[6]K. Tamai and T. Sato, Tohoku University Bubble Chamber Memorandum No. TUBC MEMO-5, 1968 (unpublished).

[7]K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 503 (1963).

GAUGE-FIELD ALGEBRA WITH SYMMETRY BREAKING*

I. Kimel
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois
(Received 20 May 1968)

It is shown that in gauge-field models for the electromagnetic and weak hadronic currents, restrictions are imposed on the way the symmetry is broken if the algebra of currents is to hold. A modified form of Weinberg's second sum rule is proposed, and relations among vector-meson masses are obtained on the assumption of nonet symmetry for the currents.

The spectral-function sum rules derived by Weinberg[1] for chiral SU(2)⊗SU(2) have been used to obtain some very good relations. Two outstanding examples are the ρ-A_1[1] mass ratio and the $\pi^+-\pi^0$ mass difference.[2] When extended to chiral SU(3)⊗SU(3) these sum rules take the form

$$\int[\mu^{-2}\rho_{\alpha\beta}^{(1)}(\mu^2)+\rho^{(0)}(\mu^2)]d\mu^2=S_1\delta_{\alpha\beta}, \qquad (1)$$

$$\int\rho_{\alpha\beta}^{(1)}(\mu^2)d\mu^2=S_2\delta_{\alpha\beta}. \qquad (2)$$

The functions $\rho_{\alpha\beta}^{(1)}(\mu^2)$ and $\rho_{\alpha\beta}^{(0)}(\mu^2)$ are the spin-1 and -0 spectral functions for vector or axial-vector currents. The constants S_1 and S_2 cannot be calculated but the important thing is that they are equal for both types of currents.

There are several ways of deriving Eqs. (1) and (2): (i) They follow from current algebra plus the assumption of c-number Schwinger terms.[1,3] Equation (2) requires extra assumptions about the high-momentum behavior of the currents.

(ii) If SU(3)⊗SU(3) is a good asymptotic symmetry, convergence or superconvergence conditions can be imposed on invariant coefficients of vector and axial-vector propagators leading to (1) and (2).[4]

(iii) The simplest and most elegant derivation of the sum rules is provided by the so called algebra-of-fields model[5] which explicitly gives equal c-number Schwinger terms for both kinds of currents.

Equations (1) and (2) already contain symmetry-breaking effects for spin-1 mesons in the sense that they can accommodate, for instance, $m_\rho \neq m_{A_1}$, but that is not enough and further symmetry breakings have to be considered. Note that one of the results of Eqs. (1) and (2) would be $m_{K^*}=m_\rho$ if the scalar (κ) excitation is ignored.

Different points of view have been expressed as to how the symmetry is broken. Okubo pointed out[6] that the Weinberg sum rules are model dependent and that even if we restrict ourselves to gauge-field models, there is still freedom in choosing particular forms of symmetry breaking.

In the original algebra-of-fields model the bare masses of the gauge fields are assumed to be equal[5] which leads to too high a symmetry implied by the second sum rule (2). Since the mass term in the Lagrangian violates gauge invariance, Okubo argues that it is reasonable to suppose the bare masses are not common. By assuming that the bare masses satisfy a Gell-Mann–Okubo relation and renormalizing his currents, Okubo obtains a modified second sum rule:

$$\int\rho_{\alpha\beta}^{(1)}(\mu^2)d\mu^2=b\delta_{\alpha\beta}+cd_{8\alpha\beta}. \qquad (3)$$

In the present note we first want to show that the reasoning leading to Eq. (3) cannot hold in a model of gauge fields if the current-algebra relations are retained at the same time. For the sake of clarity let us consider only the octet of vector currents. A Lagrangian with the features assumed by Okubo has to be of the form

$$\mathfrak{L}=-\tfrac{1}{4}F_{\mu\nu}^{\alpha}F^{\alpha\mu\nu}$$

$$-\tfrac{1}{2}m_0^2(1+D'd_{8\alpha\alpha})v_\mu^\alpha v^{\alpha\mu}, \qquad (4)$$

with

$$F_{\mu\nu}{}^{\alpha}=\partial_{\mu}v_{\nu}{}^{\alpha}-\partial_{\nu}v_{\mu}{}^{\alpha}-f_0 f_{\alpha\beta\gamma}v_{\mu}{}^{\beta}v_{\nu}{}^{\gamma}. \quad (5)$$

This Lagrangian leads to the following sum rules for the spectral functions of the v's:

$$\int[\mu^{-2}\rho_{\alpha\beta}{}^{(1)}+\rho_{\alpha\beta}{}^{(0)}]d\mu^2=\frac{\delta_{\alpha\beta}}{m_0{}^2(1+D'd_{8\alpha\alpha})}, \quad (6)$$

$$\int\rho_{\alpha\beta}{}^{(0)}d\mu^2=\delta_{\alpha\beta}. \quad (7)$$

If we renormalize the currents by

$$v_{\mu}{}^{\alpha}=\frac{S^{1/2}}{m_0(1+D'd_{8\alpha\alpha})^{1/2}}\bar{v}_{\mu}{}^{\alpha}, \quad (8)$$

we find that the spectral functions for the $\bar{v}$'s indeed satisfy Eqs. (1) and (3). The trouble now is that the $\bar{v}$'s do not satisfy the algebra of currents.

