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THERMOMAGNETIC EFFECTS IN DIRTY TYPE-II SUPERCONDUCTORS

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We reconsider microscopically and thermodynamically the heat current associated with the motion of vortex lines in a dirty type-II superconductor and show that the heat-current operator $\vec{j}_0^h$ employed previously by Caroli and Maki has to be replaced by $\vec{j}^h$ in the presence of a constant magnetic field H: $\vec{j}^h = \vec{j}_0^h + H\vec{j}^M$, where $\vec{j}^M$ may be called the magnetization current. We then recalculate the entropy associated with a single vortex line S_D at all temperatures (and in the high-field region), which vanishes like T at low temperatures.

The thermomagnetic effects associated with the motion of vortex lines are of current interest. Recent experiments in the resistive state of type-II superconductors show the existence of thermomagnetic effects such as the Peltier,[1] Ettingshausen,[2] and Nernst effects.[3,4]

Employing a phenomenological model Stephen[5] considered previously the entropy carried by a single vortex line with limited success. More recently Caroli and the present author[6] formulated the problem microscopically in terms of the moving order parameter and arrived at the surprising result that the entropy carried by a individual vortex line S_D diverges like T^{-1} at low temperatures. On the other hand, the thermodynamical reasoning shows that S_D vanishes at $T = 0$°K.[7] The purpose of this note is to clarify the origin of this difficulty and present a new expression for the entropy free of the above defects.

In I we used the following expression for the heat current in the calculation of S_D:

$$\vec{j}_0^{\,h} = -\frac{1}{2m}\sum_\alpha \left(\nabla\frac{\partial}{\partial t'} + \nabla'\frac{\partial}{\partial t}\right) \times \psi_\alpha^\dagger(\vec{r}', t')\psi_\alpha(\vec{r}, t)\Big|_{\vec{r}'=\vec{r},\, t'=t}, \quad (1)$$

where $\psi_\alpha^\dagger(\vec{r}, t)$ and $\psi_\alpha(\vec{r}, t)$ are electron field operators.

The above expression has been used in the discussion of the thermal conductivity in superconductors.[8] Equation (1) can be also decomposed as

$$\vec{j}_0^{\,h} = \vec{j}^E - \mu\vec{j}, \quad (2)$$

where $\vec{j}^E$ is the energy current, $\vec{j}$ is the mass current, and μ is the chemical potential of the electron.

In the presence of a magnetic field, however, we have to generalize the heat-current operator somewhat. This follows from the thermodynamical relation

$$\delta Q = T\delta S = \delta E - \mu\delta N + H\delta M, \tag{3}$$

where δQ is the variation of the heat energy, δS is the variation of the entropy, δN is the variation of the electron number, and δM is the variation in the magnetization. Correspondingly the heat current in this general situation can be written as

$$\vec{j}^h = \vec{j}^E - \mu\vec{j} + H\vec{j}^M = \vec{j}_0^{\,h} + H\vec{j}^M. \tag{4}$$

Here $\vec{j}^M$ may be called the magnetization current and is related to the local magnetization $M(\vec{r}, t)$ by

$$\nabla\cdot\vec{j}^M(\vec{r}, t) + \partial M(\vec{r}, t)/\partial t = 0. \tag{5}$$

In order to calculate $\vec{j}^h$ it is necessary to know $\vec{j}_0^{\,h}$ and $\vec{j}^M$. Fortunately these quantities have been obtained in I. First let us consider a dirty type-II superconductor, where a magnetic field, H (slightly smaller than H_{c2}), is applied in the Z direction and an electric field, E, in the x direction. The order parameter $\Delta(\vec{r}, t)$ then moves uniformly in the y direction,[6]

$$\partial\Delta(\vec{r}, t)/\partial t = u\,\partial\Delta(\vec{r}, t)/\partial y, \quad u = E/H. \tag{6}$$

In this situation we have[6]

$$(j_0^{\,h})_x = 0,$$

$$(j_0^{\,h})_y = MEL_D'(t), \tag{7}$$

$$-4\pi M(\vec{r}, t) = \frac{e\tau_{tr}N}{mT}|\Delta(\vec{r}, t)|^2\psi^{(1)}(\tfrac{1}{2}+\rho), \tag{8}$$

and

$$L_D' = [2 + \rho\psi^{(2)}(\tfrac{1}{2}+\rho)/\psi^{(1)}(\tfrac{1}{2}+\rho)], \tag{9}$$

where $\psi^{(1)}(z)$ and $\psi^{(2)}(z)$ are the tri- and the tetragamma functions. ρ is determined by

$$-\ln t = \psi(\tfrac{1}{2}+\rho) - \psi(\tfrac{1}{2}), \tag{10}$$

where $t = T/T_0$ and $\psi(z)$ is the di-gamma function. From Eqs. (6) and (8) we get

$$\partial M(\vec{r}, t)/\partial t = u\,\partial M(\vec{r}, t)/\partial y. \tag{11}$$

Substituting this into Eq. (5) we have

$$j_x^{\,M} = 0, \quad j_y^{\,M} = -uM(\vec{r}, t). \tag{12}$$

Finally, the heat current is given by

$$j_x^{\,h} = 0, \quad j_y^{\,h} = MEL_D(t), \tag{13}$$

where

$$L_D(t) = [1 + \rho\psi^{(2)}(\tfrac{1}{2}+\rho)/\psi^{(1)}(\tfrac{1}{2}+\rho)]. \tag{14}$$

The magnetization M may be rewritten as

$$-4\pi M = \frac{(H_{c2}-B)}{\{1.16[2\kappa_2^{\,2}(t)]+1\}}, \tag{15}$$

where B is the induction.

