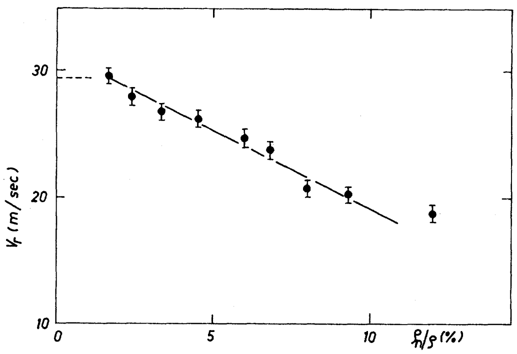

FIG. 3. The critical velocity V_γ for production of vortex rings by positive ions as a function of the normal-to-total density ratio ρ_n/ρ. The dotted line summarizes the results at low temperatures (Ref. 2), where V_γ is constant. The solid line is drawn with the same slope of that representing the critical velocity of the periodic discontinuities of positive ions as a function of ρ_n/ρ (Ref. 10).

and fields. A typical result, obtained at 1.41°K with the method (B), is plotted in Fig. 1. The drift velocity is a decreasing function of the electric field, and follows the dynamics of the charged vortex ring.

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MODEL FOR THE CORE OF A QUANTIZED VORTEX LINE IN HELIUM II*

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We propose that a quantized vortex line in helium II has a central core of normal fluid extending over a distance of a few angstroms. Surrounding this core is a "tail," a region of excess density of rotons whose momenta are predominantly aligned oppositely to the direction of circulation of the superfluid. This model is used to calculate the drag on a negative ion trapped on a vortex line, and appears to account for experimental results satisfactorily.

Perhaps the least satisfactory aspect of the classical theory of vortices is the treatment of the core. Now that vortices in liquid helium have provided fresh impetus for such studies, it is not surprising that there should be considerable interest in sorting out effects associated with the core structure. This Letter reports a model for the structure of the core with emphasis on the behavior of the excitations comprising the normal component. Models for the superfluid based on the behavior of a condensate have already appeared.[1]

Studies of ion motion in liquid helium have included three classes of experiment which shed light on vortex structure. The motion of large vortex rings demonstrates the quantization of circulation and gives a measure of vortex strength.[2] The trapping and escape of ions from vortices are sensitive to the distribution of superfluid near the core.[3] The motion of ions along quantized vortex lines is sensitive to excitations localized about the core.[4] We shall be concerned with the interpretation of such experiments by recourse to the Landau model applied to the vortex lines.

The dominant excitations at the relatively high temperatures of our experiments are rotons. Their excitation spectrum is approximated near the roton minimum by the familiar relation

$$\epsilon(p) = \Delta + (p-p_0)^2/2\mu_0, \qquad (1)$$

where ϵ is the energy, p the momentum, Δ the energy gap, and μ_0 the effective mass of the rotons. When relative motion is occurring between the superfluid and the rotons, the number of rotons per unit phase space is given by

$$n(\vec{p}) = h^{-3}\{\exp[(kT)^{-1}][\epsilon(\vec{p}) - \vec{p}\cdot(\vec{v}_n - \vec{v}_s)] - 1\}^{-1}. \tag{2}$$

Consider now a vortex filament having a velocity field in the superfluid specified by $\vec{v}_s = \vec{V}(r) = h(2\pi m r)^{-1}\vec{e}_\theta$, where $\vec{e}_\theta$ is a vector tangential to the direction of circulation. The distribution of rotons near the line is given by

$$N(T, V) = \int \frac{1}{h^3}\{\exp[(kT)^{-1}][\Delta + (p-p_0)^2/2\mu_0 + pV\cos\theta] - 1\}^{-1} d^3p$$
$$= \frac{2\pi kT}{h^3 V}\int_0^\infty p \ln\left\{\frac{1-\exp\{-(kT)^{-1}[\Delta + (p-p_0)^2/2\mu_0 + pV]\}}{1-\exp\{-(kT)^{-1}[\Delta + (p-p_0)^2/2\mu_0 - pV]\}}\right\} dp, \tag{3}$$

since the rotons are fixed and $\vec{v}_n = 0$. An immediate consequence of Eq. (3) is the existence of a Landau critical velocity at some radius R_c where rotons exist with zero energy—that is, where

$$[\Delta + (p-p_0)^2/2\mu_0 - pV]_{\min} = 0. \tag{4}$$

Within this radius the excitations may no longer be considered as rotons and form a stagnant central core.

