Carovillano, Phys. Rev. 122, 1185 (1961); K. E. Lassila, Phys. Rev. 135, A1218 (1964).

[8]L. C. Himmel and P. R. Fontana, Phys. Rev. 162, 23 (1967).

[9]H. Metcalf, thesis, Brown University, 1967 (unpublished).

[10]We have done the calculations of Ref. 8 independently and have also used the matrix elements presented by S. J. Brodsky and R. G. Parsons, Phys. Rev. 163, 134 (1967), to calculate both the shape of the signal and the positions of the two hfs crossing points. All three calculations of the position of the center of the level crossing signal agree to within 2 ppm and the calculation of the hfs doublet separation agrees to within 0.1%.

[11]The short term stability of the power supply is about 10 ppm.

[12]Two identical Lorentzians, each with a small admixture of dispersion to allow for various geometric imperfections in the experiment.

[13]If there is a contribution to the asymmetry which is not dispersion-like, then the error in the center frequency introduced by treating it as a dispersion is very small. For a linear type of asymmetry (a ramp multiplying the signal) this error is $\beta^3\times$ (linewidth). All expected contributions to the asymmetry are either linear or dispersive.

EFFECTS OF $\Delta S=-\Delta Q$ ON CP-NONCONSERVATION PHENOMENOLOGY IN K^0 DECAY*

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Incorporating the present experimental value of the $\Delta S=-\Delta Q$ amplitude into the K^0 CP-nonconservation phenomenology, we find that solution II (η_{00} and η_{+-} in opposite quadrants) is as viable as solution I (η_{00} and η_{+-} in the same quadrant), though both have poor fits. Thus, if a $\Delta S=-\Delta Q$ amplitude is allowed, a clear choice between these two solutions cannot yet be made.

Present experiments on K^0 decays indicate the possibility of a nonzero $\Delta S=-\Delta Q$ amplitude in the weak current.[1] The purposes of this paper are to indicate the ways in which this amplitude enters into the K^0 CP-nonconservation phenomenology and, making use of this expanded formalism, to show that there are two possible solutions to the phenomenology. Specifically we find, using the present experimental value for the $\Delta S=-\Delta Q$ amplitude, that "solution II" (η_{00} and η_{+-} in opposite quadrants, $|\epsilon|<|\epsilon'|$)[2] and "solution I" (η_{00} and η_{+-} in the same quadrant, $|\epsilon|>|\epsilon'|$)[2] have comparable, though poor, fits. Further, the choice between solution I and solution II is relatively independent of $|\eta_{00}|$ and φ_{+-} and depends primarily on better further determination of φ_{00}, $\frac{1}{2}\delta$, x, and $\delta_0-\delta_2$. If we set $x=0$, the best fit for solution I is changed very little while solution II is essentially eliminated.

We follow the notation of Lee and Wu[3]:

$$|K_L^{\,0}\rangle\equiv[2(1+|\epsilon|^2)]^{-1/2}[(1+\epsilon)|K^0\rangle-(1-\epsilon)|\bar{K}^0\rangle],$$

where

$$\epsilon=\frac{(\Gamma_{12}-\Gamma_{12}{}^*)+i(M_{12}-M_{12}{}^*)}{(\gamma_S-\gamma_L)+2i(m_S-m_L)},\tag{1}$$

γ_S and γ_L are the total decay rates of $K_S^{\,0}$ and $K_L^{\,0}$, and $\Delta m=m_S-m_L<0$. The off-diagonal elements of the decay and mass matrices are

$$\Gamma_{21}=\Gamma_{12}{}^*=\pi\textstyle\sum_F\rho_F\langle\bar{K}^0|H|F\rangle\langle F|H|K^0\rangle,\tag{2}$$

$$M_{21}=M_{12}{}^*$$
$$=\langle\bar{K}^0|H|K^0\rangle$$
$$+\textstyle\sum_n\mathcal{P}[\langle\bar{K}^0|H|n\rangle\langle n|H|K^0\rangle/(m_{K^0}-m_{n^0})].$$

For leptonic decays we define the $\Delta S=-\Delta Q$ amplitude[4] by

$$x=|x|\exp(i\varphi_x)\equiv g/f,\tag{3}$$

where

$$\langle\pi^-l^+\nu|H|K^0\rangle=f,\quad\langle\pi^+l^-\nu|H|K^0\rangle=g^*;$$
$$\langle\pi^-l^+\nu|H|\bar{K}^0\rangle=g,\quad\langle\pi^+l^-\nu|H|\bar{K}^0\rangle=f^*.$$