To prove our point let us accept the Lagrangian (4) and renormalize the currents by[7] $j_{\mu}{}^{\alpha}=Z_{\alpha}{}^{-\frac{1}{2}}\times v_{\mu}{}^{\alpha}$. We then obtain the following once-integrated commutators:

$$[Q^{\alpha},j_i{}^{\beta}(x)]=\frac{if_0}{m_0{}^2(1+D'd_{8\alpha\alpha})}\times\left(\frac{Z_{\gamma}}{Z_{\alpha}Z_{\beta}}\right)^{1/2}f_{\alpha\beta\gamma}j_i{}^{\gamma}(x). \quad (9)$$

In order to recover the algebra-of-currents relations we are forced to put $D'=0$. Thus a model based on Lagrangian (4) will not be acceptable.

Symmetry breaking in the kinetic terms of the Lagrangian does not suffer from these defects. With the Lagrangian

$$\mathcal{L}=-\tfrac{1}{4}(1+Dd_{8\alpha\alpha})F_{\mu\nu}{}^{\alpha}F^{\alpha\mu\nu}-\tfrac{1}{2}m_0{}^2v_{\mu}{}^{\alpha}v^{\alpha\mu}, \quad (10)$$

the j's obey the current-algebra relations provided all the Z's are equal:

$$Z_{\alpha}=Z=(f_0/m_0{}^2)^2. \quad (11)$$

Weinberg's first sum rule (1) is still valid, but instead of (2) we now have

$$\int\rho_{\alpha\beta}{}^{(1)}(\mu^2)d\mu^2=\frac{S_2'}{(1+Dd_{8\alpha\alpha})}\delta_{\alpha\beta}. \quad (12)$$

Equation (12) together with Eq. (1) gives a Gell-Mann–Okubo relation for the inverse mass squared of the vector mesons of the type first considered by Coleman and Schnitzer.[8]

Everything we have said so far can be applied to the octet of axial currents. In particular the sum rule (12) will be valid for the axial spectral functions, too. The constants S_2' and D should be the same for both types of currents since Eq. (2) seems to be a good sum rule for SU(2)⊗SU(2).[6,9]

To study the ω-φ mixing problem we have to take into account the baryon number current $j_{\mu}{}^{\beta}$. The sum rule (1) can be extended to the (8 + 1) currents (the singlet current is $j_{\mu}{}^{0}=\sqrt{\tfrac{3}{2}}j_{\mu}{}^{\beta}$):

$$\int[\mu^{-2}\rho_{\alpha\beta}{}^{(1)}+\rho_{\alpha\beta}{}^{(0)}]=S_1\delta_{\alpha\beta}+S_1'\delta_{\alpha 0}\delta_{\beta 0}. \quad (13)$$

We could in principle have another term $S_1''\times(\delta_{\alpha 0}\delta_{\beta 8}+\delta_{\alpha 8}\delta_{\beta 0})$ in the right-hand side of Eq. (13), but if only medium-strong symmetry violations are taken into account, we can show that $S_1''=0$. Consider for instance the following commutator at equal times:

$$[j_0{}^0(x),[Q^6,j_i{}^7(y)]]=i\frac{\sqrt{3}}{2}[j_0{}^0(x),j_i{}^8(y)]-\frac{i}{2}[j_0{}^0(x),j_i{}^3(y)]. \quad (14)$$

By the Jacobi identity the left-hand side is equal to

$$[Q^6,[j_0{}^0(x),j_i{}^7(y)]]+[j_i{}^7(y),[Q^6,j_0{}^0(x)]]. \quad (15)$$

Since $j_0{}^0(x)$ is a unitary singlet density, $[Q^6,j_0{}^0(x)]=0$. Also $[j_0{}^0(x),j_i{}^3(y)]=[j_0{}^0(x),j_i{}^6(y)]=0$; nonzero Schwinger terms in these commutators would imply the existence in Eq. (13) of still other terms like $S_1{}^{03}(\delta_{\alpha 0}\delta_{\beta 3}+\delta_{\alpha 3}\delta_{\beta 0})$ and $S_1{}^{07}\times(\delta_{\alpha 0}\delta_{\beta 7}+\delta_{\alpha 7}\delta_{\beta 0})$, giving rise to medium-strong ω-ρ and ω-K^* mixing. Thus we must have $[j_0{}^0(x), j_i{}^8(y)]=0$ and consequently $S_1''=0$.