Three effects must now be included. First, neutron data indicate that Δ decreases with increasing temperature.[5] We interpret this temperature dependence as a manifestation of roton-roton attractive interaction and use an empirical formula of Bendt, Cowan, and Yarnell,[6]

$$\Delta(N) = \Delta - 0.7383\times 10^{-37} N, \tag{5}$$

to obtain a first-order estimate of this interaction. Second, we should properly apply the statistical mechanics to the eigenfunctions of the rotons which are, in effect, bound to the vortex line by the reduction in energy due to superfluid circulation. Let us attempt to correct the energy of free rotons given by Eq. (1) by noting that a roton wave packet confined to a dimension r must by the uncertainty principle contain a spread of momenta of order $\hbar/r$. This spread in momentun leads to an increase in energy of order $\Delta\epsilon \approx \hbar^2/8\mu_0 r^2$. Finally, the roton parameters of Eq. (1) are pressure dependent, and may be described by the following formulas[7]:

$$\mu_0' = \mu_0(1 - 0.0217P),$$
$$p_0' = p_0(1 + 0.0029P),$$
$$\Delta' = \Delta(1 - 0.0075P), \tag{6}$$

where P is the applied pressure in atmospheres and $\mu_0 = 0.16m$, $p_0/\hbar = 1.91$ Å^{-1}, and Δ is given by Eq. (5). Equation (1) now reads

$$\epsilon(p, P) = \Delta' + (p-p_0')^2/2\mu_0' + \hbar^2/8\mu_0' r^2, \tag{7}$$

and Eq. (3) becomes

$$N(V,T,P) = \frac{2\pi kT}{h^3 V}\int_0^\infty p \ln\left\{\frac{1-\exp\{-(kT)^{-1}[\Delta' + (p-p_0')^2/2\mu_0' + pV + \hbar^2/8\mu_0' r^2]\}}{1-\exp\{-(kT)^{-1}[\Delta' + (p-p_0')^2/2\mu_0' - pV + \hbar^2/8\mu_0' r^2]\}}\right\} dp. \tag{8}$$

Equation (8) has been numerically integrated and solved simultaneously with Eq. (5) by iteration on an IBM computer.

The results of this analysis are shown in Fig. 1 which was calculated for $T = 1.6$°K, $P = 0$. The upper curve shows the roton density which rises from an equilibrium density at large distances and changes very rapidly near $R_c = 4.25$ Å. The middle curve shows the effective well formed as the minimum value of the quantity $\Delta' + (p-p_0')^2/2\mu_0' - pV + \hbar^2/8\mu_0' r^2$ falls near R_c. The lower curve shows the variation of ρ_s/ρ obtained by calculating ρ_n in a manner analogous to the calculation of N, then subtracting from an assumed constant density ρ to find ρ_s. These results, characteristic of those found at all temperatures, suggest that a quantized vortex line has a central core of normal fluid whose radius is temperature and pressure dependent (increasing with both), and a "tail" in which the roton density (and also ρ_n) increases as R_c is approached. The rotons in the tail are highly polarized because the majority of them have momentum (but not group velocities) aligned along a direction close to $-\vec{V}$. There is no net rotation of either core or tail.

The temperature[4] and pressure dependence of the motion of negative ions along vortex lines has

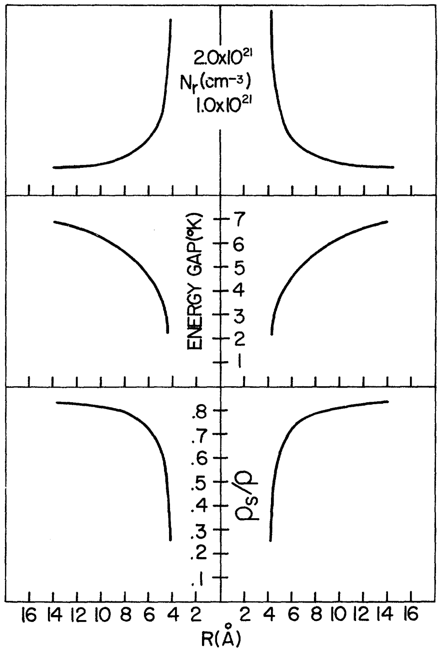

FIG. 1. Behavior of three quantities near a vortex line located at $r=0$. Here $R_c = 25$ Å.

been measured in this laboratory with results shown in Figs. 2 and 3. Let us compute the transit time $\tau(T,P)$ of an ion using the model of a vortex outlined above. We assume that the ion is a sphere of radius ~16 Å at $P=0$ and with a known pressure variation.[3,7,8] The extra density of excitations shown in Fig. 1 (where the abscissa has the dimensions of the diameter of the ion) will contribute an additional retarding force on the ion, in excess of that experienced by a free ion. The trapped-ion transit time has two contributions: τ_t due to the tail and τ_c due to the core.