From Eq. (1) we can relate $\mathrm{Im}\epsilon$ and $\mathrm{Re}\epsilon$.[5] Let $\epsilon=\epsilon_\Gamma+\epsilon_M$, where $\epsilon_\Gamma=D^{-1}(\Gamma_{12}-\Gamma_{12}{}^*)$, $\epsilon_M=iD^{-1}(M_{12}-M_{12}{}^*)$, and $D=(\gamma_S-\gamma_L)+2i(m_S-m_L)$. The contributions to ϵ_Γ are from on-the-mass-shell intermediate states, while ϵ_M has contributions from off-the-mass-shell states. Since M is Hermitian, $(M_{12}-M_{12}{}^*)$ is purely imaginary and the phase of ϵ_M, determined by D^{-1}, is $43.2°\pm1.2°$. If $\epsilon_\Gamma=0$, then

$$\mathrm{Im}\epsilon=\frac{-2(m_S-m_L)}{\gamma_S-\gamma_L}\mathrm{Re}\epsilon=(0.94\pm0.04)\,\mathrm{Re}\epsilon,\tag{4}$$

where the uncertainty comes primarily from Δm.

Now ϵ_Γ can be written as $\epsilon_\Gamma = \epsilon_\Gamma(l) + \epsilon_\Gamma(3\pi) + \epsilon_\Gamma(2\pi, I=2)$, where the three terms describe intermediate leptonic, three-pion, and $I=2$ two-pion states, respectively. In particular,

$$\epsilon_\Gamma(l) = D^{-1}[(\Gamma_{12}-\Gamma_{12}^*)_{\pi^-l^+\nu} + (\Gamma_{12}-\Gamma_{12}^*)_{\pi^+l^-\nu}] = D^{-1}[4\pi i \sum \rho_l |F|^2 \,\mathrm{Im}x], \tag{5}$$

where ρ_l is the density of final leptonic states. Normalizing to

$$\gamma_L(\mathrm{lept}) = 2\pi\sum\rho_l[|\langle\pi^-l^+\nu|H|2^{-1/2}|K^0-\bar{K}^0\rangle|^2 + |\langle\pi^+l^-\nu|H|2^{-1/2}|K^0-\bar{K}^0\rangle|^2]$$
$$= 2\pi\sum\rho_l|f|^2|1-x|^2, \tag{6}$$

where terms $O(\epsilon)$ have been neglected, we obtain

$$\epsilon_\Gamma(l) = \frac{\gamma_L(\mathrm{lept})}{D}\frac{2i\,\mathrm{Im}x}{|1-x|^2}. \tag{7}$$

Using the arguments of Lee and Wu[6] to introduce upper limits on $|\epsilon_\Gamma(3\pi)|$ and $|\epsilon_\Gamma(2\pi, I=2)|$ and assuming $\gamma_S(3\pi) \leq 0.14\gamma_L(3\pi)$,[7] we find $|\epsilon_\Gamma(3\pi)| \leq 0.15\times10^{-3}$ and $|\epsilon_\Gamma(2\pi, I=2)| \simeq 0.045|\epsilon'|$. Then incorporating Eq. (7) and these supper limits into Eq. (4), we find the limits of $\mathrm{Im}\epsilon$ to be

$$\mathrm{Im}\epsilon = \frac{-2(m_S-m_L)}{\gamma_S-\gamma_L}\mathrm{Re}\epsilon + \frac{\gamma_L(\mathrm{lept})}{\gamma_S}\frac{2\,\mathrm{Im}x}{|1-x|^2} \pm \Delta(\epsilon_\Gamma) = (0.94\pm0.04)\,\mathrm{Re}\epsilon + 2.14\times10^{-3}\frac{\mathrm{Im}x}{|1-x|^2} \pm \Delta(\epsilon_\Gamma), \tag{8}$$

where

$$\Delta(\epsilon_\Gamma) = \frac{|D|}{\gamma_S}[|\epsilon_\Gamma(3\pi)| + |\epsilon_\Gamma(2\pi, I=2)|] \leq 0.21\times10^{-3} + 0.062|\epsilon'|.$$

Note that $\Delta(\epsilon_\Gamma)$ is an upper limit and is never exceeded in our fits.

