BINDING ENERGIES AND TWO-PARTICLE SPECTRA*

George F. Bertsch
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 4 October 1968)

The average interaction of particles in a closed shell is consistent with the spectrum of the nucleus with two particles in the shell only if the correlations which are present in the two-particle case but not in the closed-shell case are considered. Realistic forces which induce large correlations give a sizable effect which is observable in the $d_{5/2}$, $d_{3/2}$, and $f_{7/2}$ shells. This may be interpreted as a repulsive effective three-body force.

A consistent picture of nuclear forces in nuclei would allow the interaction energy of closed shells to be related to the interaction energy between two particles, determined from the spectrum of a nucleus with two valence particles in the shell. Specifically, the shell-model interaction energy of a closed j shell is[1]

$$\mathcal{V} = (2j+1)(j+1)\bar{V}_{T=0} + 3j(2j+1)\bar{V}_{T=1}, \tag{1}$$

where

$$\bar{V}_{T=0} = \sum_{J\ \mathrm{odd}} (2J+1)V_J \Big/ \sum_{J\ \mathrm{odd}} (2J+1),$$

$$\bar{V}_{T=1} = \sum_{J\ \mathrm{even}} (2J+1)V_J \Big/ \sum_{J\ \mathrm{even}} (2J+1). \tag{2}$$

The two-particle effective interaction $V_J = \langle (jj)^J | \times V | (jj)^J \rangle$ is to be determined from the binding of the state J in the nucleus with two particles in the j shell. To separate the interaction from single-particle energies and core energies, the total binding of several nuclei must be compared:

$$-V_J = E_A((j^2)J) - 2E_{A-1}(j) + E_{A-2}(0^+\mathrm{core}). \tag{3}$$

In this naive form, the consistency is not found, and this has been a long-standing puzzle. It is the purpose of this note to show that when certain correlations are taken into account, fairly accurate agreement is possible.

Most two-particle correlations are independent of other particles in the shell. Writing the two-particle wave function as

$$\psi((j^2)J) = \alpha |(jj)^J\rangle + \sum_{j',j''} \beta_{j'j''}{}^J |(j'j'')^J\rangle, \tag{4}$$

we would get the same correlations in the "closed shell" by writing the wave function as

$$\varphi \sim |\,\mathrm{closed\ shell}\rangle + \sum_{j'j''JT} [(2J+1)(2T+1)]^{\frac{1}{2}} (\beta_{j'j''}{}^J/\alpha) | \{(j^{-1}j^{-1})^{J,T}(j'j'')^{JT}\}^{00}\rangle + O(\beta^2) + \cdots. \tag{5}$$

Evaluating the expectation of (5) with a Hamiltonian will give Eq. (1) with the V_J determined from the expectation of the Hamiltonian in the two-particle wave function (4), except for terms of third and higher order in the interaction.

However, some of the two-particle correlations cannot be included in the many-particle wave function this way. If j' or $j''=j$, we cannot write the same orbital as both particle and hole, so the configuration in the second term would have to be $|(j^{-1}j')^0\rangle$. By angular-momentum restrictions, this is not possible within the same major shell. Thus to compare with closed-shell energies, we should subtract out the effect of correlations where one particle remains in the j shell and the other particle is excited to a different shell.

In perturbation theory this is easily seen to be an exclusion effect. The correlations contribute to the effective interaction by means of diagrams such as in Fig. 1(a), where a particle in the jm

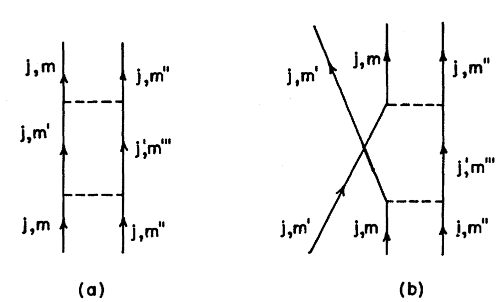

FIG. 1. Perturbation diagrams corresponding to correlations discussed in text. (a) is a typical contribution to the two-particle effective interaction. In a many-particle wave function this is quenched by the three-body diagram (b).

Table I. Change of eigenvalue of the two-particle Hamiltonian matrix caused by omitting configurations with one particle excited to a different shell. (Units are MeV.)

Shell state	$d_{5/2}$ particles	$d_{3/2}$ holes	$f_{7/2}$ particles
$J=0$	none	none	none
1	2.795	1.096	1.121
2	0.772	0.184	0.177
3	3.064	0.300	0.804
4	0.427		0.127
5	none		0.746
6			0.094
7			none

orbit is scattered to the jm' orbit. If the m' orbit is already occupied, the contribution with $m = m'$ must be cancelled by another contribution from the linked diagram as shown in Fig. 1(b). This is a three-body diagram, repulsive because it is an exclusion effect.

What makes possible the calculation of correlations to reasonable accuracy is the recent appearance of sophisticated nuclear potentials.[2,3] These give large off-diagonal matrix elements and therefore substantial correlations. We compute the effect of the correlations with Kuo's matrix elements, considering only the correlations within a major shell. The correlations where a particle has a $2\hbar\omega$ excitation should be considered also, but will approximately cancel when both the excitations to the j shell are computed.

The excitations from the j shell to the next higher j shell of the same parity, i.e., with one extra radial node, give the change in force due to the change in size of the orbitals, which over many shells must go as $A^{-1/2}$. However, within a single shell this effect seems to be much weaker, and we do not believe that there is an increase in the volume of the orbitals.

The method of calculation is to diagonalize the two-particle Hamiltonian matrix with and without configurations of the type $|jj'\rangle$. The correction to the observed interaction in the state J of the two-particle nucleus is taken as the difference between the lowest calculated eigenvalues. In Table I we give the change in the eigenvalues for the different J states in the $d_{5/2}$, $d_{3/2}$, and $f_{7/2}$ shells. Table II shows the average interaction and binding energy results for these shells.

