National Laboratory. Professor T. B. Day of the University of Maryland was of great assistance in our initial use of the programs TVGP and SQUAW. Mr. Michael Church and Mr. David Falconer made important contributions in the programming and data collection phase of the experiment. Most of all we thank our staff of scanners and measurers for their dedicated work. Finally we thank Professor G. Kane for invaluable discussions.

---

*Work supported by the U. S. Atomic Energy Commission.

†Address unknown.

‡Present address: Carnegie-Mellon University, Pittsburgh, Pa.

[1]J. Lys, J. Chapman, D. Falconer, C. T. Murphy, and J. C. Vander Velde, Phys. Rev. Letters 21, 1116 (1968).

[2]V. Domingo, G. P. Fisher, L. Marshall Libby, and R. Sears, Phys. Letters 25B, 486 (1967).

[3]W. M. Katz, B. Forman, and T. Ferbel, Phys. Rev. Letters 19, 265 (1967).

[4]O. Czyzewski, B. Escoubes, Y. Goldschmidt-Clermont, M. Guinea-Moorhead, T. Hofmokl, R. Lewisch, D. R. O. Morrison, M. Schneeberger, and S. de Unamuno, in the Proceedings of the Sienna International Conference on Elementary Particles (Societá Italiana de Fisica, Bologna, Italy, 1963), Vol. 1, p. 271.

[5]G. Lynch, P. Eberhard, G. Kalbfleisch, J. Lannutti, B. Maglić, J. Shafer, M. L. Stevenson, and N. Xuong, Phys. Rev. 131, 1287 (1963).

[6]T. M. Church et al., "Hyperon Production in $\bar{p}p$ Collisions Near 2 GeV/c," to be published.

[7]O. Dahl, L. Hardy, R. Hess, J. Kirz, D. Miller, and J. Schwartz, Phys. Rev. 163, 1430 (1967).

[8]D. Cline, C. Moore, and D. Reeder, Phys. Rev. Letters 19, 675 (1967).

[9]A. S. Carroll, J. Fischer, A. Lundby, R. H. Phillips, C. L. Wang, F. Lobkowicz, A. C. Melissinos, Y. Nagashima, C. A. Smith, and S. Tewksbury, Phys. Rev. Letters 21, 1282 (1968).

[10]J. K. Kim, Phys. Rev. Letters 19, 1079 (1967).

[11]V. Barger and D. Cline, Phys. Letters 25B, 415 (1967).

[12]Comparison of our data with the preliminary results of these other experiments and speculations on the possible existence of a resonance in the system may be found in C. T. Murphy, Argonne National Laboratory Report No. ANL/HEP 6812 (unpublished). Complete details pertaining to the analysis of this experiment are available in Michigan Bubble Chamber Research Note 81/68.

DIRECT EVIDENCE FOR THE MULTIPLET ASSIGNMENTS OF $\Lambda(1520)$ AND $\Lambda(1405)$†

Robert D. Tripp, Roger O. Bangerter, Angela Barbaro-Galtieri, and Terry S. Mast
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 31 October 1968)

A measurement has been made of the relative signs of the resonant $K^-p \to \Sigma\pi$ reaction amplitudes coming from partial waves corresponding to $\Sigma(1385)$, $\Lambda(1405)$, and $\Lambda(1520)$. From this it is shown that $\Lambda(1405)$ and $\Lambda(1520)$ are to be described as predominantly SU(3) singlets.

One of the triumphs of SU(3) has been the consistently correct prediction of the relative signs of resonant reaction amplitudes as derived from formation experiments. In particular, all the better established Y^* resonances formed in K^-p reactions and placed into singlets, octets, and decuplets according to mass formulas have correct relative signs between resonant amplitudes as measured in their $\Sigma\pi$ and $\Lambda\pi$ decay modes.[1,2] Breaking of SU(3) often alters the predicted decay rates considerably; however, unless it is severe, the relative signs are unaffected. In this Letter we investigate the interferences between the $J^P = \frac{3}{2}^+$ resonance $\Sigma(1385)$, the $J^P = \frac{1}{2}^-$ resonance $\Lambda(1405)$, and the $J^P = \frac{3}{2}^-$ resonance $\Lambda(1520)$ as measured in the reaction $K^-p \to \Sigma\pi$. Taking $\Sigma(1385)$ to be in a decuplet, we shall find that $\Lambda(1405)$ and $\Lambda(1520)$ are consistent with their conventional assignments as SU(3) singlets.