Our fitting procedure then employs Eq. (8), the expression for the K_L^0 leptonic decay asymmetry,

$$\tfrac{1}{2}\delta = \mathrm{Re}\epsilon\frac{1-|x|^2}{|1-x|^2}, \tag{9}$$

and the equations for the CP-nonconserving amplitudes,

$$\eta_{+-} = |\eta_{+-}|e^{i\varphi_{+-}} = \epsilon + \epsilon',$$
$$\eta_{00} = |\eta_{00}|e^{i\varphi_{00}} = \epsilon - 2\epsilon', \tag{10}$$

where $\epsilon' = 2^{-\frac{1}{2}}ie^{-i(\delta_0-\delta_2)}(\mathrm{Im}A_2/A_0)$, δ_0 and δ_2 are the S wave, $\pi\pi$ phase shifts for $I=0, 2$. These equations and the world averages[8] given below produce a system of two constraints (independent of whether x is specified by the experimental measurements or constrained to be zero):

$$|\eta_{+-}| = (1.96\pm0.06)\times10^{-3},$$
$$\varphi_{+-} = 63°\pm10°,$$
$$|\eta_{00}| = (3.62\pm0.38)\times10^{-3},$$
$$\tfrac{1}{2}\delta = (1.18\pm0.22)\times10^{-3},$$
$$\delta_0-\delta_2 = +57°\pm18°,$$
$$\mathrm{Re}x = +0.08\pm0.10,$$
$$\mathrm{Im}x = -0.18\pm0.10.$$

The best fits for solutions I and II, $x\neq0$ and $x=0$, are indicated in Table I and Figs. 1 and 2. Note that solution II, $x=0$ is extremely unlikely, whereas the other three solutions have comparable, though poor, fits.[9] The dramatic improvement in solution II, obtained by incorporating the present world average value for x into the phenomenology, occurs because both the negative value of $\mathrm{Im}x$ and the positive value of $\mathrm{Re}x$, as they enter into $\epsilon_\Gamma(l)$ and the relationship between $\frac{1}{2}\delta$ and $\mathrm{Re}\epsilon$, conspire to improve the fit to solution II.

Selection between solutions I and II can most directly be made by a measurement of φ_{00}. Lacking such a measurement, we find that the choice between these solutions is sensitive to changes in the measurements of $\frac{1}{2}\delta$, x, and $\delta_0-\delta_2$, but is relatively insensitive to changes in $|\eta_{00}|$ and φ_{+-}.

Table I. Summary of the best fits for solutions I and II, $x \neq 0$ and $x = 0$. The values for $|\eta_{+-}|$, $|\eta_{00}|$, $\frac{1}{2}\delta$, Reϵ, and Imϵ are in units of 10^{-3}. Since the uncertainties in γ_S, γ_L, and Δm affect the fittings only slightly, these parameters have not been displayed in the table. For each fit, the expected value of χ^2 is 2.0.

	Fitted parameters								Derived parameters		
	$\lvert\eta_{+-}\rvert$	φ_{+-}	$\lvert\eta_{00}\rvert$	$\frac{1}{2}\delta$	$\delta_0-\delta_2$	Rex	Imx	χ^2	Reϵ	Imϵ	φ_{00}
Exptl value	1.96 ±0.06	63° ±10°	3.62 ±0.38	1.18 ±0.22	57° ±18°	+0.08 ±0.10	−0.18 ±0.10				
$x \neq 0$											
Solution I	1.95	54°	3.54	1.50	72°	−0.06	−0.07	8.1	1.71	1.76	37°
Solution II	1.98	51°	3.45	0.92	20°	+0.20	−0.18	8.8	0.66	−0.11	261°
$x = 0$											
Solution I	1.94	55°	3.33	1.59	69°	0	0	10.3[a]	1.59	1.77	40°
Solution II	1.98	59°	3.07	0.48	19°	0	0	21.8[a]	0.48	0.13	259°

[a]Including $\chi^2(\vec{x} = 0) = 3.8$.

Decreased values of $\frac{1}{2}\delta$ and $\delta_0-\delta_2$, a more negative value of Imx, and an increase in Rex would make solution II more probable, while opposite changes in these quantities would increase the likehood of solution I. The fits to both solutions I and II would be improved by smaller values of $|\eta_{00}|$ and φ_{+-}.