For the $T=1$ interaction in the $f_{7/2}$ shell, Federman and Talmi[4] performed an analysis including an additional correlation in Ca^{42} not present in Ca^{48}, the coupling to deformed states. Agreement for Ca^{48} binding can be produced with a reasonable value for this coupling. The prediction which their modified value of the $T=1$ interaction gives for the Ni^{56} binding is shown in parentheses in Table II. Agreement is better. However, this leaves little room for admixtures of deformed $T=0$ states, which probably are present at low excitation in the Sc^{42} spectrum.[5]

There are two additional problems that account of the correlations helps to understand. First, average pairing energies are larger than predicted from the two-particle spectrum.[1] Since the correlations that we consider affect only the $J \neq 0$ interaction, their removal increases the sep-

Table II. Binding energies and average interaction energies of particles in various shells. All energies have single particle and Coulomb energies removed. Empirical bindings are from J. E. Mattauch et al., Nucl. Phys. 67, 1 (1964). Units are in MeV.

Shell and Nucleus with 2-particle Spectrum			Average Interaction from Spectrum	Modified Interaction	Closed Shell Nucleus	Binding from 2-particle Spectrum	Binding Corrected for Correlations	Experimental Binding of Shell
$d_{5/2}$	T=1	O^{18}	-1.11	-0.60				
	T=0	F^{18}	-4.14	-2.72	Si^{28}	137.1	84.1	86.1
$d_{3/2}$	T=1	Ar^{38}	-0.60	-0.45	S^{36}	3.60	2.68	1.82
	T=0	K^{38}	-2.40	-1.86	S^{32}	34.8	26.6	27.1
$f_{7/2}$	T=1	Ca^{42}	-0.47	-0.35	Ca^{48}	13.13	9.91	7.03
	T=0	Sc^{42}	-2.14	-1.66	Ni^{56}	116.3	89.5 (80.8)	75.0

aration of the $J=0$ energy and the average $J\neq 0$ energy.

Second, it is found that the average Coulomb interaction across a shell is weaker than the average Coulomb interaction of two protons in the nucleus with only two valence particles.[6] Any mechanism which quenches attractive correlations would decrease the average Coulomb interaction. Of the 47 keV per pair to be accounted for in the $d_{5/2}$ shell,[6] these correlations are responsible for 30 keV.

*Work supported in part by the U. S. Atomic Energy Commission.

[1]A. de-Shalit and I. Talmi, Nuclear Shell Theory (Academic Press, Inc., New York, 1963), p. 452.

[2]T. T. S. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966), and A114, 241 (1968).

[3]J. P. Elliott, A. D. Jackson, H. A. Mavromatis, B. Singh, and E. A. Sanderson, to be published.

[4]P. Federman and I. Talmi, Phys. Letters 22, 469 (1966).

[5]R. Sherr, private communication.

[6]G. F. Bertsch, Phys. Rev. 174, 1313 (1968).

HIGH-ENERGY PICKUP REACTIONS AND NUCLEAR CORRELATIONS*

William A. Friedman
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 4 November 1968)

A calculation of the forward-angle differential cross section for the reaction $p+A \to (A-2)+p+d$ indicates that processes involving nucleons which are uncorrelated when the projectile initially interacts with the target are as significant as those processes involving correlated nucleons.

For many years there has been a hope that pickup reactions yielding high-energy deuterons might provide direct information about short-range nuclear correlations. It was felt that these experiments would detect the presence of the high-momentum components of the nuclear wave function which are related to these correlations. Work dating back almost 20 years has encouraged this hope. In 1950 Chew and Goldberger[1] derived an expression for the high-energy cross section for the deuteron pickup reaction. The expression they obtained related the pickup differential cross section to the probability that the target nucleus contain nucleons with momentum equal to the vector difference between the momenta of the incoming proton and the outgoing deuteron. They suggested that this cross section could thus be used to probe the momentum distribution of the target nucleons. At about the same time, Heidmann[2] derived a similar expression in which he introduced a two-body wave function representing the relative motion between two nucleons in the nucleus. He claimed that the pickup reaction provided a measurement of the probability distribution for the relative momentum between the two nucleons of the pair. In 1955 Brueckner, Eden, and Francis[3] repeated arguments similar to Heidmann's in suggesting a means for experimentally verifying the existence of the nucleon-nucleon correlations which were then being predicted and calculated by Brueckner and others. These old papers seem to have laid a basis of high expectations for pickup reactions.

At the large proton accelerators at CERN[4] in 1960 and at Brookhaven National Laboratory[5] two years later, using 30-BeV protons and several targets, a surprisingly large number of high-energy deuterons were observed. At Cambridge Electron Accelerator,[6] reactions initiated by high-energy electrons again produced a copious supply of high-momentum deuterons. More recently, experiments done at 1 BeV on the Cosmotron[7] revealed relatively large cross sections for high-momentum (forward angle) deuterons.

A pickup mechanism is probably involved in some, if not all, of these experimental observations. In light of this, two conclusions could be drawn. Either the pickup formalisms cited above were relevant and there was direct evidence that the probability for high-momentum components in the nuclear wave functions was greater than had been thought,[8] or else the analysis, linking pickup cross sections so directly to highly excited target nucleons, required re-examination at the high energies which were being encountered.[9] The work reported here derives from the second conclusion. Following a suggestion by Brown, several mechanisms contributing to the pickup process are investigated, and their significance is evaluated in the context of the recent 1-BeV experiments.

The diagrams in Fig. 1 indicate schematically