresponding to the 1^+ particle $B(1220)$. The fit to the data then gives values for the trajectory and coupling parameters of the B which are of the same order of magnitude $[\alpha_B(0) \approx \alpha_\rho(0)]$ as those of the ρ. This is not pleasing, since the B does not seem to contribute strongly to other reactions. For a trajectory passing through m_B^2 one can only have $\alpha_B(0) \approx \alpha_\rho(0)$ if the slope of the B trajectory is very small compared with most Regge-pole slopes; conversely, with a normal slope one would have $\alpha_B(0) \approx -\frac{2}{3}$, and at high energies (e.g., $s \approx 10$ GeV^2) there should be little contribution from B exchange. Although one cannot exclude the B, we shall show that one can do well without it.

We shall calculate the cut correction by assuming in the conventional way that the absorption gives rise to diffraction elastic scattering which is spin independent and equal in the initial and the final state (ωp elastic scattering is not known).

We write the full amplitude in the form

$$M = M^\rho + \lambda M^{\rho p},$$

where M^ρ is the s-channel ρ Regge-pole exchange,