Oakes and Sakurai[10] have shown that Eq. (13) is inconsistent with mass mixing. In the absence of a crossed ($S_1''\neq 0$) term in Eq. (13), mass mixing has to be ruled out. We now show that in the framework of the algebra of fields, mixing of the "current" type is the only one compatible with SU(3) commutation relations and furthermore leads to Eq. (13). To that effect let us first consider a Lagrangian with mass mixing:

$$\mathcal{L}=\mathcal{L}_8+\mathcal{L}_0-\tfrac{1}{2}\epsilon' m_0\nu(v_{\mu}{}^{8}v^{0\mu}+v_{\mu}{}^{0}v^{8\mu}). \quad (16)$$

$\mathcal{L}_8$ is a symmetric Lagrangian for the octet, $\mathcal{L}_0$ is the free Lagrangian for the singlet current $v_{\mu}{}^{0}$ with a bare mass ν.

From the Lagrangian (16) we can derive the once-integrated commutators for the nine renormalized currents:

$$[Q^\alpha, j_i^\beta(y)] = i(1-\delta_{\alpha 8}\epsilon')^{-1} f_{\alpha\beta\gamma} j_i^\gamma(y), \tag{17}$$

with $\alpha,\beta,\gamma = 0, 1, \cdots, 8$ and $f_{0\alpha\beta} = 0$. Equation (17) agrees with the algebra of currents only if $\epsilon' = 0$.

The arguments given above lead us to a model Lagrangian with an ω-φ mixing term of the "current" type:

$$\mathcal{L} = \mathcal{L}(10) + \mathcal{L}_0 - \tfrac{1}{4}\epsilon(F_{\mu\nu}{}^8 F^{0\mu\nu} + F_{\mu\nu}{}^0 F^{8\mu\nu}), \tag{18}$$

where $\mathcal{L}(10)$ is the Lagrangian of Eq. (10) and the bare masses in $\mathcal{L}(10)$ and $\mathcal{L}_0$ are equal. Lagrangian (18) is consistent with the SU(3) commutation relations and from it we can easily derive Eq. (13) and a modified second sum rule:

$$\int \rho_{\alpha\beta}{}^{(1)}(\mu^2)d\mu^2 = \frac{S_2\delta_{\alpha\beta} + S_2'\delta_{\alpha 0}\delta_{\beta 0} + S_2''(\delta_{\alpha 0}\delta_{\beta 8} + \delta_{\alpha 8}\delta_{\beta 0})}{1 + D(d_{8\alpha\alpha} - \delta_{\alpha 8}/\sqrt{3}) - (\delta_{\alpha 0} + \delta_{\alpha 8})\epsilon^2}. \tag{19}$$

Assuming vector-meson dominance of the spectral functions in Eqs. (13) and (19) we get[11]

$$\tfrac{1}{3}(4m_{K^*}{}^{-2} - m_\rho{}^{-2}) = m_\varphi{}^{-2}\cos^2\theta + m_\omega{}^{-2}\sin^2\theta. \tag{20}$$

It is easy to incorporate nonet symmetry into the model with a Lagrangian (Greek indices run from 0 to 8):

$$\mathcal{L} = \tfrac{1}{4}(1 + Dd_{8\alpha\beta})F_{\mu\nu}{}^\alpha F^{\alpha\mu\nu} - \tfrac{1}{2}m_0{}^2 v_\mu{}^\alpha v^{\alpha\mu}. \tag{21}$$

The d's are the usual symmetric coefficients for the octet plus

$$d_{0\alpha\beta} = (\tfrac{2}{3})^{1/2}\delta_{\alpha\beta}. \tag{22}$$

The Lagrangian (21) leads to the sum rules for the nonet:

$$\int [\mu^{-2}\rho_{\alpha\beta}{}^{(1)}(\mu^2) + \rho_{\alpha\beta}{}^{(0)}(\mu^2)]d\mu^2 = S_1\delta_{\alpha\beta}, \tag{23}$$

$$\int \rho_{\alpha\beta}{}^{(1)}(\mu^2)d\mu^2 = S_2\frac{\delta_{\alpha\beta} - (D/\sqrt{3})\delta_{\alpha 0}\delta_{\beta 0} - (\tfrac{2}{3})^{1/2}D(\delta_{\alpha 8}\delta_{\beta 0} + \delta_{\alpha 0}\delta_{\beta 8})}{1 + D[d_{8\alpha\alpha} - (D/\sqrt{3})\delta_{\alpha 0}] - \tfrac{2}{3}D^2(\delta_{\alpha 0} + \delta_{\alpha 8})}. \tag{24}$$

These sum rules are valid for the nonets of vector and axial-vector currents; the axial currents can be incorporated into the Lagrangian (21) in a straightforward manner.

