METHOD OF MEASURING THE BETA-DECAY COUPLING CONSTANT OF THE RHO MESON

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It is proposed that one measure the diffraction production of the ρ meson by high-energy neutrinos, i.e., the process $\nu \to \mu^- + \rho_{\text{virtual}}^+$ followed by ρ^+ diffraction scattering. By comparison with $\gamma + p \to \rho^0 + p$, the β-decay coupling constant can then be obtained. The process is distinguishable experimentally because of the characteristic low momentum transfer to the nucleon. Numerical calculations indicate that this process should be observable in large heavy-liquid bubble chambers.

Photoproduction of rho mesons [Fig. 1(a)] has been discussed by Ross and Stodolsky.[1] Recent experimental results are available from DESY[2] indicating that this diagram indeed dominates photoproduction of ρ mesons. From analyzing their results, DESY groups obtain a $\rho N \to \rho N$ total cross section of 31 ± 2 mb. The differential cross section is of the form e^{-at}, where t equals the square of the four-momentum transfer from the initial to final nucleon. Their results can be parametrized by taking

$$a = 5.6, \quad |S| < 5;$$

$$a = 1.8 + 0.758|S|, \quad 5 < |S| < 8.5;$$

$$a = 8.25, \quad 8.5 < |S|,$$

where S = total energy squared in the center-of-mass system.

There exist current-algebra estimates[3] of the $\rho\mu\nu$ coupling constant Gf_ρ with $G = 10^{-5}/M_p^2$ and $f_\rho^2 = 4.02 m_\rho^2 M_p^2$. Figure 1(c) will define our notation. The matrix element for this figure may be written as

$$T_{fi} = \frac{1}{\sqrt{2}} G f_\rho [\bar{u}_{\text{muon}} \gamma_\mu (1+\gamma_5) v_{\text{neutrino}}] \frac{\delta_{\mu\nu} + k_\mu k_\nu / M_\rho^2}{k^2 + M_\rho^2 - i\epsilon} T_{\nu\nu'} \epsilon_{\nu'}^{\alpha}, \tag{1}$$

where ϵ^α = polarization vector of final ρ, and $T_{\nu\nu'}$ = matrix element for $\rho p \to \rho p$.

In order to calculate a cross section for Fig. 1(b) and compare it with the cross section for Fig. 1(a) we must assume a form for $T_{\nu\nu'}$. We ignore the spin of the target nucleon. For both processes 1(a) and 1(b), $T_{\nu\nu'}$ must correspond to a conserved current, i.e., $k_\nu T_{\nu\nu'} = 0$. The most general matrix element that can satisfy this and also conserve parity is a sum of five terms:

$$\begin{aligned} T_{\nu\nu'} = {} & A[(p\cdot k)(p'\cdot k')\delta_{\nu\nu'} + (k\cdot k')p_\nu p_{\nu'}' - (k'\cdot p')p_\nu k_{\nu'} - (k\cdot p)k_\nu' p_{\nu'}'] + G[k_\nu' k_{\nu'} - (k\cdot k')\delta_{\nu\nu'}] \\ & + H[k^2 k'^2 p_\nu p_{\nu'}' - k'^2 (k\cdot p) k_\nu p_{\nu'}' - k^2 (k'\cdot p') p_\nu k_{\nu'}' + (k\cdot p)(k'\cdot p') k_\nu k_{\nu'}'] \\ & + I[(k\cdot p) k'^2 k_\nu k_{\nu'} - k^2 k'^2 p_\nu k_{\nu'} + k^2 (k\cdot k') p_\nu k_{\nu'}' - (k\cdot p)(k\cdot k') k_\nu k_{\nu'}'] \\ & + J[(k'\cdot p') k^2 k_\nu' k_{\nu'}' - k^2 k'^2 k_\nu' p_{\nu'}' + k'^2 (k\cdot k') k_\nu p_{\nu'}' - (k'\cdot p')(k\cdot k') k_\nu k_{\nu'}']. \end{aligned} \tag{2}$$