The rotons in the tail—except very near R_c—comprise a dilute gas, and a simple mobility analysis will be expected to be valid. The drag is then proportional to the number density of rotons in the tail and we can write

$$\frac{\tau_t(P,T)}{\tau_t(P=0,T)} = \frac{\int_{R_c(P,T)}^{R_i(P)} 2\pi r N_r(P,T,r)dr}{\pi R_i^2(P=0)N(P=0,r=\infty,T)}, \quad (9)$$

where $R_i(P)$ is known experimentally[7,8] and the free-ion transit time is determined experimentally simultaneously with τ.[4]

The contribution τ_c from the core is calculated by assuming that the core material resembles helium I with properties extrapolated to the temperature and pressure of this experiment. The drag is computed on the viscous analysis outlined by Landau and Lifshitz[9] with integration only over the forward cap of the ion, i.e., over an angle R_c/R_i in the forward direction. The core

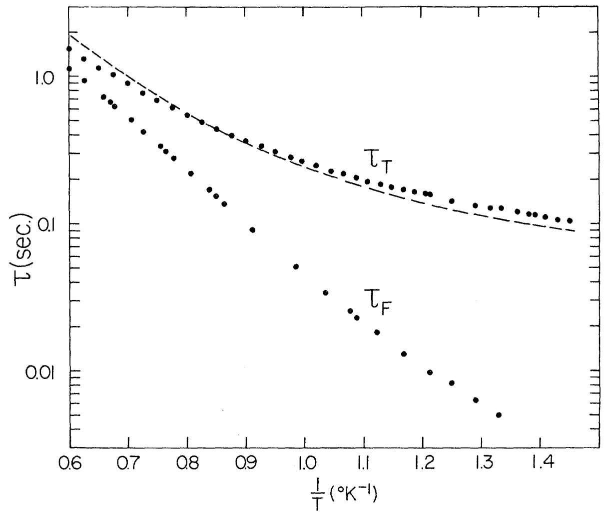

FIG. 2. Temperature dependence of the transit time of negative ions over a distance $L = 4.93$ cm and at the vapor pressure, $P=0$. The upper curve is the trapped-ion time; the lower curve, the free-ion time. The theory given by Eqs. (9), (11), and (12) is shown by the dashed line.

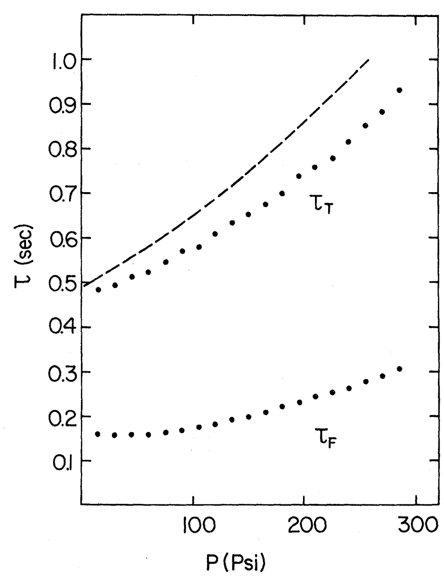

FIG. 3. Pressure dependence of the transit time of negative ions over a distance $L = 4.93$ cm at $T = 1.2$°K. The upper curve is the trapped-ion time; the lower curve, the free-ion time. Note the minimum in the free-ion time at low pressures. The theory is shown by the dashed line.

material is not expected to flow viscously over the ion surface, but probably scatters on impact with the ion. Assuming a solid-sphere boundary condition we obtain for $R_c \ll R_i$

$$F_c = \tfrac{3}{2}\pi\eta u R_c^2/R_i, \tag{10}$$

where η is the viscosity of the core, and u the velocity of the ion. The transit time over a distance L due to F_c is

$$\tau_c(P,T) = \frac{3\pi}{2}\frac{L}{eE}\frac{R_c^2(P,T)}{R_i(P)}\eta(P,T), \tag{11}$$

where E is the electric field in the apparatus and $\eta(P,T)$ is obtained by extrapolation of the data of Tjerkstra[10] ignoring the λ anomaly.

The total transit time on this model comes by adding the contributions of Eqs. (9) and (11):

$$\tau(P,T) = \tau_t + \tau_c, \tag{12}$$

and these are displayed in comparison with our data in Figs. 2 and 3. There is excellent qualitative agreement with experiment and fair quantitative agreement, which is gratifying in an analysis having no adjustable parameters. Note in particular that both the theoretical and experimental trapped-ion arrival times exhibit no initial decrease with pressure and exhibit large change from 0 to 25 atm (much larger than the corresponding change for free ions). The absence of a pronounced dip in the trapped-ion data persists at least as far down as 0.9°K and is evidence that the energy gap (and hence the rotons) dominates the scattering of trapped ions. Vortex waves apparently contribute very little in the experimentally accessible temperature range.