The entropy S_D carried by a single vortex line is related to $j_y^{\,h}$ by

$$j_y^{\,h} = -nTS_D u, \tag{16}$$

where $n = H/\varphi_0$ is the number of vortex lines and φ_0 is π/e. Solving Eq. (16) for S_D, we find the entropy for a vortex line to be given by

$$S_D(T) = \frac{1}{4eT}\frac{(H_{c2}-B)}{\{1.16[2\kappa_1^{\,2}(t)-1]+1\}}L_D(t), \tag{17}$$

where $L_D(t)$ has been given in Eq. (14). Since $L_D(t)$ vanishes like T^2 at low temperatures it is easy to see that S_D tends to zero like T as T vanishes.

It is of some interest to note that the function $L_D(t)$ appears also in the expression for the thermal conductivity in a dirty type-II superconductor,[9]

$$K_M = K_n - \frac{1}{2e}\frac{(H_{c2}-B)}{1.16[2\kappa_2^{\,2}(t)-1]+1}\rho L_D(t), \tag{18}$$

where the subscript n denotes the normal state.

A similar consideration will also apply to the calculation done in the pure limit.[6] In fact, in the pure limit we have the following expression for the entropy:

$$S_D(t) = \frac{1}{4eT}\frac{(H_{c2}-B)}{\{1.16[2\kappa_2^{\,2}(t)-1]+1\}}L_P(t), \tag{19}$$

where $L_P(t)$ is now given by

$$L_P(t) = \left\{2\int_0^1\frac{dz}{(1-z^2)^{1/2}}\int_0^\infty d\zeta\frac{\zeta^2(1-\rho^2\zeta^2)\exp(-\rho^2\zeta^2)}{\sinh[\zeta(1-z^2)^{-1/2}]}\right\}\left\{\int_0^1\frac{dz}{(1-z^2)^{1/2}}\int_0^\infty d\zeta\frac{\zeta^2\exp(-\rho^2\zeta^2)}{\sinh[\zeta(1-z^2)^{-1/2}]}\right\}^{-1}. \tag{20}$$

It is easy to show that $S_D(t)$ for a pure superconductor also vanishes like $T\ln T$ as T tends to zero. However, it turns out that in the pure limit there is another contribution to the heat current which is not considered in I and a more elaborate discussion is necessary.[10]

Finally, we mention that the extension of the present consideration to the high-field type-II superconductor[11] is almost evident. In this case the corrected entropy for a vortex line, can be expressed by

$$S_D(t) = \frac{1}{4eT}\frac{(H_{c2}-B)}{\{1.16[2\kappa_2^2(t)-1]+1\}}L(t)A(t), \tag{21}$$

where

$$L(t) = \left\{1 + \frac{\frac{1}{2}[1+b/(b^2-I^2)^{1/2}]\rho_-\psi^{(2)}(\frac{1}{2}+\rho_-) + \frac{1}{2}[1-b/(b^2-I^2)^{1/2}]\rho_+\psi^{(2)}(\frac{1}{2}+\rho_+)}{\frac{1}{2}[1+b/(b^2-I^2)^{1/2}]\psi^{(1)}(\frac{1}{2}+\rho_-) + \frac{1}{2}[1-b/(b^2-I^2)^{1/2}]\psi^{(1)}(\frac{1}{2}+\rho_+)}\right\},$$

$$A(t) = \frac{M_d}{(M_d+M_p)}, \quad \rho_\pm = \frac{1}{2\pi T}[\epsilon_0 + b \pm (b^2-I^2)^{1/2}], \tag{22}$$

$\epsilon_0 = 2DeH_{c2}(t)$, $b=(3T_{\rm so})^{-1}$, and $I=\mu H$. Here μ is the Bohr magneton and $T_{\rm so}$ is the spin-orbit lifetime. $S_D(t)$ given in Eq. (21) also vanishes like T as T tends to zero.

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EVIDENCE OF BAND CONDUCTION AND CRITICAL SCATTERING IN DILUTE Eu-CHALCOGENIDE ALLOYS

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Resistivity and Hall-effect measurements in $Eu_{0.95}Gd_{0.05}S$ give evidence for the onset of a well defined conduction band with critical scattering coexisting with magnetic impurity states.

The anomalous properties of Eu-chalcogenide alloys, $Eu_{1-x}R_xX$ (where R is a trivalent rare earth and X is one of the following: O, S, Se, Te) have been the subject of considerable experimental and theoretical investigation.[1,2] Among these, perhaps the most striking are the anomalous temperature and field dependence of the resistivity.[3] In particular, an explanation of the data on single-crystal samples with $x \leq 0.01$ has been given based on a magnetic impurity state (MIS) and conduction by means of hopping. The activation energy for this process is determined by