In this analysis we follow the procedure adopted by Watson, Ferro-Luzzi, and Tripp[3] of parametrizing the low-momentum K^-p coupled-channel amplitudes by constant complex scattering lengths and constant reaction phases, apart from the D_{03} amplitudes corresponding to $\Lambda(1520)$ which are written as Breit-Wigner resonances. All partial waves through $J = \frac{3}{2}$ are included.[4] The old experimental data[3] spanning the momentum region 250-513 MeV/c (c.m. energy 1470-1570 MeV) have been greatly augmented (about one-hundredfold at 390 MeV/c) by our more recent bubble-chamber experiment in the region 300-450 MeV/c. This partially completed experiment has so far yielded new angular distributions in the $\bar{K}^0n$,

Table I. Coupling coefficients of Y^* for various SU(3) representations.

		$g_{N\bar{K}Y^*}$	$g_{\Sigma\pi Y^*}$
Y_1^* (Σ)	Decuplet	$-(\frac{1}{6})^{1/2}g_{10}$	$(\frac{1}{6})^{1/2}g_{10}$
Y_0^* (Λ)	Singlet	$\frac{1}{2}g_1$	$(\frac{3}{8})^{1/2}g_1$
	Octet (D and F)	$(\frac{1}{10})^{1/2}g_{8D}+(\frac{1}{2})^{1/2}g_{8F}$	$-(\frac{3}{5})^{1/2}g_{8D}$
	27-plet	$(3/20)^{1/2}g_{27}$	$-(1/40)^{1/4}g_{27}$

$\Sigma^{\pm,0}\pi^{\pm,0}$, and $\Lambda\pi$ channels, and polarizations in the channels containing Σ^+, Σ^0, and Λ hyperons.[5] Although the question of relative phase of the resonant amplitudes enters for this particular study only in the $K^-p\to\Sigma\pi$ reactions, we have included other channels in the analysis in order to establish simultaneously the best values for the amplitudes.

The reaction amplitudes for $K^-p\to\Sigma\pi$ may be written in terms of the $I=0$ and $I=1$ $\Sigma\pi$ reaction amplitudes[6] as

$$T_{\Sigma^\pm\pi^\mp}=6^{-1/2}T_0\pm\tfrac{1}{2}T_1,$$
$$T_{\Sigma^0\pi^0}=-(6)^{-1/2}T_0. \qquad (1)$$

If the amplitude is resonant, it is proportional to

$$g_{N\bar{K}Y^*}g_{\Sigma\pi Y^*}/(E_R-E-\tfrac{1}{2}i\Gamma), \qquad (2)$$

where the coefficient of the coupling of each channel to the Y^* may be obtained from the Clebsch-Gordan coefficient of SU(3).[7] Table I shows each of these couplings for a decuplet Y_1^*, i.e., $\Sigma(1385)$, and for a Y_0^* belonging to a singlet, octet, or 27-plet. The subscripts on these real coupling constants indicate to which multiplet each belongs. We see that for the reaction $\bar{K}N\to\Sigma(1385)\to\Sigma\pi$ the numerator in the amplitude in (2) is negative: $-g_{10}^2/6$. Similarly for $\bar{K}N\to\Lambda(1520)\to\Sigma\pi$ if $\Lambda(1520)$ is a unitary singlet, the numerator is $(\frac{3}{32})^{1/2}g_1^2$, and if $\Lambda(1520)$ is a 27-plet, it is $-(3/800)^{1/2}g_{27}^2$. Thus in the interference between a decuplet and a singlet the numerators are of opposite sign; between a decuplet and a 27-plet they are of the same sign. For an octet Y_0^* the sign of the numerator depends on the relative strengths of the D and F couplings. For $\Lambda(1520)$ the only known $J^P=\frac{3}{2}^-$ octet into which it could possibly fit would make the sign[1] the same as for the decuplet. For the possibility of an octet $\Lambda(1405)$, the partially completed $J^P=\frac{1}{2}^-$ octet has an insufficient number of known decay rates[1] to fix the D and F couplings and therefore to predict the relative sign.