If we wish to avoid explicit poles in the denominator (i.e., k^2, k'^2, $k\cdot p$, etc.), we cannot pull common factors out of these expressions. The H, I, and J terms all vanish if the mass of the initial particle goes to zero, i.e., for a photon. This is true for any linear combination of these three terms as long as we ask for no singularities in the above sense. These three terms may well be present. However, if they are present, then they "decouple" the ρ and the photon since we would have terms present for one case and not for the other. We wish to assume a ρ-photon analogy and therefore drop H, I, and J. We note that element G should not stand alone as it implies zero forward scattering for photons. We

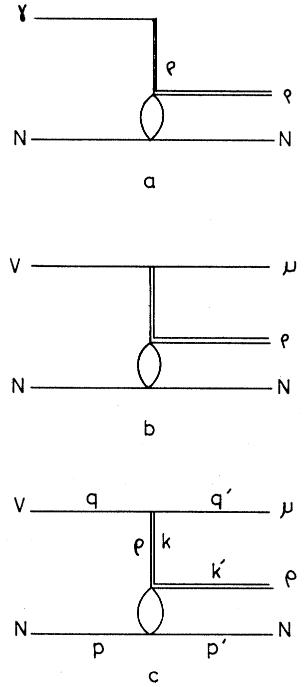

FIG. 1. Diagrams showing (a) photoproduction and (b) weak production of rho mesons; (c) definition of symbols.

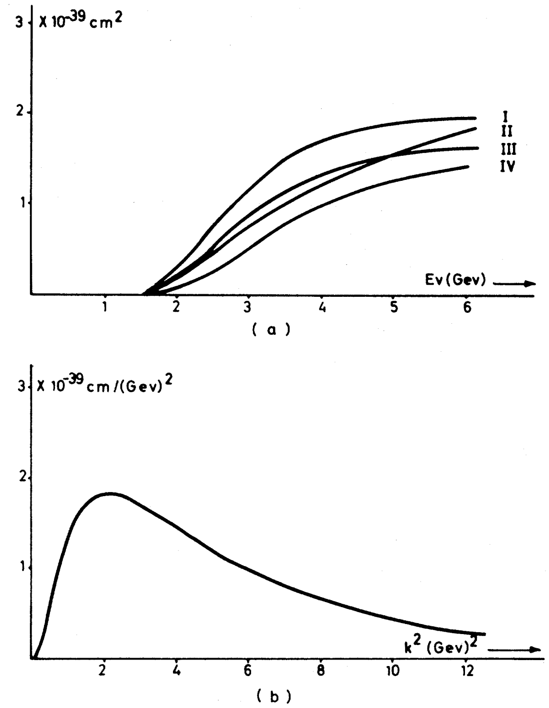

FIG. 2. (a) Cross section for $\nu p \rightarrow \rho^+\mu^-p$ for various parameter values. I: $X=0.868$; a=fit to DESY data. II: $X=0.868$; $a=7$. III: $X=0$; a=fit to DESY data. IV: $X=5$; a=fit to DESY data. (b) Differential cross section for $\nu p \rightarrow \rho^+\mu^-p$ for $E_\nu=3$ GeV, $X=0.868$, a=fit to DESY data.

choose A and G such that for foward scattering with initial and final ρ mesons on the mass shell we have equal scattering of transversely and longitudinally polarized ρ mesons, as would be predicted for diffraction scattering. This is satisfied if

$$G=-AX[(p\cdot p')(k\cdot k')+(p\cdot k)(p'\cdot k')],$$

where $X=(2m_\rho{}^2)^{-1}$. In order to compare with ρ photoproduction data, to obtain A for a given $k'p',p$ one must extrapolate in k. We perform this extrapolation keeping $E=k(4), t, k'^2, p^2, p'^2, \vec{p}$ fixed. We ignore any dependence of A on k in this extrapolation.