The vortex core model outlined here has several important consequences. Mutual friction calculations may depend on the details of the variation of N_r near the core, and should be reconsidered.[11] Recently Roberts and Donnelly[12] have shown that the expressions for the velocity and energy of vortex rings depend upon the entropy associated with the core. On our model, the core has three contributions to entropy: vortex waves, that extra entropy associated with the central core, and that associated with the tail. At higher temperatures when R_c grows larger (e.g., $R_c = 12.35$ Å at 2.1°K) and the tail spreads out, the total entropy may be sufficiently large to affect models of nucleation by substantially lowering the free energy barrier.[13]

There are obviously many uncertainties in both our model of the core and the transit-time calculations. For example, the energy localization estimate is only roughly known. Only qualitative calculations are possible, however, but these show that the effects of localization are small and hence the crudeness of the approximation is not too important to our final results. Another problem arises in the apparent exclusion of superfluid inside R_c (cf. Fig. 1). This occurs because our model is essentially an excitation model and does not properly account for the superfluid, which probably is present all the way to $r=0$. In calculating the substitution energy for ions near the core[3] we feel the condensate model of the superfluid vortex is still the most reliable. A unified model of superfluid and excitations near the core should be developed. Our assumption of constant total density could be replaced by a more rigorous criterion in, for example, a variational calculation for the total energy per unit length of vortex line.

Recently Douglass has published measurements at the vapor pressure of trapped-ion mobilities which extend to 0.8°K.[14] His results, which have an accuracy of roughly ±10%, are in satisfactory agreement with ours, which have an accuracy of roughly ±3%. Douglass has calculated the drag on the ion owing to vortex waves and shows that this contribution is important only at the lowest temperatures. The data of Fig. 2, which extend to 0.69°K, can be explained without reference to vortex-wave drag. Douglass has made some interesting comments on the role of trapped roton states without developing a detailed model.

A full discussion of this problem is in preparation. In spite of uncertainties in our model we can say that the excitation density increases dramatically near R_c, and R_c thus represents the scale over which superfluidity breaks down. In the spirit of this last statement we believe that our model adequately accounts for our observed ion transit times.

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NEUTRON SCATTERING IN FERROMAGNETIC DILUTE ALLOYS

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A new basis for the analysis of the neutron elastic-diffuse-scattering data in ferromagnetic dilute alloys is formulated and Low and Holden's data on PdFe are analyzed from this viewpoint. We especially discuss the impurity-concentration dependence of the range of the conduction-electron spin polarization about the magnetic impurity.

From measurements of the neutron elastic diffuse scattering in ferromagnetic dilute alloys one obtains information concerning the spatial distribution of the conduction-electron spins as well as the spin on the impurity atom.[1,2] The alloys of Fe or Co in Pd are especially interesting since the conduction-electron spin polarization has been shown to have a very long range, ~10 Å.[3] This exceedingly long range of magnetic perturbation due to a magnetic impurity is attributed to the large exchange enhancement of the Pd matrix[4] and the interatomic nature of the electron-electron interaction.[5]

Another interesting aspect of the problem, although it has received less attention, is that the range of conduction-electron spin polarization around a magnetic impurity decreases sharply with increasing concentration of magnetic impurities, i.e., the neutron scattering experiment shows that increasing the Fe or Co concentration from ~0.5 to 4.0 at.% reduces the range from 10 to 1 Å.[3] In this Letter we present an analysis of the concentration dependence of the conduction-electron spin polarization in ferromagnetic dilute alloys. The basis of our analysis is as follows: In exchange enhanced metals, the spatial extent of the spin polarization around an impurity disturbance may decrease sharply as the spin splitting of the host matrix increases.[6] Increasing the impurity concentration in dilute ferromagnetic alloys leads to an increase in the spin splitting of the host matrix bands. We have derived an expression for the conduction-electron spin polarization which self-consistently includes the concentration dependence of the spin splitting of the host matrix and accounts for the sharp concentration dependence observed in the neutron-scattering experiments by Low and Holden.[3] In analyzing the neutron-scattering experiments we show that simple parabolic bands with 0.36 hole/atom[7] are a very poor approximation for the $3d$ hole bands of Pd. This is in agreement with recent band-calculation results.[8] Further, the falloff with q of the susceptibility function for the actual band is expected to be significantly more rapid than that for a parabolic band.

The Fourier component of the conduction-electron spin polarization $\langle\sigma_z(q)\rangle$ $(q\neq 0)$ in terms of which the neutron scattering cross section will be described [see Eq. (6) below] is calculated starting with a finite number of magnetic impurities in the system. We obtain the following expression using the random phase approximation assuming the spins are ferromagnetically ordered