Figure 1 shows the Argand diagrams for the S_{01}, P_{13}, and D_{03} amplitudes (the subscript notation is $L_{I,2J}$) in the $\bar{K}N$ and $\Sigma\pi$ channels. The trajectory of the D_{03} amplitude in energy is shown by the dashed circle with a vector indicating its value at resonance; the vectors labeled S_{01} and

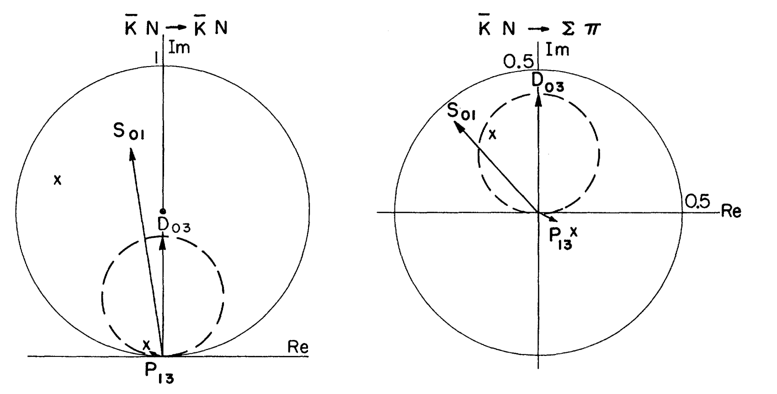

FIG. 1. Amplitudes for the reactions $\bar{K}N\to\bar{K}N$ and $\bar{K}N\to\Sigma\pi$ shown inside the unitary circles for the three partial waves of interest in this Letter. The correspondences are $\Sigma(1385)=P_{13}$, $\Lambda(1405)=S_{01}$, and $\Lambda(1520)=D_{03}$. The latter is parametrized as a resonance whose trajectory in energy is shown as a dashed circle. The overall arbitrary phase for the reaction amplitude $\bar{K}N\to\Sigma\pi$ is fixed by taking D_{03} to be imaginary at resonance. The S_{01} and P_{13} amplitudes are here parametrized as constant complex scattering lengths. Their phases in the $\Sigma\pi$ channel are left free to be determined by this experiment. It is these phases which show $\Lambda(1405)$ and $\Lambda(1520)$ to have the same relative sign, and $\Sigma(1385)$ and $\Lambda(1520)$ to have the opposite relative sign. Crosses near the heads of the arrows show where the S_{01} and P_{13} amplitudes would reach if parametrized as Breit-Wigner resonances, as described in the text.

P_{13} are shown at 395 MeV/c (corresponding to the energy of the D_{03} resonance). The $\bar{K}N$ amplitudes in Fig. 1(a) are not relevant to this discussion since the relative signs are always positive. However, they do show that the elastic channel amplitudes do have the correct orientations to be described by the behavior beyond resonance of $\Lambda(1405)$ and $\Sigma(1385)$. The $\Sigma\pi$ amplitudes in Fig. 1(b) display the signs with which we are concerned in this Letter, namely that relative to the D_{03} amplitude of $\Lambda(1520)$, the S_{01} amplitude representing the high-energy tail of $\Lambda(1405)$ has the same sign. The P_{13} amplitude given by the high-energy tail of $\Sigma(1385)$, however, shows that at its resonant energy this amplitude is negative imaginary, i.e., opposite in sign to $\Lambda(1520)$, which demonstrates that the signs of the $\Sigma\pi$ amplitudes for $\Lambda(1520)$ and $\Lambda(1405)$ relative to $\Sigma(1385)$ are as expected from unitary singlets. It also excludes the assignment of $\Lambda(1520)$ in the $\frac{3}{2}^-$ octet or the assignments of $\Lambda(1520)$ or $\Lambda(1405)$ into 27-plets.[8]