By the above method the total cross section σ and differential cross sections $d\sigma/dk^2$ have been calculated for E_ν between 1.5 and 6 GeV. Results are given in Figs. 2(a) and 2(b). The cross section was integrated over the neutrino spectrum of the 1967 CERN bubble-chamber run. In that run (chamber filled with C_3H_8) we expected 8.3 events which divide into 1.8 events on hydrogen, 6.2 quasifree events on carbon, and 0.3 coherent events on carbon.

For the quasifree events, an effective nucleon number of 9 was assumed for carbon.[4] For the coherent events, the total ρC cross section was taken as geometric (237 mb) and a was taken as $(12)^{2/3}$ times the value for a obtained in fitting the DESY data. This large slope inhibits the coherent process when combined with the finite threshold for t at our low neutrino energies (3 of the above 4.5 events are for neutrino energies between 1.5 and 3.5 GeV).

With Gargamelle it is feasible to obtain more than an order of magnitude increase in the number of events at CERN, not including possible proton-synchrotron improvements. At Serpukhov, the higher energy neutrino spectrum will give a still further increase. Hence in the next few years it is very likely that the experiment can be done.

An important experimental point is that the above process should be separable from all others experimentally by imposing the double requirement that the $\pi^+\pi^0$ effective mass be in the ρ-meson band, and that Δ^2 (the four-momentum transfer to the nucleon) be small because of the diffraction scattering.

There are several inadequacies in the above calculations:

(1) The isospin dependence of the ρ-N scattering cross section may be important. A compari-

son of γ-n production would be helpful, as well as ν-producing-ρ^+ and $\bar{\nu}$-producing-ρ^- comparisons. It would seem possible to sort this question out experimentally.

(2) The simple form assumed for ρ elastic scattering (Be^{-at}) is certainly not valid at all energies. In Fig. 3, results are shown for the $\nu p \rightarrow \rho\mu p$ total cross section for two choices of a. A photoproduction experiment in propane should be able to calibrate this point except for possible polarization effects. There is an experimental problem that may arise here. A ρ meson produced by a ν of a given energy is characteristically of much lower momentum than one produced by a γ of the same energy, because the muon takes off some of the available energy. This is no problem in calibrating the reaction, but may mean that the ρ-N scattering is not dominated by diffraction scattering for some portion of the events. Since the diffraction scattering with low Δ^2 transfer to the nucleon is one of the hallmarks of this process, one could have some trouble in distinguishing these events.

(3) Stodolsky[5] has pointed out that other diagrams with the intermediate particle a π or A_1 meson could contribute. The former, however, is hoped to be small because of the smallness of the physical $\pi p \rightarrow \rho p$. The process $A_1 p \rightarrow \rho p$ is also probably smaller than the ρ elastic scattering cross section at 1 or 2 GeV. The larger mass in the propagator would tend to damp the A_1 contribution even further.

(4) Off-mass-shell effects are hard to estimate. We have extrapolated by changing k and holding $E = k(4), t, p^2, k'^2, p'^2, \vec{p}$ fixed, and have ignored form-factor effects in the matrix element during this extrapolation. The extrapolation is roughly from $k^2 = 0$ to around $k^2 = 0.2$, where the differential cross section peaks. The physical ρ lies at $k^2 = -0.58$. Our choice of matrix element $T_{\mu\nu}$ is rather arbitrary. We have examined the effect of letting X vary from the value $X = \frac{1}{2}M_\rho^2 = 0.868$ with the results shown in Fig. 2(a). Letting X vary from 0 to 5 gives about a 50% effect in the cross section. Rousset[6] has pointed out that off-mass-shell effects could perhaps be studied by electroproduction of ρ mesons using either intial electrons or muons.

In conclusion, it would appear that with the possible exception of the off-mass-shell effect all the corrections can be sorted out experimentally. It would be extremely valuable to have further photoproduction and electroproduction experiments done in order to answer some of the above questions.