An appreciable decrease in $|\eta_{00}|$ and φ_{+-} would allow a good fit to solution I yielding experimental values compatible with the "superweak" model.[10] However, without a measurement of φ_{00}, such a set of experimental parameters also provides a reasonable fit to solution II, $x \neq 0$. For example, $|\eta_{00}| = (1.96 \pm 0.20) \times 10^{-3}$ and $\varphi_{+-} = 43° \pm 6°$ give χ^2 (solution II, $x \neq 0$) = 2.8 for two constraints.[11] Even assuming solution I, such experimental values are consistent not only with the "superweak" model, but also with other models, including a $\Delta S = -\Delta Q$ model in which all violations of CP are ascribed to an amplitude with Re$x = 0$ and Im$x \simeq -(0.1$ to $0.2)$.[12]

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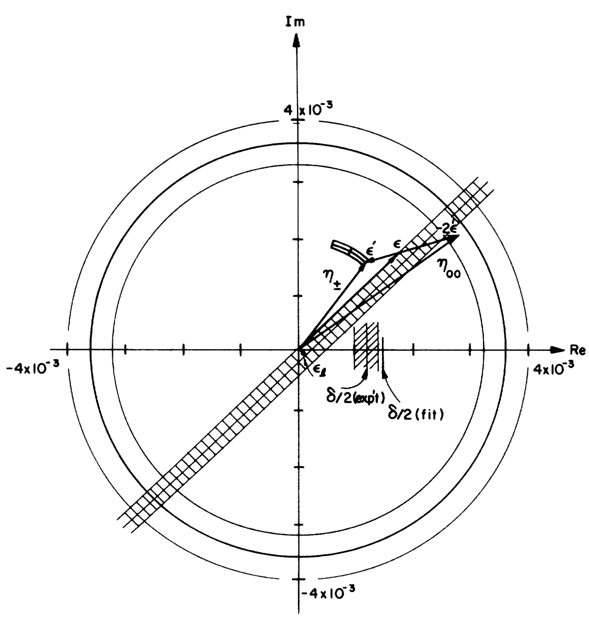

FIG. 1. The best fit for solution I, $x \neq 0$. $\frac{1}{2}\delta$ (best fit) = 0.88Reϵ. Diagonal cross-hatched area indicates the allowable values of ϵ. [See Eq. (8).]

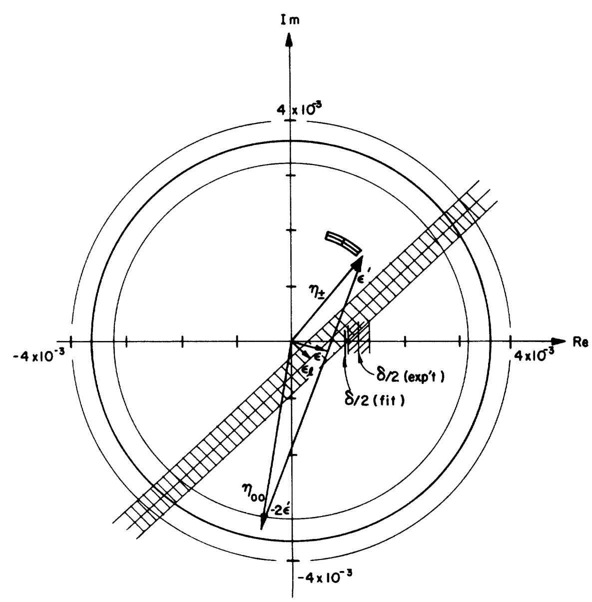

FIG. 2. The best fit for solution II, $x \neq 0$. $\frac{1}{2}\delta$ (best fit) = 1.39Reϵ. The diagonal cross-hatched area indicates the allowable values of ϵ. [See Eq. (8).]

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[1]See, for example, D. G. Hill et al., Phys. Rev. Letters 19, 668 (1967).

[2]E. Yen, Phys. Rev. Letters 18, 513 (1967).

[3]T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 16, 511 (1966).

[4]Using our notation, an excess of leptonic decays in the first few $K_S{}^0$ lifetimes after the creation of a pure $|K^0\rangle$ state (see, for example, Ref. 1) gives $\mathrm{Re}x>0$ and $\mathrm{Im}x<0$. Note that the convention used in Ref. 1 assumes $m_S-m_L>0$ and therefore produces a sign of $\mathrm{Im}x$ opposite to ours. We are grateful to these authors for pointing out to us a misprint in footnote 10 of their paper. It should read "$\delta=+0.58\hbar/c^2\tau_S$" rather than "$\delta=-0.58\hbar/c^2\tau_S$" as printed (where $\delta=m_S-m_L$).

[5]For a similar treatment, see B. R. Martin and E. de Rafael, Phys. Rev. 162, 1453 (1967).

[6]T. D. Lee and C. S. Wu, Phys. Rev. 162, 533 (1967).

[7]T. J. Devlin and S. Barshay, Phys. Rev. 19, 881 (1967).