Having seen that the S_{01} and P_{13} amplitudes at our energy have phases in both $\bar{K}N$ and $\Sigma\pi$ which are quite consistent with an interpretation that they are the high-energy tails of their respective resonances,[9] we have altered the parametrization of these states from the previous constant-scattering-length description to that of simple Breit-Wigner resonances. Threshold dependences were given by appropriate phase-space and centrifugal-barrier factors. Reminimization of all amplitudes yielded fits with higher χ^2 than the previous parametrization,[10] but nonetheless gratifyingly good considering the simplicity and highly constrained nature of this model, in which there are no free phases for the resonant amplitudes. In these fits we have fixed the masses and partial widths at resonance of $\Sigma(1358)$ and $\Lambda(1405)$ at their experimental values,[11] whereas the $\bar{K}N$ partial widths (of course below 1423 MeV) were left free to be determined by this experiment. For $\Sigma(1385)$ the ratio of the reduced $\bar{K}N$ width thus obtained to that expected by SU(3) was 1.5, and for $\Lambda(1405)$ it was 8.0. This indicates that deviations from SU(3) symmetry are small for $\Sigma(1385)$ but are quite appreciable for $\Lambda(1405)$, as has been noted previously.[12]

It is perhaps worthwhile to exhibit the data that are most relevant to the sign determination. The interference between $\Sigma(1385)$ and $\Lambda(1520)$ appears most clearly in the A_3 coefficient of the Legendre polynomial expansion of the angular distribution, since this coefficient arises only from P_3-D_3 interference. From Eq. (1), if we take the difference in the A_3 coefficients between $\Sigma^+\pi^-$ and $\Sigma^-\pi^+$, the interference occurs only between states of different isospin. Since the $I=1$ D amplitude is small, this quantity comes almost purely from $\Sigma(1385)$-$\Lambda(1520)$ interference. Figure 2(a) displays the data expressed as a ratio A_3/A_0 (we do not yet have measurements of the partial cross sections) and the curves corresponding to the two choices of relative sign. The large excursion from positive to negative, passing through zero at 385 MeV/c (just before resonance), requires the P_{13} amplitude to be as indicated in Fig. 1(b). Similarly the $I=0$ $\Sigma^0\pi^0$ angu-

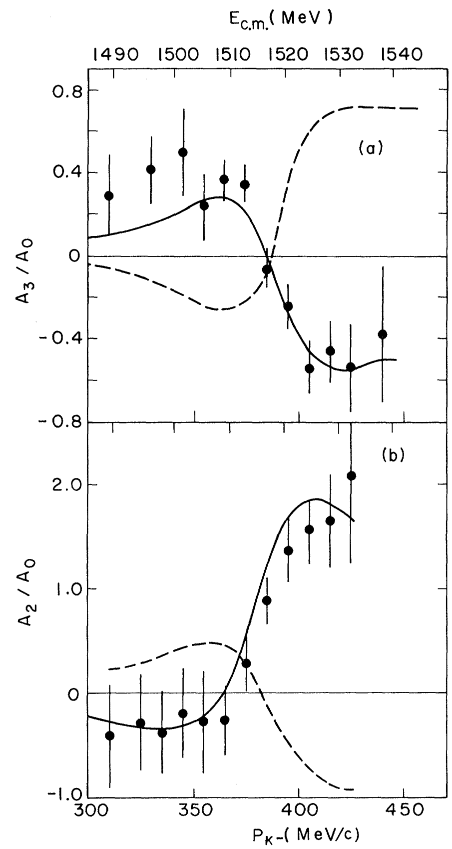

FIG. 2. Data from this experiment showing most clearly the interference between $\Lambda(1520)$ and the high-energy tails of $\Sigma(1385)$ and $\Lambda(1405)$. (a) The difference between the A_3/A_0 coefficient of the Legendre polynominal expansion of the $\Sigma^+\pi^-$ and $\Sigma^-\pi^+$ angular distributions. This arises primarily from P_{13}-D_{03} interference. The solid curve shows the best fit to all the data measured in the experiment; the dashed curve results from reversing the P_{13} phase. (b) A_2/A_0 for $\Sigma^0\pi^0$. This arises primarily from S_{01}-D_{03} interference. The solid curve is the best fit to all data; the dashed curve is the fit with the phase of S_{01} reversed.

lar distribution best shows the interference between $\Lambda(1405)$ and $\Lambda(1520)$. Figure 2(b) shows the A_2/A_0 ratio that arises primarily from the S and D states and their interference, since the P amplitudes are small compared with S and D. The curves correspond to the two choices of relative sign of $\Lambda(1405)$ with respect to $\Lambda(1520)$. It is clear from the data that beyond resonance the two amplitudes must be nearly in phase, as is required if both resonances are SU(3) singlets.

---

*Work supported under the auspices of the U. S. Atomic Energy Commission.