The second sum rule Eq. (24) looks simpler if written as

$$(\delta_{\alpha\gamma} + Dd_{8\alpha\gamma})\int d\mu^2 \rho_{\gamma\beta}{}^{(1)}(\mu^2) = S_2\delta_{\alpha\beta}. \tag{25}$$

The new relations among the vector-meson masses are

$$\underset{(1.40)}{\tfrac{1}{3}(2m_{K^*}{}^{-2} + m_\rho{}^{-2})} = \underset{(1.47)}{m_\varphi{}^{-2}\sin^2\theta + m_\omega{}^{-2}\cos^2\theta}, \tag{26}$$

$$\underset{(1.70)}{m_\rho{}^{-2}} = \underset{(1.75)}{m_\varphi{}^{-2}(3\sin^2\theta - 1) + m_\omega{}^{-2}(3\cos^2\theta - 1)}, \tag{27}$$

$$m_\omega^{-2}+m_\varphi^{-2}=2m_{K^*}^{-2}. \tag{28}$$
$$(2.59) \qquad (2.51)$$

These relations are not all independent; by adding up Eqs. (20) and (26) we get Eq. (25). This last one is satisfied[12] even better than the corresponding relation for the direct masses squared as given by Okubo's nonet model.[13]

Equation (20) is more general than nonet symmetry; its derivation does not require any special relation between the symmetry breaking in the octet and the 0-8 mixing nor an equality between the octet and singlet bare masses. So we compute θ from Eq. (17) which gives[10] $\theta = 28.2°$ and verify that Eqs. (26) and (27) are well satisfied.[12] Thus nonet symmetry for the currents based on Lagrangian (21) seems to be a valid assumption.

It is a pleasure to thank Professor J. J. Sakurai for suggesting this problem and for many stimulating discussions.

*This work supported in part by U. S. Atomic Energy Commission.

[1]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[2]T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters 18, 759 (1967).

[3]S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967).

[4]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967).

[5]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

[6]S. Okubo, in Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester, New York, 1967, edited by C. R. Hagen et al. (Interscience Publishers, Inc., New York, 1968).

[7]The Z's here are not the usual (dimensionless) renormalization constants. Our renormalization allows for change in dimension when going from the v's in the Lagrangian to the actual currents j.

[8]S. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964).

[9]J. J. Sakurai, in Proceedings of the International Theoretical Physics Conference on Particles and Fields, Rochester, New York, 1967, edited by C. R. Hagen et al. (Interscience Publishers, Inc., New York, 1968).

[10]R. J. Oakes and J. J. Sakurai, Phys. Rev. Letters 19, 1266 (1967).

[11]θ is the ω-φ (current) mixing angle as defined in Ref. 10, where Eq. (20) was also deduced.

[12]We have written the numerical values in BeV^{-2} for both sides under Eqs. (26)-(28). The masses were taken from A. H. Rosenfeld et al., Rev. Mod. Phys. 40, 77, (1968).

[13]S. Okubo, Phys. Letters 5, 165 (1963).

QUANTUM DYNAMICS IN PHASE SPACE*

G. S. Agarwal and E. Wolf
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 22 May 1968)

After a brief summary of recently derived general results relating to the mapping of functions of noncommuting operators on functions of c numbers, equations are given which describe the time evolution of the c-number equivalents (phase-space representations) of the density operator and of a Heisenberg operator. The evaluation of time-ordered functions of operators by c-number techniques is also briefly discussed.

Since the publication of the pioneering papers of Wigner,[1] Groenewold,[2] and Moyal[3] on the representation of quantum-mechanical systems in terms of generalized phase-space distribution functions, a considerable use has been made of such representations in the treatment of various problems. In the last few years, generalized phase-space descriptions have become of central importance in quantum optics, especially in the study of coherence properties of light[4] and in the theory of the laser.[5] As is well known, the phase-space representation of a quantum-mechanical system is not unique; it depends on the rule that is adopted for ordering of functions of noncommuting operators. We have recently developed a general technique for a systematic treatment of problems in this field, based on the use of certain new class of ordering operators. In this note we present the phase-space form of the basic quantum-mechanical equations of motion, which we have derived by the use of this technique.

We will first briefly explain the notation and summarize some of the main results given elsewhere.[6] We consider a correspondence between a function $F(z, z^*)$ of complex c-number vari-