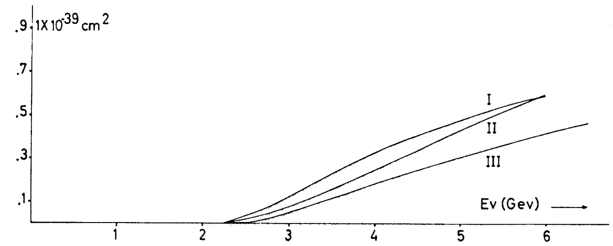

FIG. 3. Cross section for $\nu p \rightarrow A_1^+\mu^- p$ for various parameter values. I: Nonconserved current, a = fit to DESY ρ data. II: Nonconserved current, $a = 7$. III. Conserved current, $a = 7$.

The cross section varies somewhat as a function of the mass of the final state ρ. This has been investigated, and the ρ-mass peak should be shifted down by 10 to 15 MeV for ρ production by neutrinos.

The neutrino production calculation was repeated for the A_1 in an attempt to get a rough idea of the size of this cross section. The A_1 is an axial-vector particle, and the photoproduction comparison is not available. In the absence of data $\sigma_{\text{tot}}(A_1 N \rightarrow A_1 N)$ for zero-mass initial A_1 was taken equal to $\sigma_{\text{tot}}(\rho N \rightarrow \rho N)$, a was taken as 7, and f_{A_1} was taken equal to f_ρ as suggested by Weinberg.[7]

If $T_{\mu\nu}$ is taken to be the same as for ρ production (conserved current), one gets the results shown in Fig. 3. Here, as Stodolsky has noted, other diagrams such as the one with an intermediate pion may well play an important role, and the choice of matrix element can certainly be improved since $T_{\mu\nu}$ for the A_1 does not correspond to a conserved current.

An attempt to calculate A_1 production was also made with a very simple nonconserved current $T_{\mu\nu} = \delta_{\mu\nu}$. The results are also shown in Fig. 3. If the size calculated here for this cross section is roughly correct, it might be rather difficult to find the A_1 in the next CERN or Brookhaven National Laboratory neutrino experiments.

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[1]M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966).

[2]E. Lohrmann, in Proceedings of the Topical Conference on High-Energy Collisions of Hadrons, CERN,

1968 (Scientific Information Service, Geneva, 1968), p. 556; E. Lohrmann, DESY Report No. DESY 67/40, 1967 (unpublished).

[3]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).

[4]B. Margolis, private communication.

[5]L. Stodolsky, private communication.

[6]A. Rousset, private communication.

[7]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

ERRATA

ASYMPTOTIC BEHAVIOR OF INFINITELY RISING TRAJECTORIES. R. W. Childers [Phys. Rev. Letters 21, 868 (1968)].

Eq. (2) should read

$$\mathrm{Re}\,\alpha(s) \to s^{1/2}\ln s, \quad \text{as } s \to +\infty.$$

This does not alter any of the conclusions of the paper. Due to an error in Ref. 8, the term

$$-\frac{t}{\pi s}\sin\pi\alpha\cos\left(2\frac{\alpha\sqrt{-t}}{\sqrt{s}}-\frac{\pi}{4}\right)$$

should be added to the right side of Eqs. (6), (7), and (8). As a result the phrase at the bottom of the first column of p. 869 and the first sentence at the top of the second column of p. 869 are incorrect. The main conclusions are unchanged.

SELF-FOCUSING EFFECTS ASSOCIATED WITH LASER-INDUCED AIR BREAKDOWN. V. V. Korobkin and A. J. Alcock [Phys. Rev. Letters 21, 1433 (1968)].

At the top of p. 1435 the expression for determining n_2 should read:

$$n_2 = (k^2r^2E^2)^{-1}$$

and not

$$n_2 = (k^2r^2E^2)-1.$$