[1]R. Tripp et al., Nucl. Phys. B3, 10 (1967). An updated version of these SU(3) comparisons of baryon resonances will appear in the Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968, to be published.

[2]The method of employing relative signs for SU(3) classification was first used by A. Kernan and W. Smart, Phys. Rev. Letters 17, 832 (1966).

[3]M. Watson, M. Ferro-Luzzi, and R. Tripp, Phys. Rev. 131, 2248 (1963).

[4]There are no indications that higher partial waves appear in detectable amounts at the low energies considered here. We have altered slightly the analysis method of Ref. 3 by employing the appropriate c.m. momentum for each channel in the decay widths of the resonant amplitudes, using a fixed radius of interaction of 1 F in the expression for the centrifugal barrier.

[5]Σ^+ polarizations from this experiment have been published in R. Bangerter et al., Phys. Rev. Letters 17, 495 (1966).

[6]We reverse the convention used in Ref. 3 to make it more appropriate for discussion of unitary symmetry.

[7]J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

[8]From the known branching fractions of $\Lambda(1520)$ and $\Lambda(1690)$ (a presumed member of the $\frac{3}{2}^-$ octet) there is reasonably good evidence for some singlet-octet mixing between these two states, as noted by Ref. 1 and by G. B. Yodh, Phys. Rev. Letters 18, 810 (1967). In order to alter the sign of the resonant $\Lambda(1520)$ amplitude the mixing angle would have to exceed about 45 deg. The estimate for this angle is about 16 deg. For $\Lambda(1405)$ there is evidence for much stronger SU(3) symmetry breaking, as discussed later in this Letter. The conclusion of our experiment concerning $\Lambda(1405)$ is that the symmetry breaking is not so severe as to alter the sign of the resonant amplitude.

[9]J. Kadyk, Y. Oren, G. Goldhaber, S. Goldhaber, and G. Trilling [Phys. Rev. Letters 17, 599 (1966)] have investigated the $\Lambda\pi$ channel in the reaction $K_2^0p \rightarrow \Lambda\pi^+$ at low energies and find that the angular distributions require a substantial P_{13} amplitude whose magnitude is consistent with the tail of $\Sigma(1385)$. A similar interpretation based on their data and those of Ref. 3 has been put forward by J. K. Kim, Phys. Rev. Letters 19, 1074 (1967). We may also note that there are at present no other known P_{13} resonances which could contribute a substantial amplitude in the vicinity of 1520 Mev.

[10]The ratio $\chi^2/\langle\chi^2\rangle$ (where $\langle\chi^2\rangle$ is the expected chi squared) for the constant-scattering-length parametrization is 1.23, indicating either that detectable differences with this constraining parametrization are beginning to appear, or that the data in their preliminary form have some additional biases. The resonant parametrizations have yielded ratios of 1.38 for P_{13} parametrized as a resonance and 1.85 for S_{01} parametrized as a resonance.

[11]Particle Data Group, University of California Radiation Laboratory Report No. UCRL-8030 Revised, 1968 (unpublished).

[12]C. Weil, Phys. Rev. 161, 1682 (1968). For this $\Lambda(1405)$ ratio which should be 1 in exact SU(3) Weil obtains 11.1 by use of the S_{01} scattering lengths of J. K. Kim [Phys. Rev. Letters 14, 29 (1965)]. A more recent analysis by J. K. Kim and F. von Hippel (to be published) yields 6.8 ± 1.0 for this ratio.

VENEZIANO FORMULA WITH TRAJECTORIES SPACED BY TWO UNITS*

Stanley Mandelstam
Department of Physics, University of California, Berkeley, California
(Received 30 October 1968)

By representing a scattering amplitude as a sum of terms of the Veneziano type, we can cancel alternate trajectories and remain with trajectories spaced by two units of angular momentum. This result is obtained without imposing a supplementary condition and without introducing poles in the Regge residues at nonsense wrong-signature integers.

Veneziano[1] has proposed a formula for a scattering amplitude which has Regge asymptotic behavior in all channels. The Regge trajectories in such an amplitude are normally spaced at unit distance from one another in α, but Veneziano has proposed a supplementary condition[2] on the trajectory parameters which would remove alternate trajectories. A somewhat different formula for an amplitude with Regge asymptotic behavior in all channels has been proposed by Virasoro.[3] Virasoro's amplitude has trajectories spaced by two units, even though he does not impose a sup-