[8]In our analysis we have also used $\gamma_S=(1.159\pm0.008)\times10^{10}$/sec, $\gamma_L=(19.0\pm0.4)\times10^6$/sec, $\Delta m=m_S-m_L=(-0.47\pm0.02)\gamma_S$, and $\gamma_L(\mathrm{lept})=12.4\times10^6$/sec. In all cases of internally inconsistent data, the uncertainties have been increased to external consistency. The data have been taken from the following references: $|\eta_{+-}|$, compilation by T. J. Devlin, Princeton-Pennsylvania Accelerator Conference on K-Mesons, Princeton, November, 1967 (unpublished). φ_{+-}, compilation by J. M. Gaillard, Princeton-Pennsylvania Accelerator Conference on K-Mesons, Princeton, November, 1967 (unpublished). For $|\eta_{00}|$ the input values in units of 10^{-3} are the following: 3.92 ± 0.30, J. W. Cronin et al., Princeton-Pennsylvania Accelerator Conference on K-Mesons, Princeton, November, 1967 (unpublished); $4.3_{-0.8}^{+1.1}$, J. M. Gaillard et al., Phys. Rev. Letters 18, 20 (1967); 3.2 ± 0.6, S. Parker et al., Bull. Am. Phys. Soc. 13, 31 (1968); $2.0_{-2.0}^{+0.7}$, T. Kamae et al., Bull. Am. Phys. Soc. 13, 31 (1968). For $\frac{1}{2}\delta$ the input values in units of 10^{-3} are the following: 2.00 ± 0.68, D. Dorfan et al., Phys. Rev. Letters 19, 987 (1967); 1.12 ± 0.018, S. Bennett et al., Phys. Rev. Letters 19, 993 (1967). For $\delta_0-\delta_2$, using the measured value of $\gamma(K^+\to\pi^+\pi^0)$ and the present (internally inconsistent) measurements of the $K_S{}^0$ branching ratio, we follow the arguments of Lee and Wu (Ref. 3, p. 528) and obtain $|\delta_0-\delta_2|=73_{-40}^{+33}$ deg. We use the weighted average of this and the value $\delta_0-\delta_2=+53°\pm20°$ obtained from a $\pi\pi$ phase-shift analysis by W. D. Walker et al., Phys. Rev. Letters 18, 630 (1967) (and private communication). The values of $\mathrm{Re}x$ and $\mathrm{Im}x$ were calculated from a compilation by G. Manning, Princeton-Pennsylvania Accelerator Conference on K-Mesons, Princeton, November, 1967 (unpublished). Because it is not obvious that calculating weighted averages from the present data is a valid technique, the uncertainties have been arbitrarily increased from ±0.07 (as calculated) to ±0.10.

[9]We note with interest that the Steinberger group, by measuring the leptonic asymmetry with a regenerator in a $K_L{}^0$ beam, have preliminarily found a regeneration phase of $-33°\pm7°$ and a value of $(1-|x|^2)/|1-x|^2\simeq0.9-1.0$. [S. Bennett, Bull. Am. Phys. Soc. 13, 31 (1968).] This regeneration phase, if applicable to the data of C. Alff-Steinberger et al., Phys. Letters 21, 595 (1966), could reduce the world average of φ_{+-} by about 10°. The second tentative result constrains $\mathrm{Re}x=-0.05$ to 0.00, but is compatible with $|\mathrm{Im}x|\simeq0.20$. This would improve the fit to solution I somewhat, while worsening (though probably not eliminating) solution II.

[10]L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964).

[11]The best fit values are $|\eta_{+-}|=1.97\times10^{-3}$, $\varphi_{+-}=40°$, $|\eta_{00}|=1.92\times10^{-3}$, $\frac{1}{2}\delta=1.01\times10^{-3}$, $\delta_0-\delta_2=35°$, $\mathrm{Re}x=+0.15$, $\mathrm{Im}x=-0.18$, $\mathrm{Re}\epsilon=+0.80\times10^{-3}$, $\mathrm{Im}\epsilon=0.24\times10^{-3}$, and $\varphi_{00}=251°$.

[12]For example, $\mathrm{Im}x=-0.16$ yields $|\epsilon_\Gamma(l)|=0.24\times10^{-3}$ at $-47°$, requires $|\epsilon_M(l)|\simeq8|\epsilon_\Gamma(l)|$ to give $|\eta_{+-}|=1.96\times10^{-3}$, and predicts $\epsilon'=0$, $|\eta_{00}|=|\eta_{+-}|$, $\frac{1}{2}\delta=1.49\times10^{-3}$, and $\varphi_{00}=\varphi_{+-}=36°$.